Performance Analysis of Scheduling and Interference Coordination Policies for OFDMA Networks *

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Abstract — In orthogonal frequency division multiple access systems there is an intimate relationship between the packet scheduler and the inter-cell interference coordination (ICIC) functionalities: they determine the set of frequency channels (sub-carriers) that are used to carry the packets of in-progress sessions. In this paper we build on previous work - in which we compared the so called *random* and *coordinated* ICIC policies - and analyze three packet scheduling methods. The performance measures of interest are the session blocking probabilities and the overall throughput. We find that the performance of the so-called *Fifty-Fifty* and *What-It-Wants* scheduling policies is somewhat improved by coordinated sub-carrier allocation, especially in poor signal-to-noise-and-interference situations and at medium traffic load values. The performance of the *All-Or-Nothing* scheduler is practically insensitive to the choice of the sub-carrier allocation policy.
Keywords — Orthogonal Frequency Division Multiple Access, Radio Resource Management, Interference Coordination, Scheduling

1 Introduction

The 3^{rd} Generation Partnership Project (3GPP) has selected orthogonal frequency division multiple access (OFDMA) as the radio access scheme for the evolving universal terrestrial radio access (E-UTRA). Packet scheduling (PSC) and inter-cell interference coordination (ICIC) are important radio resource management (RRM) techniques that together determine the set of OFDMA resource blocks (essentially the sub-carriers) that are taken into use when a packet is scheduled for transmission over the radio interface [2], [3]. In broad terms, PSC is responsible for determining the session(s) that can send a packet during a scheduling interval and the number of sub-carriers that the session may use. The number of the assigned sub-carriers has a direct impact on the instantaneous bit-rate and thereby can be seen as part of the rate control mechanism. The ICIC function, in turn, is concerned with allocating the particular sub-carriers to the session taking into account the instantaneous channel conditions and

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the ICIC policy. Such ICIC policy may coordinate which sub-carriers should be taken into use by the schedulers in neighbor cells.

The impact of these two RRM functions on the session-wise and overall throughput has been for long recognized by the standardization and research communities. Sections 11.2.4 and 11.2.5 of [2] and Chapter 6.12 of [3] describe the roles of the PSC and ICIC functions and discusses their relation. From a performance analysis perspective, Letaief *et al.* developed a model that jointly optimizes the bit and power allocation in OFDMA schedulers [10] and [11]. ICIC has been the topic of research for long (for a classical overview paper, see [5]).

In order to get a basic understanding of the behavior of various RRM algorithms in cellular networks, two-cell models ([6]-[9]) as well as models that explicitly take advantage of the observation that for a specific mobile station, often there is a single dominant interferer ([19]) have been often used in the past. Two-cell models are useful, because they allow to get an insight into the exact operation of RRM algorithms.

Regarding multi-cell (and specifically OFDMA) systems, [12] analyzed a reuse partitioning scheme without modeling the behavior of the packet scheduler. The paper by Liu and Li proposed a so called "Radio Network Controller algorithm" that determines the set of allowed resources in each base station under its control, while the "Base Station algorithm" schedules packets for transmission [19] (see also Chapter 8 of [20]). These works demonstrate that already with a single dominant interfering neighbor cell, the total throughput increases when an appropriate ICIC policy is employed by the packet scheduler.

The contribution of the current paper is that we (1) explicitly take into account that traffic is elastic and (2) propose a flexible model to capture the behavior of a wide range of schedulers under two different ICIC policies. With regards to (1) we allow the bitrates of the sessions to fluctuate between the associated minimum and maximum rates. This model allows the maximum rate to be large so that the behavior of TCP-like greedy sources can be captured. When the session is slowed down (with respect to its peak rate requirement), its holding time increases proportionally (similarly to what has been analyzed in a CDMA environment in [14] and [16]). Regarding (2), we introduce the notion of the scheduler *policy vector* that specifies the probability that a session is granted a certain amount of sub-carriers when there are competing sessions in the system. We add this rather general scheduler model to the interference coordination model described in [18] and analyze the model in the following steps. First, we derive the distribution of the number of colliding and collision (i.e. co-channel interference) free sub-carriers. We then employ the theory of the *effective* signal-to-noise-and-interference (SINR) [24] that helps determine the packet error rate and thereby the session-wise (useful) throughput given that that number of in-progress sessions is known. Finally, assuming that sessions arrive according to a Poisson process and stay in the system for a throughput dependent amount of time, we derive the performance measures of interest, which are the session blocking probabilities and the average overall throughput. This performance analysis gives insight into the potential gains that inter-cell interference coordination can give when employing different packet scheduling policies.

