Modelling Mortality and Discharge of Hospitalized Stroke Patients using a Phase-Type Recovery Model

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Outline

Motivation

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- Quick treatment often decisive in degree of recovery
- Modelling patient recovery LOS is needed to limit cost while ensuring adequate provision of health care resources

Background

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- Cerebral Infarctions occur when there is a clot in a vein. If clot-busting drugs are administered quickly, recovery prospects can be very good.
- Transient Ischemic Attacks (TIAs) are the least severe of all, and are often referred to as 'mini-strokes'.

Relevant Literature on LOS Modelling

- Faddy & McClean (2000) address LOS of geriatric patients.
- Marshall & McClean (2003) introduced idea of conditional PH models for LOS modelling.
- Heterogeneity by such factors as age, type of stroke, etc considered by Marshall & McClean (2004), Faddy & McClean (2000), Harper et al (2012) to explain differences in patient flow characteristics

Summary Statistics for Our Dataset

Table: Summary by Type of Stroke and Mode of Discharge

Discharge Counts			
Mode of Discharge	Haemorrhagic	Infarction	TIA
Death	65	125	13
Nursing Home	5	59	8
Usual Residence	69	432	389
Average Lengths of Stay (days)			
Mode of Discharge	Haemorrhagic	Infarction	TIA
Death	18.3	34.6	37.5
Nursing Home	85.5	83.7	25.8
Usual Residence	51.3	31.9	8.2

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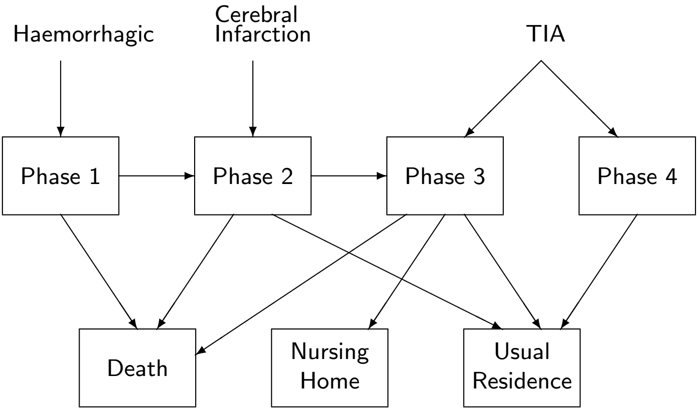
- In contrast, Infarctions are rarely 'severely ill'; for parsimony, we envisaged them as sharing the ‘moderately ill’, and ‘normal recovery’ stages with the Haemorrhagic patients.

Our Phase-type Model for Stroke Recovery (Cont'd)

- Transient Ischemic Attacks (TIAs) are even less severe, and are occasionally never diagnosed. Plots of the data revealed that a hyper-exponential mixture seemed appropriate.
- The (relatively) more severe TIAs shared the 'normal recovery' stage with the foregoing groups, while the really short TIAs had an even shorter mean duration.

The Resulting State Transition Diagram



Parameters Used in our Model

- Transition rates that are independent of age include the mortality rates μ_i, as well as discharge rates ν_i to nursing home and ρ_i to regular residence; $i = 1, 2, 3$.

- Parameters that depend upon patient age x include the probability $p(x)$ that the TIA recovery starts in stage 4, and the transition rate $\lambda_i(x)$ denotes the rate of transition from state i to $i + 1$ where $i = 1, 2$.
- The probability takes the form $p(x) = e^{-exp(\theta_0+\theta_1 x)}$. The transition rate takes the form $\lambda_i(x) = e^{\gamma_i+\beta_i x}$; $i = 1, 2$.

A Phase-type Construct That Sheds Insight

Let $\mathbf{T} = (t_{ij})$ be a 4×4 matrix of transition rates among transient states and $\mathbf{T}_A = (t_{ij})$; $i = 1, 2, 3, 4$; $j = 5, 6, 7$ be a 4×3 matrix of absorption rates to the various discharge modes (death, nursing home, and usual residence, resp.). Given an initial distribution of recovery phases $\boldsymbol{\alpha}$, one finds

$$f_X(x \,|\boldsymbol{\alpha}, \mathbf{T}, \mathbf{T}_A) = \boldsymbol{\alpha}' \exp(\mathbf{T}x)\, \mathbf{T}_A \mathbf{1}_3\,, \;\; x \geq 0\,. \tag{1}$$

The 4×3 matrix $\mathbf{P} = (-\mathbf{T})^{-1}\mathbf{T}_A$ can be interpreted as the probability of absorption into the various discharge modes (death, nursing home, or regular residence), for each of the recovery phases.

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- We then add the Infarction patients to the mix, and re-estimate the final-stage parameters while gaining initial estimates for the 'moderately ill' stage.
- We finally add the Haemorrhagic patients to the mix, and re-estimate all the foregoing parameters as well as for the 'seriously ill' stage.

Parameter Estimates

Parameter	Estimate	Std Error	Z-Stat	p-value
γ_1	6.63570	1.21893	5.44388	0.00000
β_1	-0.03652	0.01631	-2.23902	0.02515
γ_2	-3.06931	1.22697	-2.50153	0.01237
β_2	0.07153	0.01667	4.29057	0.00002
θ_0	-8.66118	1.48644	-5.82680	0.00000
θ_1	0.08801	0.01828	4.81391	0.00000
μ_1	22.10156	4.95434	4.46105	0.00001
μ_2	2.48820	0.37993	6.54912	0.00000
μ_3	1.56162	0.20294	7.69509	0.00000
ν_3	1.27849	0.17391	7.35165	0.00000
ρ_2	11.76860	0.99634	11.81180	0.00000
ρ_3	3.41989	0.38393	8.90762	0.00000
ρ_4	63.92514	4.11394	15.53865	0.00000

Ultimate Destination Percentage by Age and Type of Stroke

Age 65			
	Death	Nursing Home	Usual Residence
Haemorrhagic	38.5	4.0	57.5
Cerebral Infarction	19.4	5.2	75.5
TIA complex	24.9	20.4	54.6
TIA simple	0	0	100.0
Age 85			
	Death	Nursing Home	Usual Residence
Haemorrhagic	52.5	7.3	40.1
Cerebral Infarction	21.9	12.0	66.1
TIA complex	24.9	20.4	54.6
TIA simple	0	0	100.0

Cumulative probability of discharge by type of stroke and destination