Fluid single server queue conflict model with integral constraints

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***Abstract*—Network management systems must often deal with malicious intrusion attempts aimed at obstructing routing policy. Management techniques based on fluid models and discrete controlled random walks require modifications to deal with such intrusion attempts. In this work we propose a generalization of classic single server queue model. This approach is based on conflict control point of view and could be useful for modeling of attacking actions and response behaviour of the system.**

I. Introduction

Areas where one can apply network models are very different – information networks, telecommunications, gas transportation and energy systems, distributed production processes. Information network applications are especially important. Today networks are related to almost all sides of human activity. As a result, security and reliability of information flows directly affect the quality of service, efficiency and overall economic development of entire industries. Reliable operation of the information networks has become vital for day-to-day transactions for the most organizations. That's why the Internet becomes an attractive target for cyber crime. Financially motivated, the crime we see today becomes more distributed, sophisticated and dangerous. Distrubuted Denial of Service (DDoS) attacks has been identified as one of the most serious problem on the Internet. The aim of DDoS attacks is to prevent legitimate users from accessing resourses such as web services. During attack multiple malicious hosts that are recruited by the attacker start flooding attack against server, which cause denial of service to users. In this paper proposes an analytical model based on conflict interaction between attack and defense parties. From this point of view network attacks could be considered as a contest between two players in the other words a game or conflict controlled system. In [1] we proposed the concept of such attacks modeling. In this work we build mathematical model of the fluid single server model in the conflict case. We typically consider a linear, deterministic model. This is the simplest model that captures essential features of the system to be controlled [2]. The most important question is stability: how we can guarantee boundedness of delays for all time? If we use control-oriented approach we face with a control problem: what is the best way to develop a control function or routing and scheduling to obtain the best performance. Networks considered here consist of finite set of nodes, each containing finite buffer. Packets arrive from outside the network to various buffers. One or more servers process packets at a given node, after which a packet either leaves the network or visits another node (Fig. 1).

A general fluid model can be described by the differential equation

$$\dot{q} = Aq + \alpha(t) - Bu(t) + Cv(t) \tag{1}$$

where $q(t)$ is the n-dimensional vector of queue process, $\alpha(t) : R^n \to R$ denotes the packet arrival rate, $u(t)$ - the routing policy, $v(t)$ - attack policy. Matrix A represents an inner network construction (how different queues influence each other), matrix B - routing process and C describes attack process. The controlled differential equation (1) can be viewed as a state space model, as frequently used in control applications, with controls $u(t)$, $v(t)$ and independent function $\alpha(t)$. We assume that all mentioned function are integrable. Consider model (1) as a starting point to investigate conflict problem of interaction between a routing policy and an attacker. If a successful design is obtained, we will generalize this solution for the case of Markovian processes [3], [4]. The problem of finding of control $u(t)$ can be described as follows.

We choose a control function $u(t)$ in purpose of minimizing vector $q(t)$ under several special conditions (e.g. for minimal time) and any admissible functions $v(t)$,$\alpha(t)$. Let us fix some function $u(t)$, then solution of (1) can be found using following formula:

$$\begin{gathered} q(t) = e^{A(t-t_0)}q(t_0) - \int_{t_0}^{t} e^{A(t-s)}u(s)ds + \\ \int_{t_0}^{t} e^{A(t-s)}\alpha(s)ds + \int_{t_0}^{t} e^{A(t-s)}v(s)ds \end{gathered} \tag{2}$$

If we assume, that $A = 0$, $B = I$, $C = 0$ and , $\alpha(t) = \alpha$ (2) could be rewriting as:

$$q(t) = q(t_0) + \alpha t - \int_{t_0}^{t} u(s)ds \tag{3}$$

This is well-known single server queue fluid model if we substitute $z(t) = -\int_{t_0}^{t} u(s)ds$.