The paper is organized as follows. In the next section, we describe the scheduling and ICIC policies that we study and introduce the *policy vector* as a convenient tool to characterize these policies. Next, in Section 3 we state the performance analysis objective in terms of the input parameters and the performance measures of interest. The solution is summarized in a sequence of steps (as described above). Section 4 discusses numerical results. We highlight our findings in Section 5.

2 Scheduling and Inter-cell Interference Coordination Policies

We consider an OFDMA cell that comprises S orthogonal frequency channels (sub-carriers). The number of in-progress sessions is denoted by i and represents the state of the system. When the system is in state i, the scheduler determines the number of sub-carriers that are assigned to each session. For a particular session under study, this implies that the session is assigned s number of sub-carriers with probability P_s; $\sum_0^S P_s = 1$. We refer to the mechanism that (in each system state) establishes P_s as the *scheduling policy*. The *scheduling policy vector* is a vector of dimension $(S+1)$ whose s^{th} element specifies the probability that the session under study (i.e. *any* session) is allocated s channels, $s = 0 \dots S$. (We note that the indexing of the $(S+1)$ elements of the policy vector runs from 0 to S.) In the following subsections we describe three such scheduling polices.

Throughout we assume that the sessions belong to the same service class that is characterized by a peak rate requirement $\hat{R}$ and a maximum *slowdown factor* $\hat{a} \geq 1$. The minimum accepted (guaranteed) bit rate for a session is $R_{min} = \hat{R}/\hat{a}$. Also, when a session is granted s number of frequency channels, its ideal bit-rate (assuming a given and fixed modulation and coding scheme, MCS) and assuming zero packet error/loss rate ($PER = 0$) is denoted by R_s. When $\hat{R}$ is set to R_S (that is the peak bit-rate requirement is the bit-rate that is provided when all resources are assigned to a single session), we say that the session is *greedy*. We will also use the operator $\mathcal{S}(R)$

that returns the number of required channels in order for the session to experience R bit-rate (again assuming $PER = 0$). That is, when a session is admitted into the system, the number of allocated channels s (in the long term) must fulfil: $R_{min} \leq R_s \leq \hat{R}$. This implies that we assume that an admission control procedure operates in the system such that the maximum number of simultaneously admitted sessions remain under $\hat{I} \triangleq \lfloor \frac{S}{\mathcal{S}(\hat{R}/\hat{a})} \rfloor$. We say that state i is an under-loaded, critically loaded or overloaded state if $\mathcal{S}(i \cdot \hat{R})$ is less than, equal to or greater than S respectively.

2.1 The What-It-Wants Scheduling Policy

The What-It-Wants scheduling policy attempts to grant $\mathcal{S}(\hat{R})$ channels to the sessions as long as $i \cdot \mathcal{S}(\hat{R}) \leq S; \quad i > 0$. Otherwise, in overloaded states, it grants either $\lfloor \frac{S}{i} \rfloor$ or $\lceil \frac{S}{i} \rceil$ channels. Specifically, the What-It-Wants scheduling policy is defined by the following Policy Vector. If $i \cdot \mathcal{S}(\hat{R}) \leq S$:

$$\vec{P}_{WIW}(s) = \begin{cases} 1 & \text{if } s = \mathcal{S}(\hat{R}) \quad \text{(granting peak rate with probability 1),} \\ \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For overloaded states, we need to distinguish between two cases. If $\frac{S}{i}$ is an integer number, then:

$$\vec{P}_{WIW}(s) = \begin{cases} 1 & \text{if } s = \frac{S}{i} \quad \text{(granting an equal share with probability 1),} \\ \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

When $\frac{S}{i}$ is not an integer number, the following relations must hold. The scheduler grants $\lfloor \frac{S}{i} \rfloor$ channels with probability P_1 and $\lceil \frac{S}{i} \rceil$ number of channels with probability $1 - P_1$. Clearly:

$$P_1 \cdot \left\lfloor \frac{S}{i} \right\rfloor + (1 - P_1) \cdot \left\lceil \frac{S}{i} \right\rceil = \frac{S}{i}; \quad P_1 = \frac{\lceil \frac{S}{i} \rceil - \frac{S}{i}}{\lceil \frac{S}{i} \rceil - \lfloor \frac{S}{i} \rfloor}.$$

Thus, the policy vector in this case takes the form:

$$\vec{P}_{WIW}(s) = \begin{cases} P_1 & \text{if } s = \lfloor \frac{S}{i} \rfloor \quad \text{(granting a bit less than the exact equal share with probability } P_1\text{),} \\ \\ 1 - P_1 & \text{if } s = \lceil \frac{S}{i} \rceil \quad \text{(granting a bit more than the exact equal share with probabiliy } 1 - P_1\text{),} \\ \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

