Partial stochastic characterization of timed runs over DBM domains

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PMCCS-9
September 18, 2009 - Eger, Hungary

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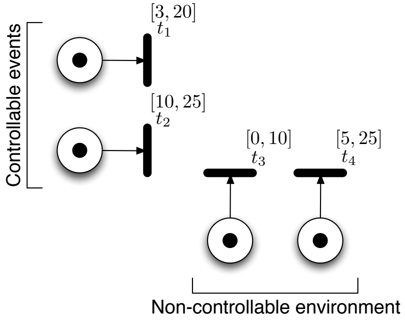
   - Timing boundaries enlargement
   - Partial stochastic characterization of timings

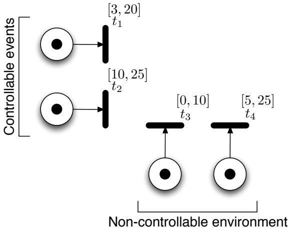
The addressed problem: an intuition

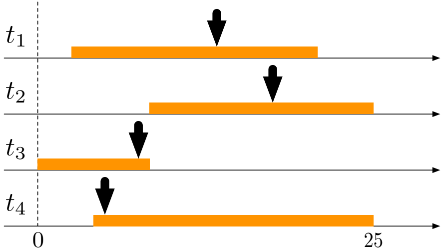
- Continuous-Time Discrete-Events Model
  - non-deterministic timings;
  - **controllable** timings are bounded within continuous intervals;
  - **non-controllable** timings are chosen by the system within a predictable range, following a given probability distribution.
  - (input/output transitions, actions/endogenous events)

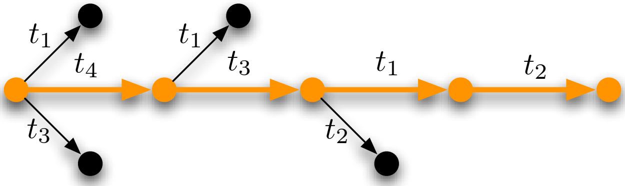


The addressed problem: an intuition

- The system can execute along different firing sequences (symbolic runs);
  - the actual sequence is determined by values assumed by timers.

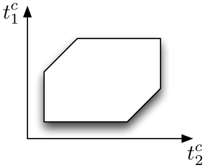




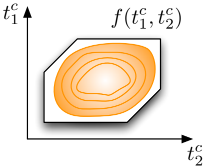


The addressed problem: an intuition

- **Problem**: force the system to run along a selected sequence.
    - controllable timers can be assigned arbitrary values;
    - success still depends upon values of non-controllable timers.
- The problem has a qualitative and a quantitative aspect:

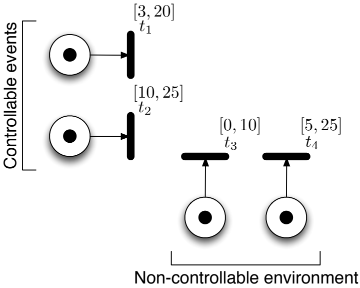


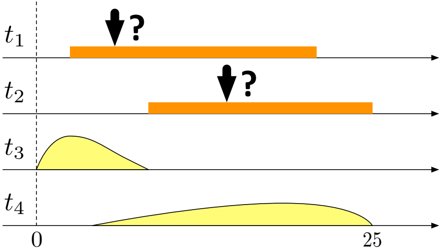
- identification of the range of valuations for controllable timers that can let the system run along the selected sequence (**qualitative** problem);



- evaluation of the success probability for every choice of controllable timers (**quantitative** problem).

An introductory example





- 4 concurrent transitions;
- t_1,t_2: controllable transitions;
- t_3,t_4: non-controllable transitions;

- **Problem**: select values for t_1 and t_4 so as to make possible/maximize the probability to execute the sequence $\rho = t_3, t_1, t_2, t_4$.