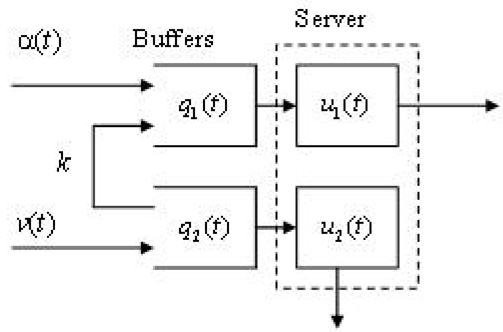

Figure 1. Network system

II. SINGLE SERVER QUEUE FLUID MODEL WITH INTEGRAL CONSTRAINTS

Let us consider (3). In the differential form:

$$\dot{q} = \alpha - u(t) \tag{4}$$

where $q_0 \in R^n_+$,$u(t) \leq \mu$ and policy $u(t)$ is, generally speaking, measurable function. Let us expand this model by following assumptions :

- Arrival rate of packets changes with time. Denote $\alpha(t)$ as an arrival rate at the time t . Suppose that $\alpha(t) \leq \alpha^{max}$.
- Buffer is finite, so $0 \leq q(t) \leq q^{max}$.
- $u(t) \leq \mu$

We will consider two constraint types for function$\alpha(t)$.

1) Geometric constraint: $0 \leq \alpha(t) \leq \alpha^{max}$.
2) Integral constraint: $\int_{t_0}^{\infty} \alpha(s)ds \leq \alpha_{int}$.

Constraint 1 means boundedness of input channel. Constraint 2 describes situation that during time period $[0, \infty]$ total amount of traffic is limited by the constant value.

The main question is: how we can achieve satisfying condition $q(t) = 0$ for all $t \geq T$. As we know from general theory the single server queue is stable ($q(t) = 0$ could be obtained) when $\alpha \leq \mu$.

Claim 1: Let us consider the equation (4) with constraints 1, 2. If $\alpha^{max} < \mu$ then for any given q_0, $\alpha(\cdot)$ there exists minimal moment of time $T < \infty$, such that $q(T) = 0$. If $\alpha^{max} \geq \mu$, than for $q(t_0) \leq q^{max} - \frac{(\alpha^{nax}-\mu)\alpha_{int}}{\alpha^{max}}$, there exists minimal moment of time $T < \infty$, such that $q(T) = 0$.

Proof. If $q(t_0) = 0, T = t_0$. For $q(t_0) > 0$ consider a solution:

$$q(t) = q(t_0) + \int_{t_0}^{t} \alpha(s)ds - \int_{t_0}^{t} u(s)ds$$

For non-idling policy

$$u(t) = \begin{cases} -\mu & q(t) > 0 \\ 0 & q(t) = 0 \end{cases}$$

and

$$q(t) = q(t_0) + \int_{t_0}^{t} \alpha(s)ds - t\mu$$

Now we use constraints 1, 2 and obtain:

$$\int_{t_0}^{t} \alpha(s)ds \leq \alpha^{max}t,\ t \leq \frac{\alpha_{int}}{\alpha^{max}}$$

$$\int_{t_0}^{t} \alpha(s)ds \leq \alpha_{int},\ t > \frac{\alpha_{int}}{\alpha^{max}}$$

It is clear that for $\alpha^{max} < \mu$ and moment of time $T \leq \frac{q(t_0)}{\mu-\alpha^{max}}$: $q(T) = 0$.

If $\alpha^{max} > \mu$, then we have two possibilities. First, let $t \leq \frac{\alpha_{int}}{\alpha^{max}}$. According to previous statement

$$q(t) \leq q(t_0) + \alpha^{max}t - t\mu$$

$$q(t) \leq q(t_0) + \frac{(\alpha^{max} - \mu)\alpha_{int}}{\alpha^{max}}$$

taking into account condition $q(t_0) \leq q^{max} - \frac{(\alpha^{nax}-\mu)\alpha_{int}}{\alpha^{max}}$ we obtain:

$$q(t) \leq q^{max}$$

for all $t \leq \frac{\alpha_{int}}{\alpha^{max}}$.

