Problems 2 - Hamming, Hadamard, binary Golay, CRC, LDPC codes

Coding Technology

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2025/10/03

Problem 1

A linear binary block code has syndrome vectors of length 7 bits, and each error group contains 32 error vectors. What are the parameters of the code?

Solution. Syndrome vectors are $n - k$ bits long, so $n - k = 7$. Each error group contains 2^k error vectors (same as the number of codewords), so $2^k = 32$ and $k = 5$. Finally, $n = 12$, so this is a C(12,5) code.

vector	length
message vector	k
codeword vector	n
error vector	n
received vector	n
syndrome vector	$n - k$
detected error	n
detected codeword	n
detected message	k

Problem 2

For a binary linear systematic code, we know the error group corresponding to one of the syndromes:

$$(111) \quad \rightarrow \quad \{(11111), (10000), (01001), (00110)\}.$$

(a) Which is the group leader?

(b) What are the parameters of the code?

(c) List the codewords.

(d) Compute the generator matrix and parity check matrix.

(e) How many errors can the code detect? How many errors can the code correct?

Problem 2

Solution.

(a) Which is the group leader?

The vector with minimal weight, so (10000).

(b) What are the parameters of the code?

Error vectors are n bits long, so $n = 5$. Each error group contains 2^k error vectors, so $2^k = 4$ and $k = 2$. This is a C(5,2) code.

(c) List the codewords.

The codewords can be obtained by subtracting (or adding) one of the error vectors from all other error vectors in the error group:

$$\begin{array}{ll} (11111) - (11111) = (00000) & (11111) - (10000) = (01111) \\ (11111) - (01001) = (10110) & (11111) - (00110) = (11001) \end{array}$$

Problem 2

(d) Compute the generator matrix and parity check matrix.

The codewords are (00000), (01111), (10110), (11001). The code is systematic, so

$$(01) \to (01111), \quad (10) \to (10110).$$

The generator matrix can be built from the codewords corresponding to the unit vectors, so

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

and then

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2

(e) How many errors can the code detect? How many errors can the code correct?

The code is linear, so

$$d_{\min} = \min_{c \neq 0} w(c) = 3,$$

so the code can detect

$$d_{\min} - 1 = 2$$

errors, and correct

$$\left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor = 1$$

error.

Hamming codes

Hamming codes can correct $t = 1$ error.

Hamming codes are perfect codes:

$$n = 2^{n-k} - 1 \quad \Longleftrightarrow \quad \sum_{i=0}^{1} \binom{n}{i} = 2^{n-k}$$

Construction of $C(n, k)$ Hamming code:

- the column vectors of the parity check matrix H run through all different nonzero binary vectors of length $n - k$.

Problem 3

A binary linear code has generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

Is this a Hamming code?

Solution. $n = 5, k = 3$, so this is a $C(5, 3)$ code. But

$$n = 5 \neq 2^{n-k} - 1 = 3,$$

so this is not a Hamming code.

(Possible Hamming code parameters are (3,1), (7,4), (15,11), (31,26), (63,57) etc. - but no Hamming code in-between!)

Problem 4

A linear binary code has parity check matrix

$$H = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Is this a Hamming code?

Solution. $n = 7, k = 4$, so $n = 2^{n-k} - 1$ holds, but columns 1 and 4 are the same, so this is not a Hamming code.

Problem 5

We have a 12-bit message to transmit through a channel with bit error probability $p_b = 0.03$.

(a) We transmit the message without coding. What is the probability that the receiver gets the message wrong?

(b) We cut the message into 3 sections of 4 bits each, then transmit each section using a $C(7,4)$ Hamming code. When can the receiver decode correctly? Compute the probability of a decoding error for the entire 12-bit message. What is the rate of this code?

(c) Compute the probability of a decoding error if we use a $C(23,12)$ Golay code for the entire message instead. What is the code rate?

(d) What is the Shannon-limit for the code rate for this channel?

Problem 5

We have a 12-bit message to transmit through a channel with bit error probability $p_b = 0.03$.

(a) We transmit the message without coding. What is the probability that the receiver gets the message wrong?

Solution. $1 - (1 - p_b)^{12} \approx 0.306$.

