On a class of dependent Sparre Andersen risk models with application.

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Risk process $\{X(t),\ t \geq 0\}$

$$X(t) = u + ct - \sum_{i=1}^{N(t)} J_k, \quad t \geq 0.$$

- $N(t) = \max\{n \in \mathbb{N} : \sum_{k=1}^{n} T_k \leq t\}$ number of claims up to time t, T_k interclaim, J_k claim size,
- $u \geq 0$ initial capital, $c > 0$ premium rate, $c\mathbb{E}[T_1] > \mathbb{E}[J_1]$,
- $\{(T_k, J_k),\ k \in \mathbb{N}\}$ i.i.d. with dependence structure, defined by

$$\mathbb{P}(T_k \in \mathrm{d}t, J_k \in \mathrm{d}x) = \alpha(\mathrm{d}t)\, e^{Rx}\, \underline{r}\, \mathrm{d}x \quad t, x \in \mathbb{R}_+,$$

where $\alpha(\mathrm{d}t) \in \mathbb{R}^{1\times m}$, is a $1 \times m$ distribution vector, $R \in \mathbb{R}^{m\times m}$ sub-generator matrix, $\underline{r} = (-R)\underline{1}$.

Ruin probability

We let $\tau := \{t \geq 0,\ X(t) < 0\}$ the ruin time and its Laplace Transform

$$\boxed{\hat{\psi}(q,u) := \mathbb{E}_u\left[e^{-q\tau}\right], \quad q \geq 0, \quad u \geq 0.}$$

$\longrightarrow$ **Goal :** Compute $\hat{\psi}(q,u)$ with efficient algorithm, with LT $\hat{\alpha}(-q) := \int_0^\infty e^{-qt}\alpha(\mathrm{d}t) \in \mathbb{R}^{1\times m}$, $q \in \mathbb{R}_+$, available.

Notation : If $Q \in \mathbb{R}^{m\times m}$ negative-definite, we extend definition of LT :

$$\boxed{\hat{\alpha}(Q) := \int_0^\infty \alpha(\mathrm{d}t)e^{Qt} \in \mathbb{R}^{1\times m}.}$$

$\hat{\alpha}(-qI)$ available for all $q \in \mathbb{R}_+$, but $\hat{\alpha}(Q)$ not explicitly computable in practice for general Q !

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Fixed point equation

Theorem

Laplace transform $\hat{\psi}(q,u)$ *verifies*

$$\hat{\psi}(q,u) = \hat{\rho}(q) e^{[R + \underline{r}\,\hat{\rho}(q)]u} \underline{1}, \quad u \geq 0,\; q \geq 0, \tag{1}$$

where $\hat{\rho}(q)$ *is a* $1 \times m$ *sub-probability vector satisfying the fixed point equation*

$$\hat{\rho}(q) = \hat{\alpha}(cR + c\,\underline{r}\,\hat{\rho}(q) - qI), \quad q > 0. \tag{2}$$

If $q = 0$ *there exists a* $1 \times m$ *sub-probability vector* $\hat{\rho}(0)$ *verifying (2) such that expression (1) holds for* $\hat{\psi}(0,u)$.

Example and issues

Main issue is solving (2), i.e.

$$\boxed{\hat{\rho}(q) = \hat{\alpha}(cR + c\,\underline{r}\,\hat{\rho}(q) - qI),}$$

with unkwown $\hat{\rho}(q) \in \mathbb{R}^{1\times m}$ subprobability vector. E.g.

- $\alpha(\mathrm{d}t) \in \mathbb{R}$ scalar, J_k exponentially distributed : Malinovskii (1998), $\hat{\rho}(q)$ scalar,
- $\alpha(\mathrm{d}t) \in \mathbb{R}$ scalar, $J_k \sim PH(\underline{r}, R)$: Asmussen and Albrecher (2010), $\hat{\rho}(q)$ scalar.

Issues here :

1. (2) does not necessarily have a unique solution,
2. $\alpha(\mathrm{d}t)$ vector,
3. need to be able to compute $\hat{\alpha}(M)$ where M is a matrix : *no explicit form*.

