Analytical TCP Throughput Model for HSDPA*

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Abstract

In this paper, an approximate, Padhye model based TCP throughput calculation method is presented for mobile data services over HSDPA. The Padhye model is defining the TCP throughput based on two input parameters: the packet loss probability and the TCP Round Trip Time. In order to provide the input parameters for the TCP throughput calculation, an equivalent queuing network model of the HSDPA system is created, which includes the congestion points and protocol layers that are having dominant impact on the delay and packet drop. The solution of the queuing network model is described in detail. Finally, the model is validated with NS2 simulations.

1 Introduction

HSDPA (High Speed Downlink Packet Access) provides a packet based downlink service for data users over the UMTS (Universal Mobile Telecommunications System) with data rates ranging up to several megabits per second. [7]

In conventional UMTS, the Layer 2 protocols of the radio protocol interface, such as Radio Link Control (RLC) and Medium Access Control (MAC) protocol are terminated in the Radio Network Controller (RNC). Physical layer protocols of the radio interface are implemented in the Node-B that is connected to the RLC via the Iub interface. In Acknowledged Mode (AM), the RLC is responsible for error-free, in-sequence delivery of the user data. This is achieved by retransmissions based on the (Automatic Repeat Request) ARQ mechanism. RLC retransmissions are increasing the Layer 2 Round Trip Time (RTT) and may trigger TCP timeouts.

HSDPA is introducing an additional protocol layer located in the Node-B (see Figure 1), namely MAC-hs, which makes possible Node-B controlled fast adaptation of the modulation and coding scheme, fast scheduling and retransmission handling with the Hybrid ARQ (HARQ) functionality. This solution is reducing the Layer 2 RTT when retransmissions are required due to erroneous data transfer. Although retransmissions are handled by the Node-B, RLC ARQ has not been removed from the system in order to be compatible with the

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Rel'99 network solutions and to provide the capability of soft handover control. Retransmissions are still handled by RLC if the maximum allowed number of MAC-hs retransmissions is exceeded or there are packet drops on the transport network. The RLC retransmissions are increasing the round trip time of the data connections using HSDPA service. These factors and the in-sequence delivery of the user packets by the RLC are causing that the TCP flow control is not able to detect and resolve the congestion situation on the Iub interface. As a result, the TCP notices the congestion only upon the expiry of the Timeout timer or when finally the RLC discards the packets that has reached the maximum number of retransmissions.

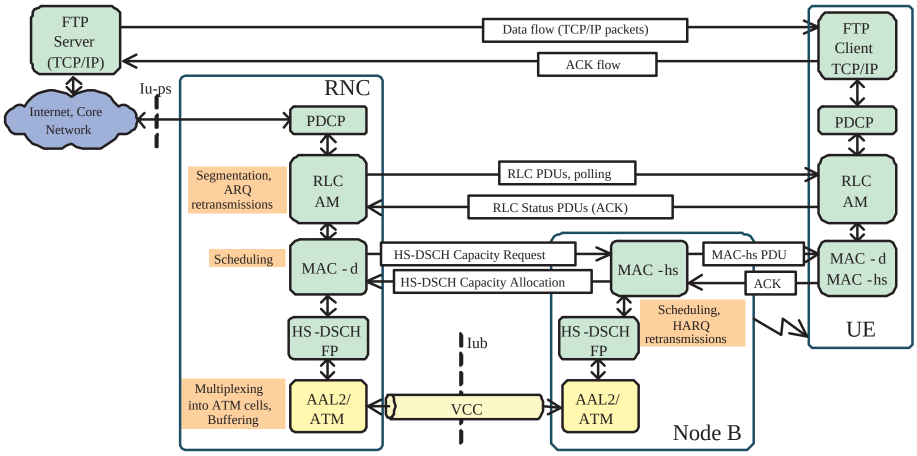

Figure 1: The overview of the protocols of HSDPA

The distribution of the radio protocol architecture between RNC and Node-B requires that a flow control algorithm – the HSDPA flow control [10] – is implemented. With this algorithm the Node-B controls the amount of data sent from the RNC in order to keep its buffers at optimal level so that the air interface capacity is not wasted, and in the same time the delay on the Node-B buffer is not too high. Typically, the HSDPA flow control is measuring the Node-B buffer size and the amount of transferred Packet Data Units (PDUs) over a sampling period without considering the available resources on the Iub transport network shared by Real Time (RT) Non Real Time (NRT) and HSDPA services.

A good indicator of the level of service an HSDPA access network can provide to the mobile users is the achievable TCP throughput. TCP performance in HSDPA has been considered in [3], [2] and [1] in the past. These papers are presenting a detailed simulation based analysis of the TCP behaviour over HSDPA systems, but the results are based on simulations. In this paper we propose an analytical throughput model for TCP connections over HSDPA.

The rest of the paper is organized as follows. Section 2 gives a short technological overview on the HSDPA UTRAN, describes how the packets are delivered from the RNC to the User Equipments (UE) and introduces a queueing network model of the system. Section 3 summarizes the concept of the approximate throughput calculation and describes in detail the solution of the queuing network model of the system. A numerical example is provided in Section 4, finally Section 5 concludes the paper.

