Sample midterm test sheet A
Coding Technology
Fall 2025

1. For each of these statements, decide if it is true or false. (5 correct answers → 10 pts, 4 correct answers → 5 pts, 0–3 correct answers → 0 pts)

   A. Any C(n, k) binary code has 2^k codewords.

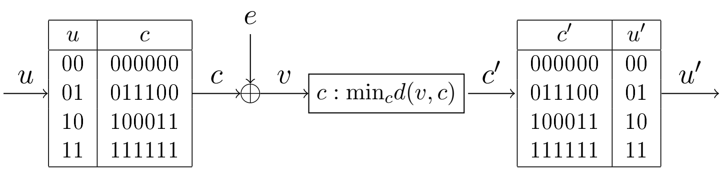
   B. For any C(n, k) binary Hamming code, $n = 2^k - 1$.

   C. If two error correction codes are equivalent, they have the same error correction capabilities.

   D. In a Galois field, every nonzero element has a multiplicative inverse.

   E. $g(x) = y^5 x + y^2 x + x^2$ generates a BCH code over GF(8).

2. Consider the following coding scheme.



   (a) What are the n and k parameters of the code? (2 pts)

   (b) How many errors can this code correct? (2 pts)

   (c) Execute the entire coding scheme for $u = (0\,1)$ and $e = (0\,0\,0\,0\,1\,0)$, that is, calculate c, v, c', u'. Is the decoding correct? (6 pts)

3. The C(16,11) extended Hamming code can distinguish between 0, 1 or 2 errors (assuming ≤ 2 errors occurred), and for 0 or 1 errors, it can decode correctly. For a channel with bit error probability $p_b = 0.01$, calculate the probability of each of the following outcomes for a single block:

   (a) 0 or 1 errors detected, correct decoding (4 pts);

   (b) 2 errors detected, no decoding (3 pts);

   (c) erroneous decoding (3 pts).

4. An LDPC code has parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

   Execute the bit-flipping algorithm for the received vector $v = (1101001100)$ to obtain the detected codeword c'. (10 pts)

5. Design a code over GF(5) that can correct 1 error by generator matrix. (Primitive elements over GF(5) are 2 and 3.)

   (a) What are the code parameters? (2 pts)

   (b) Calculate the code rate. (2 pts)

   (c) Determine the generator matrix of the code. (3 pts)

   (d) Calculate the codeword corresponding to the message vector whose digits are all 3's. (3 pts)

Sample midterm test sheet B
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1. For each of these statements, decide if it is true or false. (5 correct answers $\rightarrow$ 10 pts, 4 correct answers $\rightarrow$ 5 pts, 0–3 correct answers $\rightarrow$ 0 pts)

   A. Any $C(n, k)$ binary code has 2^n codewords.

   B. Perfect $C(n, k)$ codes have minimal code distance $d_{\min} = n - k + 1$.

   C. The code rate of a $C(n, k)$ code is k/n.

   D. For a systematic linear binary code, the rightmost $(n-k) \times (n-k)$ block of H is the identity matrix.

   E. Reed-Solomon codes are MDS codes.

2. For a systematic binary linear code, we know the error group corresponding to one of the syndromes:
$$(100) \quad \rightarrow \quad \{(10011), (01010), (00100), (11101)\}.$$
   (a) Which is the group leader? (2 pts)

   (b) What are the parameters of the code? (2 pts)

   (c) List the codewords. (2 pts)

   (d) Compute the generator matrix and parity check matrix. (2 pts)

   (e) How many errors can the code detect? How many errors can the code correct? (2 pts)

3. A binary linear code has parity check matrix
$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$
   (a) What are the code parameters? (2 pts)

   (b) Is this a Hamming code? (2 pts)

   (c) Decode the received vector $v = (0\,1\,0\,1\,1\,0\,0)$. Describe how the error vector is detected based on the syndrome, then decode the message. (6 pts)

4. A binary CRC code adds 3 bits to each message, with parameter vector $d = (011)$.

   (a) What is the codeword corresponding to the message $u = (1011101)$? (5 pts)

   (b) Does the code detect the error for $e = (0001001000)$? (5 pts)

5. Consider GF(8) with reducing polynomial $p(y) = y^3 + y + 1$. Design a code over GF(8) that can correct 1 error by generator polynomial.

   (a) Determine the generator polynomial of the code. (2 pts)

   (b) Calculate the code rate. (2 pts)

   (c) When using this code to transmit a message over a channel with digit error probability $p = 0.01$, what is the probability of a decoding error? (6 pts)