If $t > \frac{\alpha_{int}}{\alpha^{max}}$, then

$$q(t) \leq q(t_0) + \alpha_{int} - t\mu$$

$$q(t) \leq q(t_0) + \alpha_{int} - \frac{\alpha_{int}}{\alpha^{max}}\mu \leq q^{max}$$

For moment of time $T \leq \frac{q(t_0)+\alpha_{int}}{\mu}$ follows inequality holds $q(T) \leq 0$.

For large α_{int} and $\alpha^{max} < \mu$ we have obtained the same result as for single server queue. Integral constraints allow situation when $\alpha^{max} \geq \mu$.

III. CONFLICT MODEL

The single server queue is a useful model for a dynamic investigation of very different systems. In this chapter we introduce an extension of the fluid model [1]in the case of conflict process [5]. Let us consider dynamic system defined for an initial condition $q(t_0) = (q_1(t_0), q_2(t_0))$ by the system of linear equations:

$$\begin{aligned} \dot{q}_1(t) = \alpha(t) + kq_2(t) - u_1(t) \\ \dot{q}_2(t) = v(t) - u_2(t) \end{aligned} \tag{5}$$

Let us make some definitions and assumptions. Phase state is described by the phase vector $q(t) = (q_1(t), q_2(t)) \in R^2$, where $q_1(t) \in R_+$ is the queue length at time t. The queue length is subject to the linear phase constraint $0 \leq q_1(t) \leq q_1^{max}$for all $t \geq 0$. Parameter $q_2(t) \in R_+$ denote the strength function of the attack that associated with the attacker player which trying to overwhelm network (or maximize $q_1(t)$ in other words) using control parameter $v(t)$, $0 \leq v(t) \leq v^{max}$. The strength function here means the function of the factors that can cause attack party win or lose. For example, for the simple flooding attack the factors can be the volume of traffic per second or the number of hosts that participate in the attack. Similar model was introduced in [8].By choosing $v(t)$ at time t attacker sets attack power $q_2(t)$ which is subject to the linear phase constraint $0 \leq q_2(t) \leq q_2^{max}$ for all $t \geq 0$.

In model (5) we assume that:

- $v(t)$, $u(t)$, $\alpha(t)$ are continuous differentiable function of time;
- attack rate $q_2(t)$ is proportional to strength of the attack $v(t)$ and defense strength $u_2(t)$;
- attack packets inflict proportional damage to queue $q_1(t)$ with a coefficient k. ;

The other player – defender – has a control vector $u(t) = (u_1(t), u_2(t))$. He divides his control resources between two directions: $u_1(t)$ - for service of the arrived packets, $u_2(t)$ - for counteraction of the attacker activity. Control parameter $u(t)$ is subject of following constraints:

$$u_1(t) \geq 0,\ u_2(t) \geq 0,\ u_1(t) + u_2(t) \leq \mu$$

Suppose that defender has information about $\alpha(t)$, $v(t)$ and $q(t)$ at the moment of time t. Function $\alpha() : R \rightarrow R^2$ describes packets arrival service at time t. Suppose the following assumptions hold:

- $0 \leq \alpha(t) \leq \alpha_{max}$ for all $t \geq 0$;
- $\alpha(t)$ - continuous, integrable function.
- $\int_{t_0}^{\infty} \alpha(t)dt \leq \alpha_{int}$

Integral constraints are useful for modeling the situation when users have limited amount of requests during a long period of time. For example if server turns off for a night there is period of working $[0, T]$. All sessions $[T_i, T_{i+1}]$ can be considered as separated periods of activity. For every $[T_i, T_{i+1}]$ users choose function $\alpha(t)$ that defines requests distribution over time.