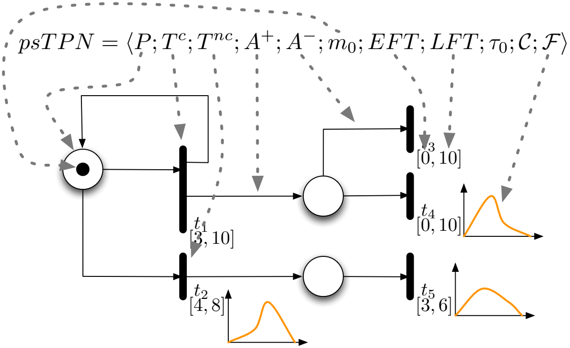
Contribution

- **partially** stochastic Time Petri Nets
  - combines non-deterministic selection of controllable timers and stochastic sampling of non-controllable timers.
- evaluation of the **execution probability** of any firing sequence:
  - **support**: set of controllable choices that can let the system execute along the sequence;
  - **function**: distribution of the success probability as a function of values given to controllable timers.

Related work

- **Real-Time test case sensitization**
  - L. Carnevali, L. Sassoli, E. Vicario: ETFA '07
  - qualitative approach: all timers are non-deterministic.
  - application in testing of real-time software (Linux RTAI).
- **stochastic Time Petri Nets**
  - G. Bucci, R. Piovosi, L. Sassoli, E. Vicario: QEST '05
  - L. Carnevali, L. Sassoli, E. Vicario: Trans. on Software Engineering, September 2009.
  - quantitative evaluation: all timers are stochastic.
- **Test case execution optimization on Timed Automata**
  - M. Jurdińsky, D. Peled, H. Qu: FATES '05
  - N. Wolowick, P. D'Argenio, H. Qu: ICST '09
  - non-controllable timers are uniformly distributed.

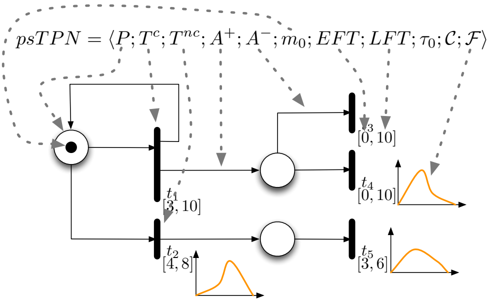
partially stochastic Time Petri Nets: Syntax



- T partitioned: T^c controllable, T^{nc} non-controllable;
- $\mathscr{F} : T^{nc} \longrightarrow \mathbb{F}$ associates each non-controllable transition with a static probability distribution $F_t()$ supported in $[EFT(t), LFT(t)]$:

$$F_t(x) = \int_0^x f_t(y)dy$$

partially stochastic Time Petri Nets: Semantics

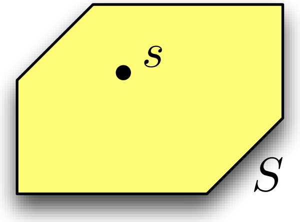


- Tokens move as in Petri Nets (logical locations);
- each transition *t* has an Earliest and a Latest Firing Time ($EFT(t)$ and $LFT(t)$), and an initial time to fire $\tau_0(t)$.
  - *t* cannot fire before it has been enabled with continuity for $EFT(t)$;
  - neither it can let time advance without firing after it has been enabled with continuity for LFT(t);
  - firings occur in zero-time.

partially stochastic Time Petri Nets: Analysis

- **state** s = marking + valuation of transitions times-to-fire
- **state class** S = marking + continuous set of times-to-fire
  - timers within the same state class range in a **Difference Bound Matrix** (DBM) zone.

$$\tau_i - \tau_j \leq b_{ij}$$



- **Remark**: every state (class) may jointly enable controllable and non-controllable transitions, thus combining **stochastic** and **non-deterministic** behavior.

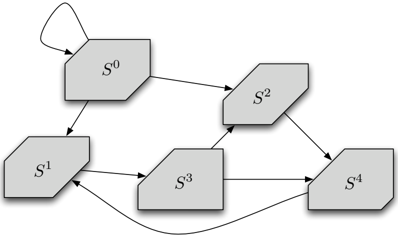
State class graph enumeration

- **AE reachability relation** between state classes:

Definition: AE reachability relation

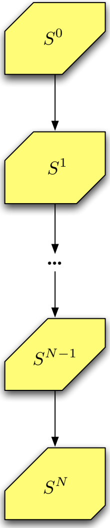
Given two state classes S and S' we say that S' is a successor of S through t_0 iff S' contains all and only the states that are reachable from some state collected in S through some feasible firing of t_0.