2 System overview and the equivalent queuing model

The overview of the radio access network configuration in case of HSDPA service is shown in Figure 1. After header compression in the Packet Data Convergence Protocol (PDCP) layer, the incoming data (TCP/IP) packets are segmented and encapsulated by the RLC AM entity. These segments (PDUs) are scheduled by the MAC-d layer according to the HSDPA flow control commands. The RLC entity is actively polling the User Equipment (UE) that is responding with Status PDUs indicating the sequence number of lost and received PDUs. Lost PDUs are retransmitted. The master of the HSDPA flow control is the MAC-hs located in the Node-B. It grants resources to the HSDPA connections (MAC-d flows) at RNC by sending High Speed Dedicated Shared Channel (HS-DSCH) Capacity Allocation message that includes the allocation size i.e. the number of PDUs and their maximum size (HS-DSCH Credits, MAC-d PDU Length); the interval the data can be sent at (HS-DSCH Interval), and the validity period of the allocation (HS-DSCH Repetition Period). This message is sent either solicited, upon reception of a HS-DSCH CAPACITY REQUEST message from the RNC, or unsolicited. The HS-DSCH Frame Protocol (FP) assembles a frame out of the scheduled PDUs as it is specified in and transfers it to the ATM Adaptation Layer type 2 (AAL2), where these frames are segmented to 45 bytes, and encapsulated into Common Part Sublayer (CPS) packets. The size of the CPS-Packet header is 3 byte thus the maximum size of one packet is 48 byte. The CPS-Packets are eventually assembled into CPS PDUs and sent to the destination via the Virtual Channel Connection (VCC). The CPS-PDU header is one byte long thus at maximum 47 CPS-Packet bytes can be fitted into one Asynchronous Transfer Mode (ATM) cell. As queues are intrinsic to the HSDPA system, a natural approach to model the TCP RTT (Round Trip Time) – that is an important parameter with impact on the overall TCP performance – is to create an equivalent queuing model. Accordingly, the potential bottleneck points that are dominating the downlink delay has to be identified (in case of mobile services the users are mainly downloading content to their mobiles causing loaded system in downlink). The developed model focuses on the downlink performance, whereas the uplink delay is modelled with a constant delay. Packets drop (p) can appear at these bottleneck points due to buffer overflow or when the maximum number of retransmissions is reached. There are three such points in the system:

- The buffers of the RLC layer where the RLC PDUs (resulted from the segmentation of the user packets) are stored until a positive acknowledgement arrives or the maximum number of retransmissions is reached and the RLC AM entity discards them. The RLC buffers are scheduled by the MAC-d layer based on the credits received from the Node-B (MAC-hs entity). These credits are calculated in order to maximize the air interface throughput not necessarily taking into consideration the congestion situation over the Iub links thus the RLC layer can easily overload the transport network. In this model it is assumed that the uplink delay of the HS-DSCH CAPACITY ALLOCATION message is zero.

- The buffers of the AAL2/ATM transport network. As the transport network is a shared and limited resource, congestion may occur leading to increased delay and eventually to packet drops. In this paper the transport network is modelled with one buffer corresponding to the bottleneck link.

- The MAC-hs buffers in Node-B. There is one buffer per each MAC-d flow (HSDPA connection) that is storing the MAC-d PDUs waiting for transmission. The amount of the MAC-d PDUs that can be transmitted in a 2 ms long TTI depends on the reported channel quality indicator (CQI). In case of transmission failure, the MAC-d PDUs are retransmitted. If the maximum number of retransmissions is reached the MAC-d PDUs are discarded by the HARQ and the RLC ARQ will handle the retransmissions.

An overview of the queueing network model of the system is shown on Figure 2. The three components of the queuing model i.e. the RLC buffers, the transport buffers and the MAC-hs buffers are located at different protocol layers. Each flow has a dedicated buffer at the RLC layer that stores the PDUs resulted from the segmentation of the TCP packets. The MAC-d schedules these buffers independently based on the credits received from the Node-B. PDUs are discarded in case of buffer overflow or when the maximum number of retransmissions is reached. Each PDU is stored in the buffer until the positive acknowledgement is received or when PDU Discard procedure is executed by the RLC.

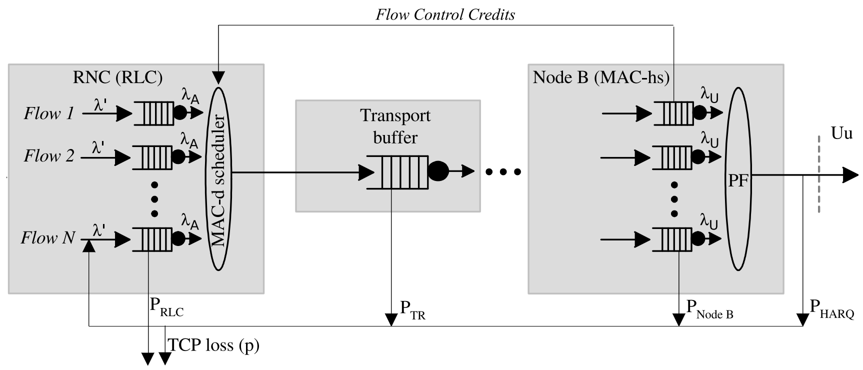

Figure 2: The overview of the queueing network model of the system

The transport network is modelled by one buffer representing the bottleneck link. ATM cells are discarded at buffer overflow. At the Node-B, each MAC-d flow has a dedicated buffer. At each 2 ms TTI the Proportional Fair (PF) scheduler is selecting the buffer to be served based on the average throughput of each flow and their instantaneous channel quality. Upon an erroneous transmission over the air interface, the PDUs are retransmitted until the maximum number of transmissions is reached.

3 The concept of the TCP throughput calculation

There are several models available to calculate the TCP throughput. The most popular one is the so-called Padhye model [11]. This model essentially gives a simple formula that expresses the TCP throughput (B) as a function of the packet loss (p) and round trip time (RTT)

$$\mathrm{B}\left(p, RTT\right) = \begin{cases} \frac{\frac{1-p}{p}+\mathrm{E}(W)+\hat{\mathrm{Q}}(\mathrm{E}(W))\frac{1}{1-p}}{RTT\left(\frac{b}{2}\,\mathrm{E}(W_u)+1\right)+\hat{\mathrm{Q}}(\mathrm{E}(W))T_0\,\frac{f(p)}{1-p}}, & \text{if } \mathrm{E}\left(W_u\right) < W_{\max} \\ \frac{\frac{1-p}{p}+W_{\max}+\hat{\mathrm{Q}}(\mathrm{E}(W))\frac{1}{1-p}}{RTT\left(\frac{b}{8}W_{\max}+\frac{1-p}{pW_{\max}}+2\right)+\hat{\mathrm{Q}}(W_{\max})T_0\,\frac{f(p)}{1-p}}, & \text{otherwise.} \end{cases} \tag{1}$$

In the formula p denotes the packet loss probability, b is the number of packets covered by one acknowledgement ($b = 1$ is assumed in this paper), T_0 is the timeout (we use $T_0 = 1.5\,\mathrm{sec}$), RTT is the Round Trip Time of the packets, $W_{\max}$ is the maximum Congestion Window size, $\mathrm{E}\left(W_u\right)$ is the mean Unconstrained Window size given by:

$$\mathrm{E}\left(W_u\right) = \frac{2+b}{3b} + \sqrt{\frac{8\left(1-p\right)}{3bp} + \left(\frac{2+b}{3b}\right)^2}.$$