2.2 The All-Or-Nothing Scheduling Policy

In the All-Or-Nothing scheduling policy all resources are assigned to the scheduled session. This type of scheduling is employed in High Speed Downlink Packet Access (HSDPA) systems when code multiplexing is not used [21]. Thus, a session with peak rate requirement $\hat{R}$ would need to be scheduled with probability $\mathcal{S}(\hat{R})/S$ in order for it to receive its peak rate. However, when there are $i \geq 1$ on-going sessions, any given session cannot get scheduled with higher probability than $1/i$. That is, in the All-Or-Nothing scheduling policy, in system state i, a session gets scheduled with probability $P_2 = Min[\mathcal{S}(\hat{R})/S, 1/i]$. The scheduling policy takes the following form:

$$\vec{P}_{AoN}(s) = \begin{cases} P_2 & \text{if } \ s = S + 1 \quad \text{(all channels are granted),} \\ \\ 1 - P_2 & \text{if } \ s = 0 \quad \text{(no channels are granted)} \\ \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

2.3 The Fifty-Fifty Scheduling Policy

The Fifty-Fifty scheduling policy can be seen as a policy in between the What-It-Wants and All-Or-Nothing policies. When there are i sessions in the system, the scheduler divides the resources (almost) equally between the competing sessions. However, similarly to the All-Or-Nothing policy, in under-loaded states this would mean that the sessions receive more resources in the long term than $\mathcal{S}(\hat{R})$. Thus, in this policy, in underloaded state i, if $\frac{S}{i}$ is not integer, a session receives $\lfloor \frac{S}{i} \rfloor$ channels with probability P_{31}, $\lceil \frac{S}{i} \rceil$ number of channels with probability P_{32} and no channels with probability $1 - P_{31} - P_{32}$. Clearly, in states for which $i \cdot \mathcal{S}(\hat{R}) < S$ and $\frac{S}{i}$ is not an integer number:

$$P_{31} \cdot \left\lfloor \frac{S}{i} \right\rfloor + P_{32} \cdot \left\lceil \frac{S}{i} \right\rceil = \mathcal{S}(\hat{R}), \quad \text{and:} \quad P_{31} : P_{32} = \left(\left\lceil \frac{S}{i} \right\rceil - \frac{S}{i} \right) : \left(\frac{S}{i} - \left\lfloor \frac{S}{i} \right\rfloor \right).$$

If $\frac{S}{i}$ is integer, the session is assigned $\frac{S}{i}$ number of channels with probability P_{33} and zero channels with probability $1 - P_{33}$:

$$P_{33} \cdot \frac{S}{i} = \mathcal{S}(\hat{R}); \quad \text{and:} \quad P_0 = 1 - P_{33}.$$

For critically and overloaded states ($i \cdot \mathcal{S}(\hat{R}) \geq S$) the channels are fully utilized ($P_{34} + P_{35} = 1$):

$$P_{34} \cdot i \cdot \left\lfloor \frac{S}{i} \right\rfloor + P_{35} \cdot i \cdot \left\lceil \frac{S}{i} \right\rceil = S.$$

In the critically loaded and overloaded states, if $\frac{S}{i}$ is integer, the number of allocated sessions for each session is $\frac{S}{i}$ with probability 1. Based on these observations, the scheduling policy vector for the Fifty-Fifty policy is straightforward to determine (although a bit tedious to formally specify it):

$$\vec{P}_{FF}(s) = \begin{cases} \mathcal{S}(\hat{R}) \cdot A \ \text{ if } \ s = \lceil \frac{S}{i} \rceil \ \text{ and } \ i \cdot \mathcal{S}(\hat{R}) < S \\ \quad \text{and} \quad (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ \mathcal{S}(\hat{R}) \cdot B \ \text{ if } \ s = \lfloor \frac{S}{i} \rfloor \ \text{ and } \ i \cdot \mathcal{S}(\hat{R}) < S \\ \quad \text{and} \quad (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 - \mathcal{S}(\hat{R}) \cdot A - \mathcal{S}(\hat{R}) \cdot B \ \text{ if } \ s = 0 \\ \quad \text{and } \ i \cdot \mathcal{S}(\hat{R}) < S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 - \frac{\mathcal{S}(\hat{R})}{\frac{S}{i}} \ \text{ if } \ s = 0 \text{ and } \frac{S}{i} = \text{Integer} \\ \quad \text{and} \quad i \cdot \mathcal{S}(\hat{R}) < S \\ \frac{\mathcal{S}(\hat{R})}{\frac{S}{i}} \ \text{ if } \ s = \frac{S}{i} = \text{Integer} \\ \quad \text{and} \quad i \cdot \mathcal{S}(\hat{R}) < S \\ \lceil \frac{S}{i} \rceil - \frac{S}{i} \ \text{ if } \ s = \lfloor \frac{S}{i} \rfloor \\ \text{and} \quad i \cdot \mathcal{S}(\hat{R}) > S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ \frac{S}{i} - \lfloor \frac{S}{i} \rfloor \ \text{ if } \ s = \lceil \frac{S}{i} \rceil \\ \text{and} \quad i \cdot \mathcal{S}(\hat{R}) > S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 \ \text{ if } \ s = \frac{S}{i} = \text{Integer} \\ \quad \text{and} \quad i \cdot \mathcal{S}(\hat{R}) \geq S \\ 0 \ \text{ otherwise,} \end{cases}$$