This model is stabilizable if $\mu > v^{max}$ for any initial position such that $q(t_0) \leq q^{max} - \frac{(\alpha^{nax}-\mu)\alpha_{int}}{\alpha^{max}}$. Time optimal strategy given by following formula $u(t) = (u_1(t), 0)$, where $u_1(t) = -\mu$ if $q(t) > 0$ and 0 otherwise. To solve (10) we should find an admissible strategy $u(t, q(t), v(t))$ and moment of time T such that $q(T) = 0$. Denote this strategy $u(t, q(t), v(t))$ as the solution of game (5). Note that this result must be achieved over all an admissible functions $v(t)$. Let us write equations (5) in more general form. Denote $A = \begin{pmatrix} 0 & k \\ 0 & 0 \end{pmatrix}$, then

$$\dot{q}(t) = Aq(t) + \begin{pmatrix} \alpha(t) \\ v(t) \end{pmatrix} - \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \tag{6}$$

Control sets U and V defined as follows $U = \{u \in R_+^2 : u_1 + u_2 \leq \mu\}$, $V = \{v \in R_+^2 :\ v_1 = 0,\ v_2 \leq \nu\}$. We solve this problem using the idea of the first direct method of Pontryagin [7], [6].

Theorem 1: Consider the conflict controlled system (6). If $\mu \geq v^{max}$ and $q_1(t_0) + \frac{k(q_2(t_0))^2}{\mu - \nu} + \alpha_{int} \leq q_1^{max}$, then we can construct the solution $u(t)$ and moment of time T, such that $q(t) = 0$ for $t \geq T$.

Proof. Without loss of generality $q(0) \neq (0, 0)$. Let us define moments of time

$$T_1 = min\{t > 0 :\ q_1(t) = 0\}$$

$$T_2 = min\{t > 0 :\ q_2(t) = 0\}$$

Then, define

$$u(t) = \begin{cases} (0, -\mu) & t \in [0, T_2] \\ (\mu - v(t), v(t)) & t \in [T_2, T_1] \end{cases} \tag{7}$$

First we show then the strategy $u(t)$ is admissible. It is clear, that the strategy $u(t)$ is defined for all $t \geq 0$ and $u_1(t) + u_2(t) = \mu$. Since condition $\mu \geq \nu$ holds, it follows that $u_1(t) \geq 0$, $u_2(t) \geq 0$ for all $t \geq 0$. Substituting (7) into (6) we obtain

For $t \in [0, T_2]$

$$\dot{q}_1(t) = \alpha(t) + kq_2(t)$$

$$q_2(t) = v(t) - \mu$$

For $t \in [T_2, T_1]$

$$\dot{q}_1(t) = v(t) + \alpha(t) - \mu$$

$$q_2(t) = 0$$

Denote $\mu - v^{max}$ as ε, then

$$\dot{q}_2(t) \leq -\varepsilon$$

$$q_2(t) \leq q_2(0) - \varepsilon t$$

From the last statement follows that $q_2(t) = 0$ for moment of time $t \leq T_2^* = \frac{q_2(0)}{\varepsilon}$. Subsequently, we obtain that $T_2 \leq T_2^*$.

Consider $q_1(t)$:

$$q_1(t) = q_1(0) + \int_0^t kq_2(\tau)d\tau + \int_0^t \alpha(\tau)d\tau$$

$$q_1(t) \leq q_1(0) + kq_2(0)t - k\varepsilon\frac{t^2}{2} + \alpha_{int}$$

$$q_1(T_2) \leq q_1(0) + \frac{k(q_2(0))^2}{2\varepsilon} + \alpha_{int}$$

$$q_2(T_2) \leq q_1^{max}$$

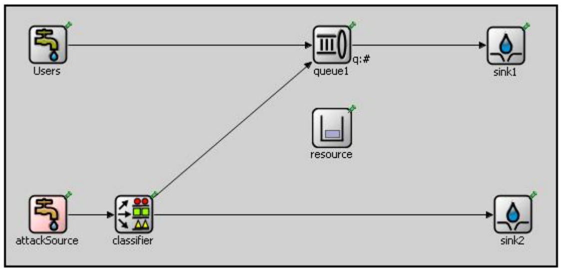

Figure 2.