(b) We cut it into 3 sections of 4 bits each, then transmit each section using a $C(7, 4)$ Hamming code. When can the receiver decode correctly? Compute the probability of a decoding error for the entire 12-bit message. What is the rate of this code?

Solution. The receiver can decode correctly if all 3 blocks are decoded correctly. The Hamming code can correct 1 error, so each block is decoded correctly if there are either 0 or 1 errors in each block.

Problem 5

(b) The probability that 1 out of the 3 blocks is decoded correctly is

$$(1 - p_b)^7 + 6p_b(1 - p_b)^6 \approx 0.958,$$

the probability that all 3 blocks are correct is

$$((1 - p_b)^7 + 6p_b(1 - p_b)^6)^3 \approx 0.879,$$

and the probability of a decoding error for the entire 3 block message is

$$1 - ((1 - p_b)^7 + 6p_b(1 - p_b)^6)^3 \approx 0.121.$$

The code rate is $4/7 \approx 0.571$.

Problem 5

(c) Compute the probability of a decoding error if we use a $C(23, 12)$ Golay code for the entire message instead. What is the code rate?

Solution. With the C(23,12) Golay code, the entire 12-bit message will be a single block. This code can correct 3 errors, so the block error probability is

$$1 - (1 - p_b)^{23} - \binom{23}{1}(1 - p_b)^{22} p_b - \binom{23}{2}(1 - p_b)^{21} p_b^2 - \binom{23}{3}(1 - p_b)^{20} p_b^3 \approx 0.00454.$$

The code rate is $12/23 \approx 0.522$.

(d) The Shannon-limit of the channel is

$$1 - (-p_b \log_2 p_b - (1 - p_b)\log_2(1 - p_b)) \approx 0.806.$$

Problem 6

We transmit a single bit through a channel with bit error probability $p_b = 0.1$ using the $C(4,1)$ extended Hamming code. Due to the SECDED property of the extended Hamming code, the possible outcomes of the decoding are the following:

- no error detected, correct decoding;
- 1 error detected, correct decoding;
- 2 errors detected, no decoding;
- 3 or more errors but no error detected, decoding error

Compute the probability of each of the above outcomes.

The error groups are:

$$
\begin{array}{ll}
(000) \to \{(0000),(1111)\}, & (001) \to \{(0001),(1110)\} \\
(010) \to \{(0010),(1101)\}, & (100) \to \{(0100),(1011)\} \\
(101) \to \{(0101),(1010)\}, & (110) \to \{(1001),(0110)\} \\
(011) \to \{(0011),(1100)\}, & (111) \to \{(1000),(0111)\}
\end{array}
$$

Problem 6

Solution.

- no error detected, correct decoding

$$P(0 \text{ errors}) = (1 - p_b)^4 = 0.9^4 = 0.6561$$

- 1 error detected, correct decoding

$$P(1 \text{ error}) = \binom{4}{1}(1 - p_b)^3 p_b = 4 \cdot 0.9^3 \cdot 0.1 = 0.2916$$

- 2 errors detected, no decoding

$$P(2 \text{ errors}) = \binom{4}{2} 0.9^2 0.1^2 = 0.0486$$

- 3 or more errors but no error detected, decoding error

$$P(3 \text{ or more errors}) = \binom{4}{3} 0.9^1 0.1^3 + \binom{4}{4} 0.1^4 = 0.0037$$

Problem 7

The $C(8,4)$ extended Hamming code has generator

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix},$$

and the $C(8,4)$ augmented Hadamard code has generator

$$G' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Are the two codes equivalent?

Problem 7

Solution. Two linear binary block codes are equivalent if the generators can be transformed into each other by a sequence of the following steps:

- permutation of the rows;
- permutation of the columns;
- adding one row to another row.

The original G is in systematic form; let's try to bring G' to systematic form too.

Targeting a leftmost I block. Exchange columns 2 and 5:

$$G' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Problem 7

Add row 2 to row 1:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Add row 3 to row 1:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Problem 7

Add row 4 to row 3:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

The goal is

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix},$$

which can clearly be obtained by a permutation of columns 5-8, so the two codes are equivalent.

Problem 8

A CRC code adds 3 parity bits and has parameter vector (101).

(a) What is the codeword corresponding to the message (1010011)?

(b) The codeword from part (a) is transmitted through a channel. The error vector is 0001100000. Execute error detection for the received codeword.