where:

$$A \triangleq 1 - \frac{i}{S} \cdot \left\lfloor \frac{S}{i} \right\rfloor,$$

$$B \triangleq \frac{i}{S} \cdot \left\lceil \frac{S}{i} \right\rceil - 1. \tag{5}$$

2.4 A Numerical Example

Consider an OFDMA cell that supports $S = 64$ sub-carriers (channels). Sessions have a peak rate requirement that corresponds to $\mathcal{S}(\hat{R}) = 4$ channels. When there are 6 in-progress sessions, the system is under-loaded ($6 \cdot 4 < 64$), the three scheduling policy vectors are as follows:

$$\underline{P}_{WIW} = [0, 0, 0, 0, 1, 0 \ldots, 0]\,; \quad \underline{P}_{AoN} = \left[\frac{60}{64}, 0, \ldots, 0, \frac{4}{64}\right]; \quad P_{FF} = \left[\frac{40}{64}, 0, \ldots, 0, \frac{8}{64}, \frac{16}{64}, 0, \ldots, 0\right], \tag{6}$$

where the P_{FF} vector has non-zero elements at positions 1, 11 and 12. Since the system is underloaded, the What-It-Wants policy grants the peak rate with probability 1 (4 channels), the All-Or-Nothing policy allocates all the 64 channels with probability 4/64. The Fifty-Fifty policy either allocates 10 or 11 channels to any given session (with probabilities 8/64 and 16/64 respectively) or it does not schedule the session (zero channels with probability 40/64). It is easy to see that all three scheduling policies allocate the peak rate (4 channels) in the long term average (in this system state).

For an overloaded example, consider the above example with $i = 20$ in-progress sessions. The system is overloaded and so the peak rate cannot be granted in this system state. However, any one of the sessions can still receive (in long term average) 64/20=3.2 channels. The three policy vectors in this case are as follows:

$$\underline{P}_{WIW} = \left[0, 0, 0, \frac{4}{5}, \frac{1}{5}, 0 \ldots, 0\right]; \quad \underline{P}_{AoN} = \left[\frac{19}{20}, 0, \ldots, 0, \frac{1}{20}\right]; \quad P_{FF} = \left[0, 0, 0, \frac{4}{5}, \frac{1}{5}, 0 \ldots, 0\right]. \tag{7}$$

In this system state ($i = 20$), the What-It-Wants policy allocates 3 channels to 16 sessions and 4 to 4 sessions. Observing a single session, this means that this session under study is allocated either 3 or 4 channels. The All-Or-Nothing policy now allocates all the channels to the session under study during 1/20 of the time. We also realize that for critically or overloaded states ($i \cdot \mathcal{S}(\hat{R}) \geq S$), the What-It-Wants and the Fifty-Fifty policies have identical policy vectors.

2.5 Comments on the Reasoning Above and the Use of the Policy Vectors

Characterizing the number of assigned sub-carriers by means of the scheduling policy vectors basically assumes that the channel conditions within the cell are such that channel dependent scheduling does not much alter the resource shares between the in-progress sessions. For instance, in the All-Or-Nothing case, it is assumed that all sessions get an equal time share of the available channels. This assumption is not unrealistic if the mobile stations require service belonging to the same service class and their radio conditions are similar. We believe that this assumption does not distort the dependency of the performance measures of interest (as we shall define these later).

Another subtle assumption, which we will make use later on, is related to the independence of sessions and their share of the available resources. Under the assumption above on the channel conditions, an observer may indeed observe the probability vectors described above. However, assuming that the scheduler resides in the base station (both in downlink and uplink), the events that Session-A is assigned s_A and Session-B is assigned s_B number of channels are not independent. We will return to the issue of how our model takes account of this fact later.

