Stability criterion of a MAP/PH-multiserver model with simultaneous service

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The Inspirator

High performance computing cluster

homogeneous machine with multiple CPUs utilizing parallel computing

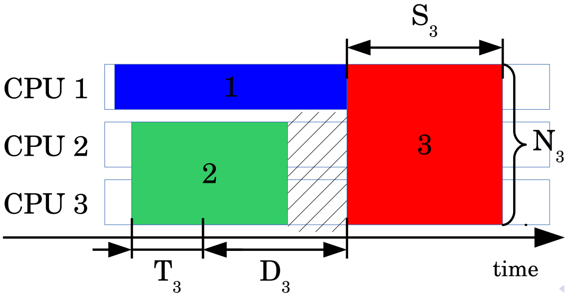
- shared by many users
- job/task uses multiple CPUs
- high utilization level
- very expensive (build, run)

- Memory/storage allocation in a PC
- Wireless channel occupation

The $MAP/M/c$-type Model

Queueing system:

- c identical servers, FCFS;
- interarrival times $T_i, i \geqslant 1$ defined by MAP (D_0, D_1) with k states (intensity $\lambda = \theta D_1 \mathbf{1}$);
- exponential service times S_i (intensity μ);
- customer i requires N_i servers at once, aka *rigid job* (distribution $\{p_j = \mathsf{P}(N = j), 1 \leqslant j \leqslant c\}$).



Idle servers with non-empty queue!

The History

1. *Kim S.S. M/M/s Queueing System Where Customers Demand Multiple Server Use: Ph. D. Dissertation, 1979.*
   Reference to the Neuts ergodicity condition.
2. *Brill P., Green L. Queues in which customers receive simultaneous service from a random number of servers: A system point approach. Management Science, 1984. Vol. 30. No. 1. P. 51–68.*
   Stability criterion for M/M/2-type system (without proof).
3. *D. Filippopoulos, H. Karatza. An M/M/2 parallel system model with pure space sharing among rigid jobs. Mathematical and Computer Modelling, 2007. Vol. 45, No. 5–6. P. 491–530.*
   Stability criterion for M/M/2-type system.
4. *S.R. Chakravarthy, H.D. Karatza. Two-server parallel system with pure space sharing and Markovian arrivals. Computers and Operations Research, 2013. Vol. 40, No. 1. P. 510–519.*
   Stability criterion for MAP/M/2-type system.

Main Result

Continuous-time QBD process $\{\Theta(t) = \{\nu(t), m(t), \varphi(t)\}, t \geqslant 0\}$,

level ν — number of customers in the system,

macrostate $m = (m_1, \ldots, m_n) \in \{1, \ldots, c\}^c =: \mathcal{M}$, m_i is a number of servers required by i-th oldest customer in the system,

MAP-phase $\varphi \in \{1, \ldots, k\}$.

Stability criterion

QBD process is positive recurrent iff

$$\rho = \frac{\lambda}{\mu} C < 1, \quad C := \sum_{m \in \mathcal{M}} \frac{\prod_{j=1}^{c} p_{m_j}}{\sigma(m)}, \tag{1}$$

where $\sigma(m) = \max_{i \leqslant c} \left\{ \sum_{j=1}^{i} m_j \leqslant c \right\}$.

Discussion

Case $c = 2$: a known earlier result

$$\frac{\lambda}{\mu} < \frac{2}{2 - p_1^2}.$$

Case $k = 1, p_1 = 1$ (classical $M/M/c$ system):

$$\frac{\lambda}{\mu} < c.$$

Case $k = 1$ (exponential arrivals):
[*A. Rumyantsev and E. Morozov. Stability criterion of a multiserver model with simultaneous service. Annals of Operations Research, 2015*]