Solution. We first extend the vectors:

(a)

$$(1010011) \to (1010011|000)$$
$$(101) \to (1101)$$

Problem 8

(a) Then

$$
\begin{array}{ccccccc|ccc}
1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & & & & & & \\
\hline
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
\rightarrow & 1 & 1 & 0 & 1 & & & & & \\
\hline
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
 & & \rightarrow & 1 & 1 & 0 & 1 & & & \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 & & & & \rightarrow & 1 & 1 & 0 & 1 & \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
 & & & & & \rightarrow & 1 & 1 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{array}
$$

$$c = (u|d) = (1010011|011)$$

Problem 8

(b)

$$v = c \oplus e = (1010011011) \oplus (0001100000) = (1011111011)$$

1	0	1	1	1	1	1	0	1	1
1	1	0	1						
0	1	1	0	1	1	1	0	1	1
$\rightarrow$	1	1	0	1					
0	0	0	0	0	1	1	0	1	1
				$\rightarrow$	1	1	0	1	
0	0	0	0	0	0	0	0	0	1

Nonzero remainder $\rightarrow$ error detected!

Problem 9

An LDPC code has parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Execute the bit-flipping algorithm for the received vector

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

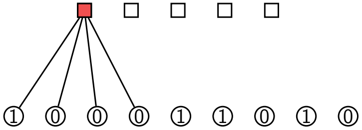
to obtain the detected codeword c'.

Problem 9

Solution.

$$H = \begin{bmatrix} \color{blue}{1} & \color{blue}{1} & \color{blue}{1} & \color{blue}{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

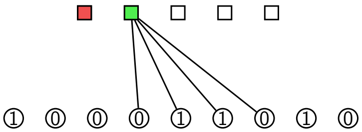


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

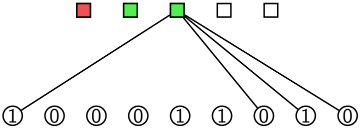


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ \color{blue}{1} & 0 & 0 & 0 & 0 & 0 & \color{blue}{1} & \color{blue}{1} & \color{blue}{1} \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

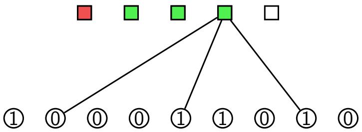


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

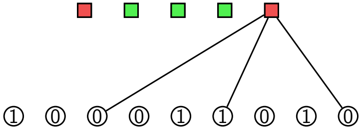


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

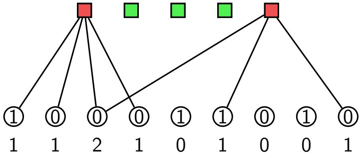


Problem 9

Solution.

$$H = \begin{bmatrix} \color{blue}{1} & \color{blue}{1} & \color{blue}{1} & \color{blue}{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & \color{blue}{1} & 0 & 0 & \color{blue}{1} & 0 & 0 & \color{blue}{1} \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

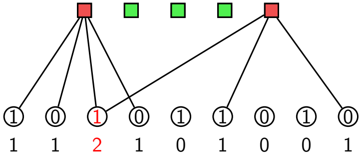


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$



Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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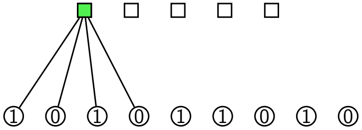
Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

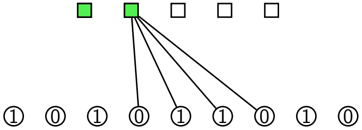


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

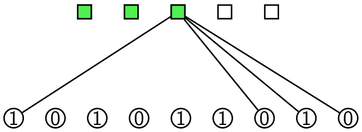


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

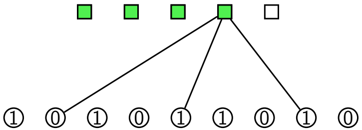


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

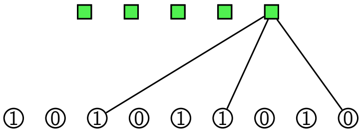


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} & 0 & 0 & {\color{blue}1} \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

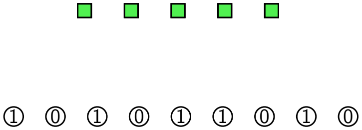


Problem 9

Solution.

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$



$$c' = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$