- Enumeration $\longrightarrow$ Timed Transition System (**state class graph**);
- DBM form is closed wrt successor evaluation;
- **symbolic runs** are paths in the state class graph.



Domain of timings along a symbolic run

- Consider a symbolic run ρ starting from class S^0, terminating in S^N;
- t_i^n is the instance of transition t_i enabled along ρ in class S^n;
  - associated to an **absolute virtual** firing time τ_i^n;
- absolute firing times feasible for ρ satisfy three kinds of constraints:
  1. model constraints;
  2. disabling constraints;
  3. sequence constraints.

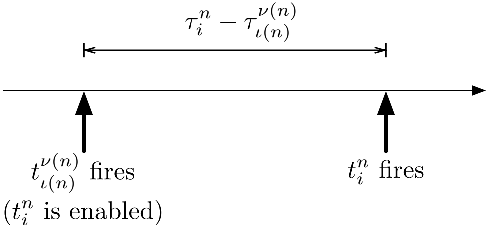


Domain of timings along a symbolic run

1 Model constraints

- time elapsed between enabling and firing of each transition t_i^n fired along ρ must range within its static firing interval:

$$EFT(t_i) \leq \tau_i^n - \tau_{\iota(n)}^{\nu(n)} \leq LFT(t_i) \text{ where } t_{\iota(n)}^{\nu(n)} \text{ enables } t_i^n$$

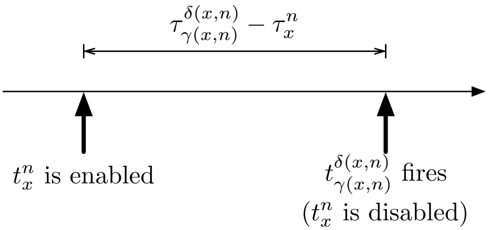


Domain of timings along a symbolic run

2 Disabling constraints

- if transition t_x^n is enabled but not fired along ρ, its absolute firing time must be greater than the one of its disabling transition $t_{\gamma(x,n)}^{\delta(x,n)}$:

$$\tau_x^n \geq \tau_{\gamma(x,n)}^{\delta(x,n)} \text{ where } t_{\gamma(x,n)}^{\delta(x,n)} \text{ disables } t_x^n$$

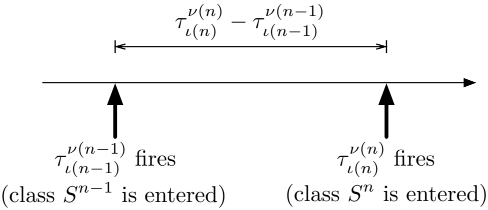


Domain of timings along a symbolic run

3 Sequence constraints

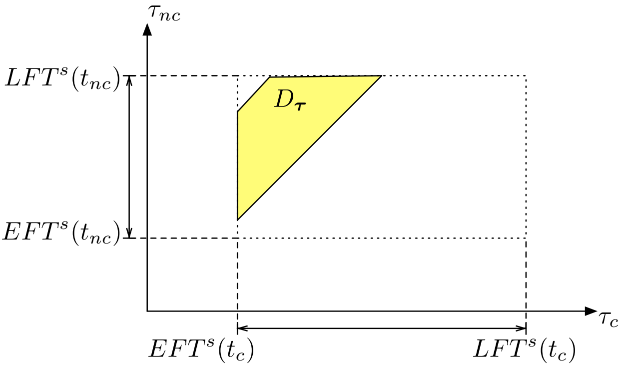
- transitions must fire in the expected sequence:

$$\tau_{\iota(n+1)}^{\nu(n+1)} \geq \tau_{\iota(n)}^{\nu(n)} \forall n \in [0, N-1]$$