Proof Sketch

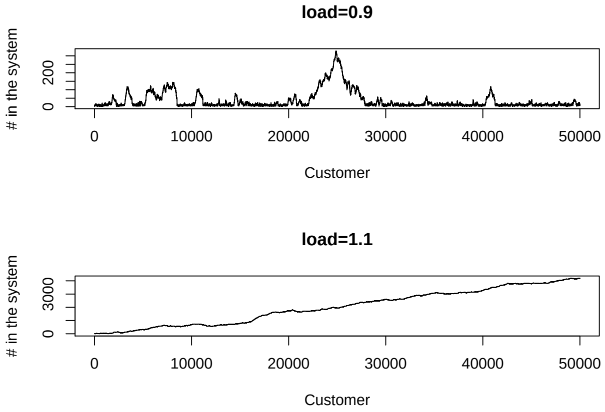
- Define infinitesimal generator for $\{\Theta(t)\}$ with finite number of phases kc^c at high levels $\nu(t) > c$, use Kroenecker sums/products and the properties of $M/M/c$-type model
- Apply the Neuts ergodicity condition $\gamma A_2 \mathbf{1} > \gamma A_0 \mathbf{1}$, where $\gamma = \alpha \otimes \theta$
- The vector α is defined componentwise

$$\alpha_m = C^{-1} \frac{\prod_{i=1}^{c} p_{m_i}}{\sigma(m)}, \quad m \in \mathcal{M},$$

  and comes from the $M/M/c$-type model
- Recall θ comes from MAP as a solution $\theta D = 0,\ \theta \mathbf{1} = 1$, where $D = D_0 + D_1$.

On $MAP/PH/c$ Model

Numerical experiments show the validity of the result for PH-type service (and even for Pareto service!). For a PH (τ, T) define $\mu = (-\tau T^{-1}\mathbf{1})^{-1}$.



Unfortunately, the proof is still in progress.

Necessary Stability Condition

It can be shown, that the system with *batches of customers* of size N_i, each having service time S_i, is minorant to HPC model. The necessary condition follows

$$\frac{\lambda}{\mu}\mathsf{E}N < c. \qquad (2)$$

One may use (2) to easily check the *instability* of the system model.

Accelerated Verification

An equivalent representation

$$C = \sum_{i=1}^{c} \frac{1}{i} \sum_{j=i}^{c} {p_j^*}^i \sum_{t=c-j+1}^{c} p_t, \qquad (3)$$

the summation is done over

- number i of customers at service,
- number j of servers serving customers,
- number t of servers required by the customer at the head of the queue.

For $c = 5000$ on my laptop: 20 sec.

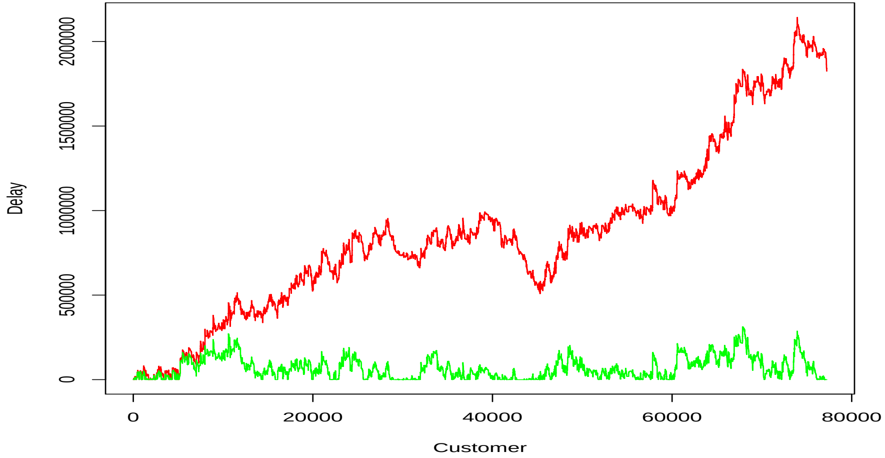
Application Example

Upgrade an unstable cluster: given $\lambda, \mu, p_1, \ldots, p_c$, find $c' > c$ s.t. $\lambda C/\mu < 1$.

Example: Cornell Theory Center (CTC) IBM SP2 cluster

- $s = 336$ processors, 77221 tasks from Workload Archive
- EASY Backfill scheduler: stable, but high delays (mean 25540 sec, max 7231000 sec)
- $\rho \approx 1.14$ (unstable *under FIFO*)

Upgrade CTC SP2 cluster



Original *(under FIFO)* $c = 336$ and Upgraded: $c' = 372$

Thank you for attention!

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