$\hat{\mathrm{Q}}\left(w\right)$ is the probability that a loss in a window of size w is due to Timeout, calculated with the formula:

$$\hat{\mathrm{Q}}\left(w\right) = \min\left(1, \frac{\left(1-\left(1-p\right)^3\right)\left(1+\left(1-p\right)^3\left(1-\left(1-p\right)^{w-3}\right)\right)}{1-\left(1-p\right)^w}\right).$$

Finally, $f\left(p\right)$ is a simplifying notation:

$$f\left(p\right) = 1 + p + 2p^2 + 4p^3 + 8p^4 + 16p^5 + 32p^6.$$

Thus, the two unknown parameters of the TCP throughput calculation are the Round Trip Time (RTT) and the packet loss probability (p). Since the major part of the RTT is spent as waiting time in the buffers of the network devices, and the packet loss occurs due to saturated buffers or air interface errors, we model the HSDPA system by a queueing network. To reduce complexity, we decided not to involve the micro-behaviour of the TCP flow control into the model. Instead, we consider the TCP traffic as a flow of packets having a constant intensity.

By assuming a constant rate TCP traffic, the RTT and p are calculated using the queueing network model of the system described in detail in the next sections. Once the RTT and p are known, the TCP traffic intensity corresponding to p and RTT can be calculated with the Padhye model. This value is not necessarily equal to the TCP rate assumed initially. In this case the initially assumed TCP rate is adjusted, and throughput calculation is repeated until the equilibrium is reached. The output of the method will be the TCP rate B^* that – when loaded into the queueing network model – results in a p and RTT such that the Padhye model provides the same TCP throughput, thus $B^* = \mathrm{B}\left(p, RTT\right)$.

3.1 Overview of the calculation algorithm

As described in Section 3, the TCP throughput over HSDPA is calculated as the load (λ_{TCP}) that carried over the network causes a Round Trip Time RTT and packet loss p such that the Padhye formula results in the same throughput i.e. $\mathrm{B}\left(p, RTT\right) = \lambda_{\text{TCP}}$.

This equilibrium is reached by a simple interval bisection method summarized in Algorithm 1. At the beginning of the algorithm, the lowest possible throughput is initialized to $a_1 = 0$. The mean TCP throughput can not be larger than the average air interface throughput, thus the upper limit of the interval bisection can be initialized to $\mathrm{E}\left(S_{\text{Node-B}}\right)$. In each step, the queueing network shown on Figure 2 is analysed, the packet loss and mean RTT is calculated. The upper and lower bounds of the interval are adjusted depending on the relation of the actual TCP throughput assumption, λ_{TCP} and the throughput resulted by the Padhye formula $\lambda_{\text{PADHYE}} = \mathrm{B}\left(p, RTT\right)$.

Algorithm 1 The TCP Throughput Calculation Algorithm

INPUT: `sysparam`//the system parameters are listed in Table 1
OUTPUT: γ//the TCP throughput

$a_1 = 0$//the lowest possible throughput value
$a_2 = \mathrm{E}\left(S_{\text{Node-B}}\right)$//the upper bound is the average air interface throughput
while $|\lambda_{\text{TCP}} - \lambda_{\text{old}}| > \epsilon$ **do** //the loop of the bisection method
  $\lambda_{\text{TCP}} = \frac{a_1+a_2}{2}$
  $p, RTT =$ `QN Analysis` $\left(\lambda_{\text{TCP}}\right)$
  $\lambda_{\text{PADHYE}} = \mathrm{B}\left(p, RTT\right) \cdot K f_T / f_M$ //Apply Padhye model as in (1)
  if $\lambda_{\text{PADHYE}} > \lambda_{\text{TCP}}$ **then**
    $a_1 = \lambda_{\text{TCP}}$
  else
    $a_2 = \lambda_{\text{TCP}}$
  $\lambda_{\text{old}} = \lambda_{\text{TCP}}$
return $\gamma = \lambda f_M$//harmonize units

The parameters p and RTT are calculated by the analysis of the queueing network (Figure 2). The users are assumed to be identical, the calculation is performed for one selected (tagged) user. Accordingly the queueing network seen by the tagged user consists of three nodes: the RLC buffer, the transport (ATM) buffer and the MAC-d buffer at the Node-B. This queueing network does not belong into the class of queueing networks for which exact solution is known, thus a traffic decomposition based approximate analysis has been developed. [4] The analysis starts with the first queue. In addition to the performance measures of interest, the output process has to be approximated, too. This approximate departure process is the arrival process to the next queue in the network, that can be analysed in the same way. The calculation is repeated until the last queuing node is analysed. As the network has feed back traffic (RLC loss is modelled as if the lost PDUs would be re-inserted into the RLC buffer), an iterative solution method has to be used. Initially it is assumed that there is no feed back traffic; the whole network is analysed and the feed back traffic (amount of PDUs that must be retransmitted) is calculated. In the next iteration step this feed back traffic is taken into consideration during the analysis of the first queue thus in the whole queuing network model. The iterations are repeated

Description	notation	value
The number of HSDPA users	K	16
The RLC buffer size [PDUs]		1000
Transport node buffer size [ATM cells]	L	2000
Node-B buffer size [PDUs]		1000
Maximum number of RLC (re)transmissions	R	6
Maximum number of HARQ (re)transmissions	M	3
TCP packets acknowledged by one ACK	b	1
TCP Timeout interval	T_0	1.5 sec
Maximum TCP Congestion Window size	$W_{\max}$	48 KB
Block Error Rate over the air interface	P_e	0.01
Prob. of two successive erroneous transmissions	P_s	0.001
Service distribution at the air interface	$\Pr\left(\hat{S}=k\right)$	from trace file
TCP packet size	f_T	1500 byte
Size of MAC-d and RLC PDUs	f_M	336 bit
Accuracy parameters	ϵ, ϵ'	1
Transport link capacity	C	

Table 1: The parameters contained in `sysparam`

Algorithm 2 $\lambda_{\text{out}} = $ `QN Analysis` (λ_{in})