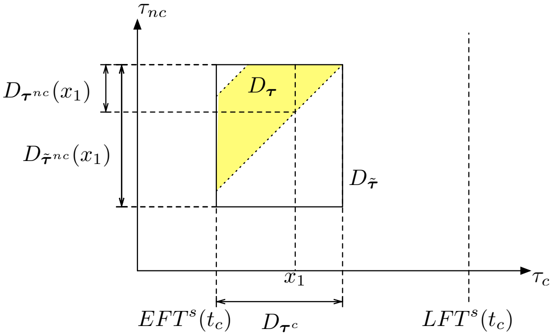


Domain of timings along a symbolic run

- timers of transitions enabled along ρ are encoded in two vectors τ^c and τ^{nc};
- the set of valuations $(\mathbf{x}, \mathbf{y})$ of timers (τ^c, τ^{nc}) that are feasible for ρ is a DBM domain D_τ:



The problem of domain enlargement



- Non-controllable timers can take values outside D_τ;
  - must be taken into account to evaluate the probability of successful execution;
- **enlarged** domain $D_{\tilde{\tau}}$ includes divergent behaviors:
  - controllable timers conform to D_τ;
  - non-controllable timers satisfy model constraints.

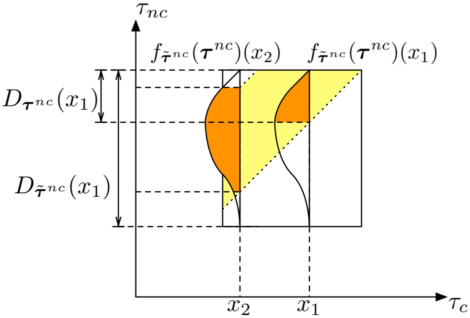
Main result: distribution of the probability of successful execution

- **family of functions** $f_{\tilde{\tau}^{nc}}(\tau^{nc})(\mathbf{x})$:
  - for each selection $\mathbf{x}$ of controllable timers, probability density function of non-controllable timers.
  - for a valuation $\mathbf{x} = \mathbf{x}_1$ of controllable timers, the integral of function $f_{\tilde{\tau}^{nc}}(\tau^c)(\mathbf{x}_1)$ over domain $D_{\tau^{nc}}(\mathbf{x}_1)$ represents the probability to execute ρ under the assumption of the choice $\mathbf{x}_1$ on controllable timers.

$$f_{\tilde{\tau}^{nc}}(\tau^{nc})(\mathbf{x}) = \prod_{\substack{t_i^n \in A^{nc} \\ t_{\iota(n)}^{\nu(n)} \in A^{nc}}} f_{t_i}(y_i^n - y_{\iota(n)}^{\nu(n)}) \quad \cdot \prod_{\substack{t_i^n \in A^{nc} \\ t_{\iota(n)}^{\nu(n)} \in A^c \cup \{t_*\}}} f_{t_i}(y_i^n - x_{\iota(n)}^{\nu(n)})$$

Distribution of the probability of successful execution

- The integral of the whole family of function over $D_{\tau^{nc}}(\mathbf{x})$ defines a new function $p(\mathbf{x})$;
  - $p(\mathbf{x})$ associates each valuation of controllable timers with the execution probability of ρ:



$$p(\mathbf{x}) = \int_{D_{\tau^{nc}}(\mathbf{x})} f_{\tilde{\tau}^{nc}}(\tau^{nc})(\mathbf{x}) \quad d(\tau^{nc}) = Prob\{(\mathbf{x},\mathbf{y}) \in D_\tau \mid \tau^c = \mathbf{x}\}$$

Conclusions

- we considered a **probabilistic extension** of Time Petri Nets;
- we introduced a partial stochastic characterization of timed runs based on the definition of **controllable** and **non-controllable** timers;
- we identified a measure of probability of successful execution of a run as a function of non-deterministic (controllable) variables.

Ongoing work

- **optimization** of $p(\mathbf{x})$ to maximize the execution probability;
- bring to application in (real) **real-time testing** (test case sensitization).