2.6 ICIC Policies: Random and Coordinated Sub-carrier (Channel) Allocation

As we noted in the Introduction, ICIC operates at a much coarser time scale than packet scheduling [13]. Basically, there are two approaches as to *how* the sub-carriers out of the available ones are selected when a session requires a certain number of sub-carriers (see Figure 1). The simplest way is to pick sub-carriers out of the ones that are

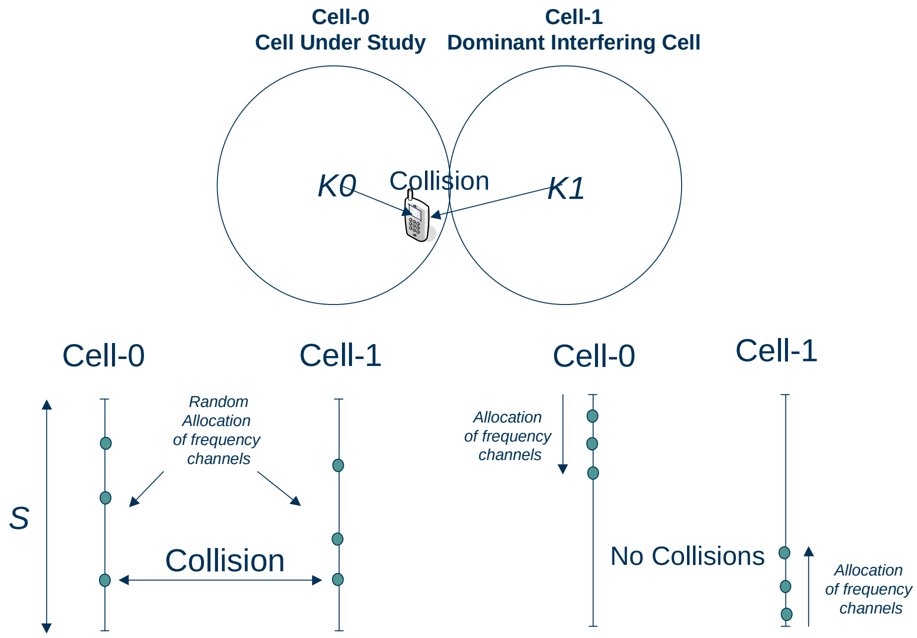

Figure 1: Random and Coordinated ICIC policies. Coordinated ICIC that operates on the hundreds of milliseconds, seconds or at even slower time scale can be realized by assigning a cell specific ordered list of the frequency channels to each cell such that the "collisions" of frequency channels are avoided as long as there are non-colliding pairs. Assuming a single (dominant) interfering cell (as in [19] and [20]), devising such ordered lists is straightforward. For many cells, coordinated ICIC implies careful frequency planning, as described in for example [5].

available (i.e. scheduled) randomly such that any available sub-carrier has the same probability to get allocated to an arriving session. *Random allocation* of sub-carriers is attractive, because it does not require any coordination between cells, but it may cause collisions even when there are free sub-carriers. In contrast, a low complexity coordination can avoid collisions as long as there are non-colliding sub-carrier pairs in the two-cell case and non-colliding tuples in the multiple-cell case. We refer to this method as *coordinated* sub-carrier allocation (also called channel segregation [5]). (Further details about these ICIC policies can be found in [18].)

3 Performance Measures of Interest and Solution Approach

3.1 Input Parameters and Performance Measures of Interest

We consider a single OFDMA cell with S channels at which sessions belonging to the same (elastic) service class arrive according to a Poisson process of intensity λ. Each session brings with itself a file whose size is an exponentially distributed random variable with parameter μ. The session requests a radio bearer that is characterized by its peak rate $\hat{R}$ (for which: $\mathcal{S}(\hat{R}) \leq S$) and minimum rate $\hat{R}/\hat{a}$, where $\hat{a} \geq 1$ is the maximum slowdown factor associated with the session. If, at the time instant of the arrival of the new session, the admission of the new session brought the system into a state in which the minimum rate (governed by the particular scheduling policy) cannot be granted, the session is blocked and leaves the system. The single cell is disturbed (interfered) by a single dominant interferer cell, such as in [19]. In this paper we characterize the load in this dominant interfering cell by the number of used sub-carriers $K_1 \leq S$. When an allocated sub-carrier in the cell under study and one of the K_1 disturbing channels use the same sub-carrier frequency, we say that the two sub-carriers *collide* and suffer from co-channel interference [5]. We reuse the co-channel interference model in [18]. This model determines the distribution of the signal-to-noise-and-interference (SINR) of the colliding subcarrier in the cell under study.

The performance measures of interest are the session-wise blocking probability and the mean file transfer time. These two quantities represent a trade-off since more admitted sessions imply lower per-session throughput and thereby longer file transfer times. This trade-off in a WCDMA environment has been investigated by Altman in [14] (see also [15]) and subsequently by Fodor *et al.* in [16] and [17].

3.2 Step 1: Determining the Distribution of the Allocated Sub-carriers

Recall that in each system state the scheduling policy vector determines the probability that a given session is allocated s channels. When a session is given s channels (which happens with $\vec{P}(s+1)$ probability), we need to calculate the conditional distribution of the number of the totally allocated number of channels in the cell (which we denote by K_0), given that the session under study is given s channels. This is because K_0 and the number of disturbing channels K_1 determine the distribution of the colliding and collision-free channels in the cell, which in turn determine the performance measures of interest.