INPUT: λ_{in} //the load generated by the TCP sources

OUTPUT: p, RTT//packet loss and mean round trip time

1: $\lambda' = \frac{\lambda_{\text{in}}}{K}$//the throughput of the tagged HSDPA user

2: **while** $|\lambda' - \lambda'_{\text{old}}| > \epsilon'$ **do** //loop to find the equilibrium value of λ'

3: $\quad (P_{\text{RLC}}, \mathrm{E}(T_{\text{RLC}}), D_{\text{RLC}}) = $ `solve rlc` (λ')//see Section 3.2

4: $\quad (P_{\text{Tr}}, \mathrm{E}(T_{\text{Tr}}), D_{\text{Tr}}) = $ `solve tr` (C, D_{RLC})//see Section 3.3

5: $\quad (P_{\text{Node-B}}, \mathrm{E}(T_{\text{Node-B}}), \lambda_U) = $ `solve node-b` (S, D_{Tr})//see Section 3.4

6: $\quad p_L \Leftarrow (P_{\text{Tr}}, P_{\text{Node-B}}, P_{\text{HARQ}})$//the loopback probability given by (15)

7: $\quad \hat{p} = \frac{\sum_{k=1}^{R}(1-p_L)^{k-1}p_L}{\sum_{k=1}^{R+1}(1-p_L)^{k-1}p_L}$//the probability that the PDU is resent by RLC

8: $\quad \lambda' = \frac{\lambda_{\text{in}}}{K} + \hat{p}\cdot p_L \cdot \lambda_A$

9: $\quad \lambda'_{\text{old}} = \lambda'$

10: $(D_u, D_s) \Leftarrow (P_{\text{Tr}}, P_{\text{Node-B}}, P_{\text{HARQ}}, \mathrm{E}(T_{\text{RLC}}), \mathrm{E}(T_{\text{Tr}}), \mathrm{E}(T_{\text{Node-B}}))$//(19)

11: $RTT = \sum_{k=1}^{R} \frac{p_L^{k-1}(1-p_L)}{1-p_L^R}\left((k-1)D_u + D_s\right)$//the RTT derived in (20)

12: $p = 1 - \frac{\lambda_U}{\lambda}$//the TCP loss probability given in (21) until the difference in the results will not exceed the accuracy parameter. The algorithm is summarized on Algorithm 2.

3.2 The model of the RLC buffer

The model of the RLC layer (referred to as `solve rlc` in line 3 in Algorithm 2) is based on the observation that the *service process* of the RLC buffer (thus, the arrival process of the RLC PDUs to the transport network) is controlled by the HSDPA flow control. To achieve efficient air interface resource usage, the Node-B grants credits to each flow based on the reported channel quality and the measured average throughput of the flows. At each HSDPA scheduling interval the MAC-d scheduler will transmit the amount of PDUs defined by the received credits. In this paper we assume that scheduling interval is 10 ms (that is a typical value), thus PDUs are scheduled at each $TTI_{\mathrm{RLC}} = 10\,\mathrm{ms}$. The calculation of the amount of PDUs that can be sent at a given scheduling instance is based on the assumption that the Node-B has a perfect knowledge on the air interface conditions, thus that the distribution of the number of MAC-d PDUs that can be transmitted over the air-interface at each 2 ms HSDPA TTI is known. (See Section 3.4).

Based on this assumption the number of PDUs the MAC-d scheduler is transferring during a 10 ms time slot (denoted by S_{RLC}) is given by:

$$S_{\mathrm{RLC}} = \sum_{1}^{5} S_{\text{Node-B}}.$$

The *arrival process* to the RLC buffer consists of the incoming traffic to the system (having an intensity of λ_{in}/K) and the PDUs that are retransmitted by the RLC AM entity (this is how RLC losses, denoted by λ_{FB} are modelled). λ_{FB} is calculated with the equation (17). At this calculation step Poisson traffic with a total arrival rate of λ' is assumed:

$$\lambda' = \frac{\lambda_{\mathrm{in}}}{K} + \lambda_{\mathrm{FB}}.$$

The distribution of the number of packets entering arriving to the RLC buffer in a 10 ms interval is calculated as follows:

$$\Pr\left(A_{\mathrm{RLC}} = k\right) = \frac{\left(\lambda' \, TTI_{\mathrm{RLC}}\right)^k}{k!} e^{-\lambda' \, TTI_{\mathrm{RLC}}} \quad k = 0, 1, 2, \ldots \tag{2}$$

The distribution is truncated at N such that the probability of the cut-off part of the distribution is reasonably small.

The *queue length* evolution embedded at TTI_{RLC} long time slots is then modelled by a discrete time Markov chain (DTMC) according to the following evolution equation:

$$X_{n+1} = \left(X_n + A_{n+1} - S_{n+1}\right)^+,$$

where X_{n+1} is the queue length, A_{n+1} is the number of arrivals and S_{n+1} is the number of packets served in the $n+1$st time slot. $(\cdot)^+$ denotes $\max(0, \cdot)$.

The ijth element of the transition probability matrix ($\boldsymbol{P}$) of the DTMC is given by:

$$p_{ij} = \begin{cases} \displaystyle\sum_{k=0}^{\infty} \Pr\left(A_{\mathrm{RLC}} = k\right) \Pr\left(S_{\mathrm{RLC}} = j - i + k\right) & i < L - N \\ \displaystyle\sum_{k=0}^{L-i} \Pr\left(A_{\mathrm{RLC}} = k\right) \Pr\left(S_{\mathrm{RLC}} = i - j + k\right) + & \\ & i \geq L - N. \\ \displaystyle + \Pr\left(S_{\mathrm{RLC}} = L + 1 - j\right) \sum_{k=L-i+1}^{N} \Pr\left(A_{\mathrm{RLC}} = k\right) & \end{cases}$$

L is the length of the RLC buffer, N is the support of the arrival distribution. In the first case the queue level is as low that no loss can happen, thus the transition probability equals the probability that there were $j - i$ more packets served than arrived. In the second case, the first (second) term corresponds to arrival sizes without (with) loss, respectively.