Last statement obtained by using condition 2. In the moment of time T_2 control switches. Since $q_2(t) = 0$, $t \geq T_2$, we can consider only $q_1(t)$

$$q_1(t) = q_1(T_2) - \int_{T_2}^{t} (v(\tau) + \alpha(\tau) - \mu) d\tau$$

$$q_1(t) \leq q_1(T_2) - \varepsilon(t - T_2) - \alpha_{int}$$

So $q_1(t) = 0$, for $t \leq \frac{q_1(T_2)+\varepsilon T_2 - \alpha_{int}}{\varepsilon}$. Therefore, we have following estimation for the time T_1:

$$T_1 \leq \frac{q_1(0)}{\varepsilon} + \frac{k(q_2(0))^2}{2\varepsilon^2} + \frac{\alpha_{int}(1-\varepsilon)}{\varepsilon} + \frac{q_2(0)}{\varepsilon}$$

This completes the proof of Theorem.

IV. Experimental modeling

Let us illustrate obtained result on the example. Consider the following model:

$$\dot{q}_1(t) = \alpha(t) + kq_2(t) - u_1(t)$$

$$\dot{q}_2(t) = v - u_2(t)$$

This model was implemented in discrete event network simulation framework OMNeT++ . It has a generic architecture, so it can be used for modeling of wired and wireless communication networks protocol modeling modeling of queueing networks modeling of multiprocessors and other distributed hardware systems validating of hardware architectures evaluating performance aspects of complex software systems. OMNeT++ itself is not a simulator of anything concrete, but it rather provides infrastructure and tools for writing simulations. One of the fundamental ingredients of this infrastructure is a component architecture for simulation models. Models are assembled from reusable components termed modules. Modules can be connected with each other via gates (other systems would call them ports), and combined to form compound modules. Modules communicate through message passing, where messages may carry arbitrary data structures. Modules can may messages along predefined paths via gates and connections, or directly to their destination; the latter is useful for wireless simulations, for example. Modules may have parameters, which can be used to customize module behaviour, and/or to parameterize the model's topology. Graphical, animating user interfaces are highly useful for demonstration and debugging purposes. Consider the following model OMNeT++ (Figure 2.).

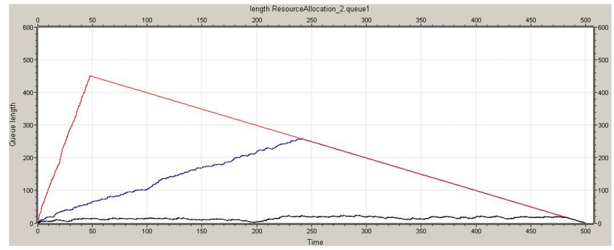

Figure 3.

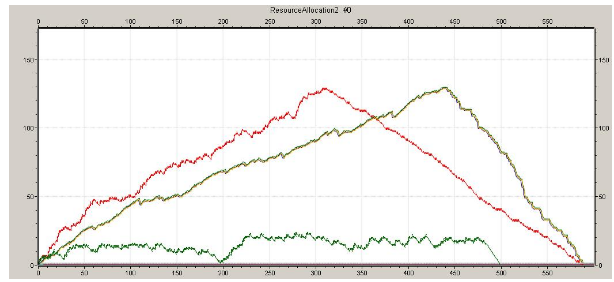

Figure 4.

Users generate traffic of requests that have to be served by server. Server consists of queue and resource pool. Illustrate system dynamic by examples. Simulation parameters given by table, queue dynamic given by Figure 3.

Example 1:

	α^{max}	α^{int}	μ	color	$T = \frac{\alpha^{max}}{\mu}$
1	2	500	1	blue	500
2	10	500	1	red	500
3	1	500	1	black	500

Attack source present attack party. It generates traffic under assumptions made in section 3. Classifier is a part of system defense. During attack phase clasifier cut off part of attack traffic. This situation described by coeficient k. On figure 4 we can see dynamic of phase variable $q_1(t)$, during attack.

Example 2:

	α^{max}	α^{int}	μ	v^{max}	k	color
1	1	300	1	0.1	0.5	green
2	1	300	1	1	0.5	yellow
3	1	300	1	1	1	red

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