We cannot give a closed form formula for the (conditional) distribution of K_0. However, in Appendix I we provide the pseudo code description of the algorithm that calculates it. The output of this algorithm is the vectors $\vec{K}_0$ and $\vec{P}_{K_0}$ and the value k_0^{MAX}. The values of $\vec{K}_0$ are the possible values of K_0 while the values of $\vec{P}_{K_0}$ are the associated probabilities. k_0^{MAX} gives the number of possible values of K_0 thereby the dimension of $\vec{K}_0$ and $\vec{P}_{K_0}$.

3.3 Step 2: Determining the Distribution of the Colliding Sub-carriers under the Random and Coordinated Sub-carrier Allocation Policies

Lemma 1 *Let S denote the total number of available sub-carriers in each cell and let $K_0 \leq S$ and $K_1 \leq S$ denote the number of allocated channels in Cell-0 and Cell-1 respectively. Let $N_1(c)$ denote the number of possible channel allocations in Cell-0 and Cell-1 such that the number of collisions is c.*

Then, the distribution and the mean of the number of collisions under the random allocation policy (γ_1) are as follows:

$$c_{MIN} = Max[0, K_0 + K_1 - S], \quad c_{MAX} = Min[K_0, K_1],$$

$$N_1(c) = \binom{S}{c} \cdot \binom{S-c}{K_0 - c} \cdot \binom{S-K_0}{K_1 - c}; \quad c = c_{MIN} \dots c_{MAX},$$

$$Pr\{\gamma_1 = c | K_0, K_1\} = \frac{N_1(c)}{TOT1},$$

$$E[\gamma_1 | K_0, K_1] = \sum_{c=c_{MIN}}^{c_{MAX}} \frac{c \cdot N_1(c)}{TOT1}, \text{ where } TOT1 = \binom{S}{K_0} \cdot \binom{S}{K_1}.$$

Proof:
The three terms of $N_1(c)$ give the number of possible channel allocations for the c colliding channels out of the S available channels, the $K_0 - c$ non-colliding channels in Cell-0 and the $K_1 - c$ non-colliding channels in Cell-1 respectively. The other results immediately follow. We note that (as a possibility for verifying this result) $TOT1$ can also be calculated as $TOT1 = \sum_{c=c_{MIN}}^{c_{MAX}} N_1(c)$. ■

Lemma 2 *Using similar notation as in Lemma 1, the distribution and the mean number of collisions under the coordinated allocation policy is given by:*

$$N_2(c) = \begin{cases} 1 & \text{if } c = c_0 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$c_0 = \begin{cases} 0 & \text{if} \quad K_0 + K_1 < S \\ K_0 + K_1 - S & \text{otherwise.} \end{cases}$$

$$Pr\{\gamma_2 = c | K_0, K_1\} = N_2(c),$$

and

$$E[\gamma_2 | K_0, K_1] = \sum_{c=c_{MIN}}^{c_{MAX}} c \cdot N_2(c).$$

Proof:
Under the assumption that we have a single dominating interfering cell, we may think of the coordinated channel allocation policy as one that allocates channels in Cell-0 and Cell-1 in "opposite order". That is, in Cell-0 channels are allocated in the order of $0, 1, \dots S$, while in Cell-1 in the order of $S, S-1, \dots 0$. Thus, for any (K_0, K_1) pair, the number of collisions is either 0 or $K_0 + K_1 - S$. ■

3.4 Step 3: Determining the Packet-wise Effective Signal-to-Noise-and-Interference-Ratio

The scheduling policy vector specifies the probability that s channels are used in Cell-0, whereas Lemmas 1-2 determine the probability that the number of colliding channels is c. We will use the following lemma to determine the probability that the number of colliding channels in a packet of size L is γ when the number of scheduled channels (for the session under study) is s and the total number of colliding channels is $c \leq s$.

Lemma 3

$$Pr\left\{\gamma \textit{ channels out of } L \textit{ are colliding}\right\} = \begin{cases} 0 & \textit{if } \gamma > c, \\ \binom{L}{\gamma} \cdot \frac{\binom{c}{\gamma}}{\binom{s}{\gamma}} \cdot \frac{\binom{s-c}{L-\gamma}}{\binom{s-\gamma}{L-\gamma}} & \textit{otherwise.} \end{cases}$$