The steady state solution ($\boldsymbol{\pi}$) of the DTMC is given by the solution of the linear equation system

$$\boldsymbol{\pi}\boldsymbol{P} = \boldsymbol{\pi}$$

$$\boldsymbol{\pi} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = 1.$$

Having the steady state solution, the loss probability at the RLC buffer is calculated as the ratio of the mean number of lost and of the mean number of arrived PDUs during a $TTI_{\mathrm{RLC}} = 10\,\mathrm{ms}$ time slot:

$$P_{\mathrm{RLC}} = \frac{\displaystyle\sum_{i=0}^{L} \pi_i \sum_{j=0}^{N} \max\left(0, i + j - L\right) \Pr\left(A_{\mathrm{RLC}} = j\right)}{\displaystyle\sum_{i=0}^{L} \pi_i \sum_{j=0}^{N} j \Pr\left(A_{\mathrm{RLC}} = j\right)}. \tag{3}$$

The system time of the PDUs in the RLC buffer is calculated using Little's theorem:

$$\mathrm{E}\left(T_{\mathrm{RLC}}\right) = \frac{\mathrm{E}\left(X_{\mathrm{RLC}}\right)}{\left(1 - P_{\mathrm{RLC}}\right)\mathrm{E}\left(A_{\mathrm{RLC}}\right)} TTI_{\mathrm{RLC}} + \frac{1}{2} TTI_{\mathrm{RLC}}, \tag{4}$$

where $\mathrm{E}\left(X_{\mathrm{RLC}}\right)$ is the mean queue length. Since this is a discrete time model but arrivals can happen in continuous time, the model does not differentiate between arrivals at the beginning of the scheduling interval and at the end of it i.e. as they would not have different system times. Assuming that the arrival instants are uniformly distributed over the scheduling interval, the system time computed from the embedded DTMC is increased with the half of the interval.

During the analysis of the queueing network, the *departure process* from the RLC buffers has to be calculated as this is the arrival process to the transport buffer. We assume that the departures are independent and identically distributed (i.i.d.), with the distribution of the number of departing packets in a TTI_{RLC} interval computed by:

$$\begin{aligned}\Pr\left(D_{\mathrm{RLC}}=k\right)=&\sum_{i=0}^{L}\pi_i\sum_{j=k+1-i}^{\infty}\Pr\left(A_{\mathrm{RLC}}=j\right)\Pr\left(S_{\mathrm{RLC}}=k\right)+\\&+\sum_{i=0}^{L}\pi_i\Pr\left(A_{\mathrm{RLC}}=k-i\right)\sum_{j=k}^{\infty}\Pr\left(S_{\mathrm{RLC}}=j\right).\end{aligned}\tag{5}$$

This expression consist of two terms: the first corresponds to the case when there are enough packets in the buffer, the number of departing packets is determined by the number of packets the server can serve whereas in the second term the server could serve more packets than the buffer content.

3.3 The model of the transport buffer

In this paper we consider an AAL2/ATM based transport network (the transport link buffer model and its solution is referred in line 4 of Algorithm 2). The AAL2 layer is multiplexing the user connections into one Constant Bit Rate (CBR) VCC, with capacity C.

The ATM switch works in continuous time in contrast with the MAC-d and PF schedulers that are working in time slotted manner. In order to avoid mixing the continuous and discrete models, we decided to apply a discrete time model for the transport buffer as well. The RLC buffer is scheduled with $TTI_{\mathrm{RLC}} =$ 10 ms transmission interval and the PF scheduler in Node-B is forwarding PDUs with a $TTI_{\text{Node-B}} = 2$ ms. The selected time slot for the transport buffer is the minimum of these two e.g. $TTI_{\mathrm{Tr}} = 2$ ms is used to approximate the transport buffer mainly because this value allows finer resolution in time than a model with 10 ms interval. Another assumption is that in the model the transport buffer stores and transmits RLC PDUs instead of ATM cells. Since the RLC PDUs are the "data units" in other parts of the network, using the same data unit in the transport buffer simplifies the calculation significantly.

The distribution of the number of *arrivals* in a time slot is derived from the distribution of the number of departures from the RLC layer (D_{RLC}). The departure process of the RLC corresponds to a 10 ms TTI_{RLC}, while the transport buffer model has a 2 ms TTI_{Tr}. Thus, as a first step a conversion has to be applied between the MAC-d scheduling interval and the transport time slot, having a departure distribution from the RLC layer in a five-times longer TTI_{RLC}. The following binomial assumption is applied:

$$\Pr\left(D^{2\,\mathrm{ms}_{\mathrm{tr}}}=k\right)=\sum_{i=k}^{\infty}\Pr\left(D_{\mathrm{RLC}}=i\right)\binom{i}{k}\left(\frac{1}{5}\right)^{k}\left(1-\frac{1}{5}\right)^{i-k},$$

where $\Pr(D^{2\,\mathrm{ms}_{\mathrm{tr}}})$ is the probability of k arriving packets in a TTI_{Tr} time period if there were i arrivals in the TTI_{RLC} time period or in other words to choose k arrivals from i with probability $\frac{1}{5}$ – the quotient of the lengths of the two kinds of TTIs.

When calculating the distribution of the number of arrivals to the transport buffer, the whole traffic aggregate has to be considered each user connection is multiplexed into one VCC:

$$A_{\mathrm{Tr}} = \sum_{1}^{K} D^{2\,\mathrm{ms}_{\mathrm{tr}}},$$

where K is the number of HSDPA users.

The *service time* of the RLC PDUs in the transport buffer is calculated as

$$D = \frac{\text{RLC packet size with overheads}}{C}.$$

The transport overheads are considered with following formula:

$$\text{RLC packet size with overheads} = f_M \cdot \underbrace{\left(\frac{53}{47}\frac{f_M + 24}{f_M}\frac{\mathrm{E}\left(D_{\mathrm{RLC}}\right) f_M + 72}{\mathrm{E}\left(D_{\mathrm{RLC}}\right) f_M}\right)}_{\text{overhead}}.$$

The overhead consists of the ATM header (40 bits) plus the 8 bit long CPS PDU Start Field (53/47), the 24 bit long CPS Packet header per an RLC PDU ($\frac{f_M+24}{f_M}$) and finally the 72 bit long HS-DSCH FP frame header that carries $\mathrm{E}\left(D_{\mathrm{RLC}}\right)$ RLC packets in an average.