Algorithm for fixed point equation

Idea : Approximating $\hat{\rho}(q)$ by $\hat{\rho}^N(q)$, $N \in \mathbb{N}$, solution to

$$\boxed{\hat{\rho}^N(q) = \hat{\alpha}^N(cR + c\,\underline{r}\,\hat{\rho}^N(q) - qI),}$$

where $\hat{\alpha}^N(Q) := \sum_{k=0}^{N} M_k(\delta)\frac{(Q+\delta I)^k}{k!}$, $\delta > 0$ large enough, and

$$M_k(\delta) := \int_0^\infty t^k e^{-\delta t}\alpha(\mathrm{d}t) \in \mathbb{R}^{1\times m}.$$

Algorithm for fixed point equation

Advantages :

$\longrightarrow$ $\hat{\alpha}^N(Q)$ computable *if the* $M_k(\delta)$*'s,* $k \in \mathbb{N}$*, are computable,*

$\longrightarrow$ Convergence :

Theorem

One has $\hat{\rho}^N(q) \longrightarrow \hat{\rho}(q)$ *as* $N \to \infty$ *for all* $q \geq 0$*. Besides, for* q *large enough :*

$$\left|\hat{\rho}(q) - \hat{\rho}^N(q)\right|_m \leq C \left[\hat{\alpha}(0).\underline{1} - \sum_{k=0}^{N} \frac{|M_k(\delta)|_m}{k!}\delta^k\right]$$

with explicit C*, and* $\hat{\alpha}(0)$ *explicit.*

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Bailout problem

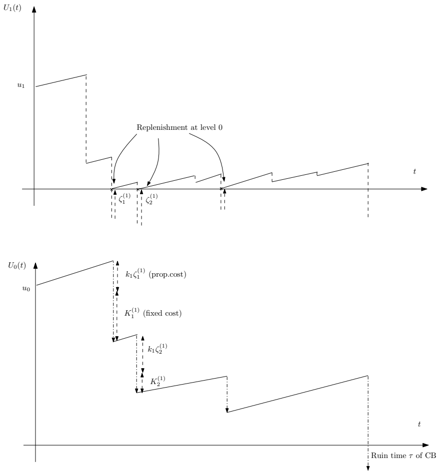

FIGURE: Sample path with proportional and fixed cost.

Bailout problem

Goal : Determine LT of ruin time τ of $\{U_0(t),\ t \geq 0\}$ (*Central Branch*) starting from $u_0 \geq 0$, when claims and interclaims for $\{U_1(t),\ t \geq 0\}$ (*subsidiary*) are *PH* distributed.

Step 1 : Identify dependence structure $\alpha(\mathrm{d}t)$ and matrix R :

$$\begin{aligned} \alpha(\mathrm{d}t) &\sim \text{ ruin time distribution of } \tau_1 \text{ jointly to phase at ruin,} \\ R &\sim \text{ same as claims of } U_1(t) \ + \ \text{independent } PH(\underline{k}, K), \end{aligned}$$

$\longrightarrow$ $\hat{\alpha}(-q)$, $q \in \mathbb{R}_+$, available.

Step 2 : Compute the $M_k(\delta)$'s, $k \in \mathbb{N}$: Ren and Stanford (2012).

Step 3 : Run the algorithm.

Queues and flushes (in progress)

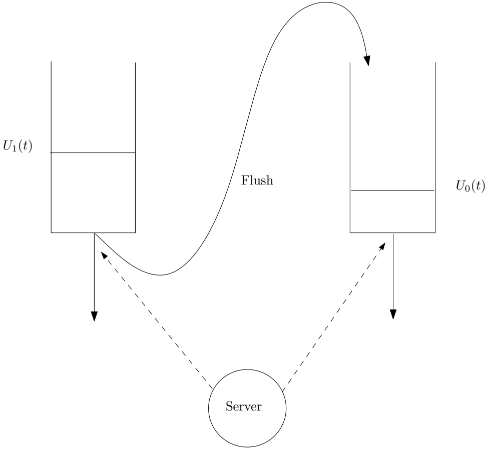

FIGURE: Flush from queue 1 to 0.

Queues and flushes

Fluid queues $\{U_0(t),\ t \geq 0\}$ and $\{U_1(t),\ t \geq 0\}$ fluid queues, fed at constant rate c_0 and c_1.

- $U_1(t)$ served *with priority* over $U_0(t)$, instantaneously, according to *PH* services,
- Content of $U_1(t)$ is occasionally *flushed* into $U_0(t)$ at time according to a Poisson process.

Goal : Determine LT of ruin time τ of $U_0(t)$ = busy period of $U_0(t)$.

Steps : Identify dependence structure $\alpha(\mathrm{d}t)$ and matrix R, and compute the $M_k(\delta)$'s, $k \in \mathbb{N}$.

Thank you !