Proof:
Given that there are c number of colliding sub-carriers out of the total s that are taken into use in Cell-1, the probability that the first picked sub-carrier is colliding is c/s, that the second is colliding is $(c-1)/(s-1)$ and so forth. Similar reasoning applies to the $L-\gamma$ non-colliding channels. We also need to take into account the number of possible combinations for the γ colliding channels within the packet that is of size L. Finally, we notice that:

$$\prod_{i=0}^{\gamma-1} \frac{c-i}{s-i} = \frac{\binom{c}{\gamma}}{\binom{s}{\gamma}} \quad \text{and} \quad \prod_{i=0}^{L-\gamma-1} \frac{s-c-i}{s-\gamma-i} = \frac{\binom{s-c}{L-\gamma}}{\binom{s-\gamma}{L-\gamma}}.$$

■

3.5 Step 4: Calculating the SINR Level in Case of Collisions for the Downlink

Lemmas 1-3 determine the probability that the number of colliding channels is γ and the number of non-colliding channels is $L-\gamma$ in a packet of a session under study. We now need to determine the impact of the collision on a channel's signal-to-noise-and-interference (SINR) ratio.

For this, we use the path loss model recommended by the 3GPP (described in [27]) and a result from [18]. Let θ be a predefined threshold and let $X \triangleq \frac{r_0}{r_1}$ be a random variable representing the ratio between the mobile station distances from its serving and disturbing base station respectively. Also, let Q_0 and Q_1 denote the power that the serving and the neighbor base station uses on the colliding channels respectively. Furthermore, let G_0 and G_1 denote the path gains from the serving base station (that is in Cell-0) and the dominant neighbor base station (that is in Cell-1) respectively to the mobile station under study. Then, the probability that the SINR remains under this threshold can be approximated as follows [18]:

$$Pr\Big(\frac{G_0 \cdot Q_0}{G_1 \cdot Q_1 + N_0} < \theta\Big) \approx \int_0^{Max[X]} \Big(f_X(x)g(x)\Big)dx; \quad g(x) \triangleq \frac{1}{2}\text{erfc}\left(-\frac{5}{b\varsigma} \cdot \frac{ln\,\frac{x^{\mu}\theta}{Q_0/Q_1}}{ln\,10}\right). \tag{8}$$

where $f_X(x)$ is the probability density function of X; b, ς and μ are the parameters of the 3GPP path loss model as described in [27]. We note that this approximation is applicable for interference limited systems. in which we can assume that the impact of the collision on the SINR is at least an order of a magnitude greater than the impact of the background noise. For instance, assuming 5 MHz reception bandwidth, and -174 dBm/Hz thermal noise density [27], N_0 can be assumed to be -107.011 dBm. The impact of collision is determined by the power employed by the disturbing base station on the disturbing channel and the path loss between the disturbing base station and the mobile station (that is the term $\frac{Q_1}{r_1^{\mu}}$) above. The channel transmission power in E-UTRA is in the magnitude of watts, and assuming at least 15 dB antenna gain, Q_1 is in the order of 50 dBm. Thus, for exterior mobiles, for all practically relevant cases in E-UTRA, we can assume that $\frac{Q_1}{r_1^{\mu}} \ggg N_0$.

The probability density function $f_X(x)$ for the important case when the mobile station is randomly placed over the entire surface area of the serving cell and there is one dominant neighbor interfering base station is derived in Appendix II.

3.6 Step 5: Calculating the Effective SINR and the Packet Loss Probability

We are now in the position that the packet loss probability in each system state can be determined.

When one or more of the channels that are used to carry a packet are hit by collisions, an efficient way to characterize the overall SINR quality of the packet is to use the notion of the *effective* SINR. This concept has been proposed in [23] and used in for instance [24], in which a method to calculate the packet error probability for a given value of the effective SINR was also proposed. A specific method to calculate the effective SINR (based on the SINR of the composing channels) that is applicable in cellular OFDM systems is also recommended by the 3GPP [22].

In this paper we employ the 3GPP method that can be summarized as follows. Suppose that there are L sub-carriers that carry a data packet and each has a SINR value of $SINR_i$. Than, the *effective* SINR that is assigned to the packet is given by:

$$SINR_{\text{eff}} = \alpha_1 \cdot I^{-1}\left(\frac{1}{L}\sum_{i=1}^{L} I\Big(\frac{SINR_i}{\alpha_2}\Big)\right), \tag{9}$$

where $I(\cdot)$ is a model specific function and $I^{-1}(\cdot)$ is its inverse. The parameters α_1 and α_2 allow to adapt the model to characteristics of the considered modulation and coding scheme. The exponential effective SINR metric proposed in [22] corresponds to:

$$I(x) = exp(-x).$$

In [24] it is shown that for QPSK and 16-QAM modulation, the parameters α_1 and α_2 can be chosen as follows: $\alpha_1 = 1$ and $\alpha_2 = 1$. In [24] a method to determine the packet error rate (σ) as a function of the effective SINR is presented. Essentially, this method maps (in a 1-1 fashion) the effective SINR onto a (modulation and coding scheme dependent) packet error rate.