In our discrete system having TTI_{Tr} long time slots the number of PDUs served in a time slot can either be $F = \left\lfloor \frac{TTI_{\mathrm{Tr}}}{D} \right\rfloor$ or $F+1$, according to the following probabilities:

$$\begin{aligned}
\Pr\left(S_{\mathrm{Tr}} = F\right) &= 1 - \left(\frac{TTI_{\mathrm{Tr}}}{D} - F\right) \\
\Pr\left(S_{\mathrm{Tr}} = F+1\right) &= \frac{TTI_{\mathrm{Tr}}}{D} - F.
\end{aligned}$$

The *queue length* can be modelled by a DTMC similar to the one we applied for the RLC buffer, i.e.,

$$X_{n+1} = \left(X_n + A_{n+1} - S_{n+1}\right)^{+}, \tag{6}$$

where X_{n+1} is the queue length, A_{n+1} is the number of arrivals and S_{n+1} is the number of PDUs served in the $n+1$st time slot.

Based on the distribution of the number of arrivals and served PDUs we can create the transition probability matrix of the DTMC such that the ijth element will be calculated in the same way as in the case of the RLC buffer:

$$p_{ij} = \begin{cases}
\displaystyle\sum_{k=0}^{\infty} \Pr\left(A_{\mathrm{Tr}} = k\right) \Pr\left(S_{\mathrm{Tr}} = j - i + k\right) & i < L - (N - F) \\
\displaystyle\sum_{k=0}^{L-i} \Pr\left(A_{\mathrm{Tr}} = k\right) \Pr\left(S_{\mathrm{Tr}} = i - j + k\right) + & \\
\displaystyle + \Pr\left(S_{\mathrm{Tr}} = L + 1 - j\right) \sum_{k=L-i+1}^{N} \Pr\left(A_{\mathrm{Tr}} = k\right) & i \geq L - (N - F).
\end{cases}$$

The computation of the loss probability is similar to the one applied at the RLC modeling:

$$P_{\mathrm{Tr}} = \frac{\sum_{i=0}^{L-F} \pi_i \sum_{j=0}^{N} \max\left(0, i+j-L\right) \Pr\left(A_{\mathrm{Tr}} = j\right)}{\sum_{i=0}^{L-F} \pi_i \sum_{j=0}^{N} j \Pr\left(A_{\mathrm{Tr}} = j\right)}. \tag{7}$$

The numerator is the expected number of lost PDUs and the denominator is the expected number of PDUs received correctly.

The system time of the PDUs in the transport buffer is calculated based on Little's theorem as (see (4)):

$$\mathrm{E}\left(T_{\mathrm{Tr}}\right) = \frac{\mathrm{E}\left(X_{\mathrm{Tr}}\right)}{\left(1-P_{\mathrm{Tr}}\right)\mathrm{E}\left(A_{\mathrm{Tr}}\right)} TTI_{\mathrm{Tr}} + \frac{1}{2} TTI_{\mathrm{Tr}}. \tag{8}$$

The departure process is calculated similar to the calculation of the same parameter in case of the RLC buffer

$$\begin{aligned} \Pr\left(D_{\mathrm{Tr}} = k\right) = &\sum_{i=0}^{L-F} \pi_i \sum_{j=k+1-i}^{\infty} \Pr\left(A_{\mathrm{Tr}} = j\right) \Pr\left(S_{\mathrm{Tr}} = k\right) + \\ &+ \sum_{i=0}^{L-F} \pi_i \Pr\left(A_{\mathrm{Tr}} = k-i\right) \sum_{j=k}^{\infty} \Pr\left(S_{\mathrm{Tr}} = j\right). \end{aligned} \tag{9}$$

3.4 The model of the MAC-hs buffers

In this paper it is assumed that the MAC-hs buffers are scheduled by a Proportional Fair algorithm, that is making the scheduling decisions based on the instantaneous channel quality and the average throughput of the users with the scope to achieve high level of resource usage and in the same time to provide high level of fairness to the users. The scheduler is selecting one user for transmission at each scheduling instance (at every $TTI_{\text{Node-B}} = 2\,\mathrm{ms}$). The reported Channel Quality Indicator (CQI) is defining the modulation and coding scheme, thus the number of MAC-d PDUs that can be transmitted during a TTI. Since the channel conditions can change quickly, temporary traffic overload can occur in the Node-B. The arriving PDUs are stored in the MAC-hs buffers (there is a separate buffer for each flow).

The modeling of the HSDPA air interface model is out of the scope of this paper. Instead, the MATLAB based tool of the Eurane project (see [6]) has been used in order to obtain the distribution of the number of MAC-d PDUs that can be transmitted in a TTI ($P(\hat{S} = k)$). This distribution has been generated by assuming saturated buffers without taking the impact of HARQ into consideration [9].

To obtain the *service process* of the MAC-hs buffer first the effect of HARQ is included in the model. According to [1,2] the probability of properly decoding the packet at the user side and thus the probability of the error free transmission after j trials is as follows:

$$P_j = \begin{cases} 1 - P_e, & j = 1 \\ P_e^{j-1} P_s^{j-2}\left(1 - P_e P_s\right), & j > 1. \end{cases}$$

The meaning and the default values of P_e and P_s are listed in Table 1. Considering that the maximal number of trials is M the expected number of retransmissions till success is as follows:

$$\mathrm{E}(H) = \sum_{j=1}^{M} j P_j + M \left(1 - \sum_{j=1}^{M} P_j \right),$$

thus the probability that the time slot is lost due to a HARQ loss is:

$$P_{\mathrm{tl}} = 1 - \frac{1}{\mathrm{E}(H)}.$$

Finally, the distribution of the number of MAC-d PDUs that can be transmitted in a TTI taking the HARQ losses also into consideration is:

$$\Pr\left(S_{\text{Node-B}} = k\right) = \begin{cases} (1 - P_{\mathrm{tl}}) \Pr\left(\hat{S} = k\right) + P_{\mathrm{tl}} & k = 0, \\ (1 - P_{\mathrm{tl}}) \Pr\left(\hat{S} = k\right) & k \neq 0. \end{cases} \tag{10}$$

The distribution of the number of *arrivals* to the MAC-hs buffer is calculated by assuming that the packets arriving from the transport network are directed to the buffer of the tagged user according to a random choice with probability $1/K$; resulting in the following binomial distribution:

$$\Pr\left(A_{\text{Node-B}} = k\right) = \sum_{i=k}^{\infty} \Pr\left(D_{\mathrm{Tr}} = i\right) \binom{i}{k} \left(\frac{1}{K}\right)^{k} \left(1 - \frac{1}{K}\right)^{i-k}.$$

Contrary to the other two nodes the *queue length* evolution of the MAC-hs buffer is

$$X_{n+1} = \left(X_n - S_{n+1}\right)^{+} + A_{n+1},$$

This means that only those MAC-d PDUs can be served by the PF scheduler that have arrived before the beginning of TTI. The ijth element of the transition probability matrix is

$$p_{ij} = \begin{cases} \displaystyle\sum_{k=0}^{j-1} \Pr\left(A_{\text{Node-B}} = k\right) \Pr\left(S_{\text{Node-B}} = i - j + k\right) + & \\ \displaystyle + \Pr\left(A_{\text{Node-B}} = j\right) \sum_{k=i}^{\infty} \Pr\left(S_{\text{Node-B}} = k\right) & i < k_m \\ \displaystyle\sum_{k=0}^{\infty} \Pr\left(A_{\text{Node-B}} = k\right) \Pr\left(S_{\text{Node-B}} = j - i + k\right) & i \geq k_m. \end{cases}$$

After the computation of the steady state solution, the loss probability is calculated as the ratio of the lost and arrived PDUs in a TTI as:

$$P_{\text{Node-B}} = \frac{\displaystyle\sum_{i=0}^{L} \pi_i \sum_{j=0}^{\infty} \max\left(0, i + j - L\right) \Pr\left(A_{\text{Node-B}} = j\right)}{\displaystyle\sum_{i=0}^{L} \pi_i \sum_{j=0}^{\infty} j \Pr\left(A_{\text{Node-B}} = j\right)}. \tag{11}$$

$P_{\text{Node-B}}$ is the loss probability of PDUs due to buffer buffer overload and tail drop at the Node-B. However, at the Node-B the buffer overload is not the only event that leads to packet loss. If the air interface quality is bad, and the HARQ mechanism fails, the MAC-hs discards the PDU from the corresponding HARQ register and the retransmission of the PDUs falls back to the RLC layer if the maximal number of retransmissions (M) has been reached. The probability of such events is denoted by P_{HARQ} and computed as:

$$P_{\text{HARQ}} = 1 - \sum_{j=1}^{M} P_j. \tag{12}$$

The system time of the MAC-hs buffer is calculated using Little's theorem as

$$\mathrm{E}\left(T_{\text{Node-B}}\right) = \frac{\mathrm{E}\left(X_{\text{Node-B}}\right)}{\left(1 - P_{\text{Node-B}}\right)\mathrm{E}\left(A_{\text{Node-B}}\right)} TTI_{\text{Node-B}} + \frac{1}{2} TTI_{\text{Node-B}}, \tag{13}$$

where $\mathrm{E}\left(X\right)$ is the mean queue length, and the addition of the extra time of half-$TTI_{\text{Node-B}}$ in the second term has the same explanation as in case of the RLC and transport network models.

For the queueing network analysis the departure intensity of the Node-B buffer is needed. The number of MAC-d PDUs per $TTI_{\text{Node-B}}$ equals the minimum of the number of packets in the buffer and the number of packets that can be served. This gives:

$$\lambda_U = \frac{1}{TTI_{\text{Node-B}}} \sum_{i=0}^{L} \pi_i \sum_{k=0}^{\infty} \Pr\left(S_{\text{Node-B}} = k\right) \min\left(i, k\right). \tag{14}$$

3.5 The feed-back link

In our queueing model the PDUs lost at the different parts of the network are considered as they where entering the RLC buffer again for repeated transmission. The feed-back link on Figure 2 "collects" these lost packets. In this section we calculate the traffic intensity on the feed-back link. This traffic (with Poisson assumption [4]) is added to the traffic entering the network during the analysis of the RLC model.

As a first step the probability of a PDU loss (due to any reason) in the network after leaving the RLC buffer is calculated. This probability is denoted by p_L and computed by:

$$p_L = P_{\text{Tr}} + \left(1 - P_{\text{Tr}}\right) P_{\text{Node-B}} + \left(1 - P_{\text{Tr}}\right)\left(1 - P_{\text{Node-B}}\right) P_{\text{HARQ}}. \tag{15}$$

It can happen that a retransmitted PDU is lost. After a given number of RLC level retransmission attempts (R) that equals the maximum number of RLC retransmissions the PDU is discarded and loss is detected by the TCP flow control. In this case this PDU does not enter the RLC buffer again (as long as the higher layer entity does not re-send it). The probability that a PDU loss did not reach the maximal number of retransmission attempts thus it increases the load of the RLC buffer is calculated with:

$$\hat{p} = \frac{\sum_{k=1}^{R} \left(1 - p_L\right)^{k-1} p_L}{\sum_{k=1}^{R+1} \left(1 - p_L\right)^{k-1} p_L} \tag{16}$$

(we assumed truncated geometrical distribution for the distribution of the number of retransmissions).

With the above considerations the traffic of the feed-back link is computed by:

$$\lambda_{\mathrm{FB}} = \hat{p} \cdot p_L \cdot \lambda_A, \tag{17}$$

where λ_A denotes the mean departure rate of the RLC buffer.

3.6 The TCP level packet loss and the RTT

In this section we describe the calculation of the TCP level performance measures based on the buffer-wise performance measures (given by equations (12), (3), (4), (7), (8), (11) and (13)).

The delay of one packet assuming that it has not been lost in the system is given by:

$$D_s = \mathrm{E}\left(T_{\mathrm{RLC}}\right) + \mathrm{E}\left(T_{\mathrm{Tr}}\right) + \mathrm{E}\left(T_{\text{Node-B}}\right). \tag{18}$$

If it has been lost somewhere, the mean delay can be computed by:

$$\begin{aligned} D_u = {} & P_{\mathrm{Tr}}\,\mathrm{E}\left(T_{\mathrm{RLC}}\right) + \left(1 - P_{\mathrm{Tr}}\right) P_{\text{Node-B}} \left(\mathrm{E}\left(T_{\mathrm{RLC}}\right) + \mathrm{E}\left(T_{\mathrm{Tr}}\right)\right) + \\ & + \left(1 - P_{\mathrm{Tr}}\right)\left(1 - P_{\text{Node-B}}\right) P_{\mathrm{HARQ}} \cdot \\ & \cdot \left(\mathrm{E}\left(T_{\mathrm{RLC}}\right) + \mathrm{E}\left(T_{\mathrm{Tr}}\right) + \mathrm{E}\left(T_{\text{Node-B}}\right)\right). \end{aligned} \tag{19}$$

If a packet has been retransmitted k times till successful transmission, the mean round trip time can be calculated as the sum of the mean delays of $k-1$ unsuccessful transmissions and one times the delay of a successful transmission. Using the geometric distribution assumption for the number of retransmission attempts again we have

$$RTT = D_{\mathrm{UL}} + \sum_{k=1}^{R} \frac{p_L^{k-1}\left(1 - p_L\right)}{1 - p_L^R} \left(\left(k-1\right) D_u + D_s\right), \tag{20}$$

where D_{UL} denotes the mean delay in uplink direction considered to be constant as the UTRAN is typically not congested in uplink direction.

The loss at TCP layer is simply calculated by one minus the ratio of the traffic entering and leaving the system:

$$p = 1 - \frac{\lambda_U}{\lambda} \tag{21}$$

4 Numerical results

The accuracy of the TCP throughput method presented in this paper has been evaluated with a numerical example. A simulation scenario has been created based on a topology consisting of one RNC and one Node-B. It is assumed that there is only one MAC-d flow and one priority queue per HSDPA user. The scheduler is Proportional Fair Scheduler. The number of HARQ processes is six; the maximum number of MAC-hs retransmissions is three, whereas the maximum number of RLC retransmissions is six. The number of HS-DSCH codes per cell is five; code multiplexing is not implemented. HSDPA users are connected to the Node-B via HS-DSCH in downlink and via DCH in uplink.

Profile	Ped-A
Speed	3 km/h
Distance	400 m
Trace length	900 s

Table 2: The parameters of the air interface profile

Each HSDPA UE is of category 5/6. The Iub interface and the user plane of the Radio Layer protocols (MAC-d, MAC-hs, RLC, PDCP) are implemented in detail. The transport network of the Iub consists of one CBR VCC. HSDPA users are originating file (ftp) downloads from servers located on the Internet. The transport protocol was TCP Reno; the maximum advertised window size was 48 kbytes; the maximum TCP/IP packet size was set to 1500 bytes. The HSDPA UE reports the observed channel quality (CQI) to the Node-B. Based on this, the amount of data to be sent to the UE is defined. The radio channel condition is simulated separately for each UE. Negative – when the Silence to Noise Ratio (SNR) is below the required threshold – or positive acknowledgement is generated upon reception of a MAC-hs frame. The CQI estimation error is modelled with a constant delay of 6 ms. Users are modelled with ITU-T Pedestrian A model, velocity 3 km/h, assuming that chase-combining is implemented in the UEs. The distance of the users from the Node-B was set to 400 m. The SNR is calculated considering the followings: distance loss according to Okumara-Hata model for urban cell with base station antenna height of 30 m, mobile antenna height of 1.5 m and carrier frequency of 1950 Mhz [8]; multi-path (fast) fading; Rake receiver assuming that channel estimation is ideal and the power levels of all paths are known; shadow (slow) fading (log-normal distribution correlated in time [5]); constant Node-B antenna gain constant (17 dBi); inter-cell interference (-70 dBm) and intra-cell interference (30 dBm).

In the analytical model the air interface trace file has been generated with the MATLAB scripts of the Eurane project (see [6]) with parameters defined by Table 2.

Next, the distribution of the number of RLC PDUs that can be transmitted by the Node-B (denoted by $\hat{S}$ in the paper) is extracted and the analysis method is executed at several link capacity settings (Figure 3).

The figure confirms that the error of the approximation is below 10%. The most important application of analytical throughput computation methods like the one presented in this paper is the transport link dimensioning. During the link dimensioning, the mean throughput (Figure 3) is calculated and the optimal transport link capacity is selected that guarantees the required level of service. The optimal link capacity is around the knee point i.e. where the TCP throughput curve in function of the transport capacity becomes horizontal. Above this point the increase of link capacity does not introduce an increase in the TCP throughput, while below this point the air interface can be underutilized. The optimal link capacity obtained from the analysis and simulation are close to each other, thus our method can be used for transport link dimensioning with a lower computational effort compared to simulations.

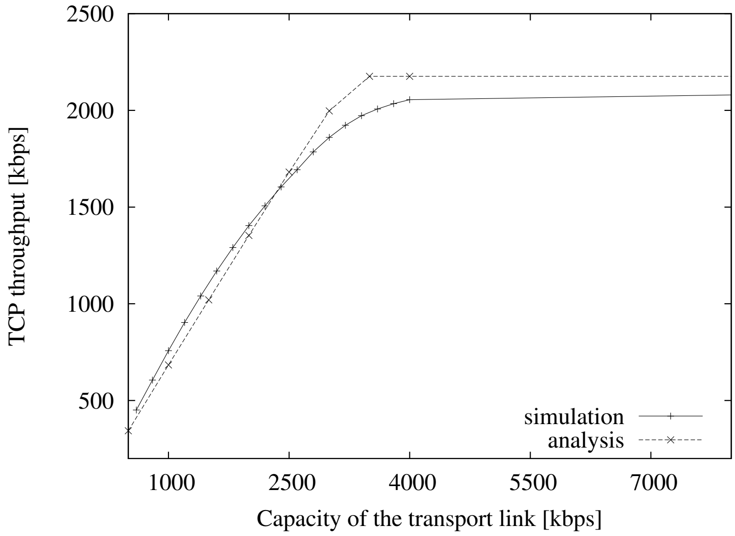

Figure 3: Comparison of the analysis and simulation results

5 Conclusion

In this paper we described an approximation model of the TCP throughput over HSDPA. We identified the relevant congestion points in the system that are having dominant impact on the TCP throughput and developed Markov models to calculate the performance measures. An iterative solution method is provided to solve the queueing network model of the system. The model have been evaluated with a numerical example to evaluate their accuracy and to show that it can be used for the transport link capacity dimensioning of the mobile backhaul.

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