3.7 Step 6: Determining the Performance Measures of Interest

We now make use of the assumption that the session arrivals form a Poisson process and that the session size is exponentially distributed. We choose the number of admitted sessions as the state variable and thus the number of states in the system is $\hat{I} + 1$. The transitions between states are due to an arrival or a departure of a session. The arrival rates are given by the intensity of the Poisson arrival processes. Due to the memoryless property of the exponential distribution, the departure rate from each state depend on the nominal holding time of the in-progress sessions, and also on the slow down factor and the packet error rate in that state. Specifically, when the slow down factor is $a_i(n)$, and the packet error rate is $\sigma(n)$ its departure rate is $(1 - \sigma_i(n))\mu_i/a_i(n)$.

The Markovian property for such systems was observed and formally proven by Altman *et al.* [25], and Nunez Queija *et al.* [26]. Thus, the system under these assumptions is a continuous time Markov chain (CTMC) whose state is uniquely characterized by the state variable n.

4 Numerical Results

4.1 Input Parameters

In accordance with the 3GPP recommendation, we here (in a somewhat simplified fashion) assume that a downlink resource block (sometimes referred to as a *chunk*) occupies 300 kHz and 0.5 ms in the frequency and time domains respectively. A chunk carries 7 OFDM symbols on each sub-carrier; therefore the downlink symbol rate is 140 symbols/chunk/0.5ms. Assuming a 10 MHz spectrum band, and considering some overhead due to measurement reference symbols and other reasons, this corresponds to 30 chunks in the frequency domain ($S = 30$), that is 8400 ksymbol/s. The actual bit-rate depends on the applied modulation and coding scheme, in this paper we do not model adaptive modulation and coding (AMC), we simply assume a fixed binary phased shift keying (BPSK) so that each symbol carries 2 bits. Sessions arrive according to a Poisson process of intensity $\lambda = 1/8\ [1/s]$. A session is characterized by the amount of bits that it transmits during its residency time in the system (we may think of this quantity as the size of the file that is to be downloaded). We assume that this file size is an exponentially distributed random variable with mean value ν.

Table 1: Model (Input) Parameters

$R_{symbol} = 280$	OFDM symbol rate per resource block;
$n_{MCS} = 2$	Number of bits per symbol (depending on the modulation and coding scheme)
$\lambda = 0.125$	Session arrival intensity
$S = 30$	Number of channels
$\mathcal{S}(\hat{R}) = 4 \dots S$	Peak channel requirement
$\mathcal{S}(R_{min}) = S(\hat{R})/\hat{a} = 2$	Minimum channel requirement
$\lambda = 1/8$	Session arrival rate
$\nu = 4 * S * R_{symbol} * n_{MCS}$	Mean file size
$SINR_{good}$ =10 dB	Signal-to-Noise-and-Interference-Ratio without collision
$Q_0 = Q_1 = 20W/5MHz$	Power applied by the serving and neighbor base stations
$SINR_{bad}$ =0...3 dB	Signal-to-Noise-and-Interference-Ratio with collision
$K_1 = S/6 \dots S$	Number of used channels in the neighbor (disturbing) cell

From the radio access network's (RAN) perspective, when a session is admitted into the system, a radio bearer associated with a minimum bit rate (also called the *guaranteed bit rate*, GBR) and a maximum bit rate (MBR) is set up. The GBR and the MBR bit rates correspond to the minimum and the maximum (peak) number of channels that the radio bearer must support. In our terminology, the GBR/MBR ratio corresponds to $\hat{a}$.

For each scheduled channel, the SINR depends on the distance between the base station and the mobile terminal, the channel conditions and whether the channel suffers from co-channel interference (collision) from neighbor cells or not. When there is no collision, we assume that the SINR value is a lognormally distributed random variable with mean 10 dB. When there is collision, the SINR value depends on the position of the mobile terminal and the applied power levels by the serving and the neighbor base stations (as described by (8) in Step 4).

The input parameters are summarized by Table 1.

4.2 Discussion of Figures 2-5

In Figures 2-5 we study the impact of the (increasing) inter-cell interference on the session blocking probability and the file download time in the case when the associated radio bearer (RB) is peak allocated ($\hat{a} = 1$) (Figures 2-3) and when the GBR is set to the half of the PBR ($\hat{a} = 2$). On the x axis we let the number of disturbing channels (i.e. the occupied channels in the neighbor cell) increase ($K1/5 = 1 \dots 6$), while the y axis shows the blocking probabilities and the mean session residency times. The left hand side figures correspond to the case when there is no channel allocation coordination between the cells, while the right hand side figures assume coordination (channel segregation). We observe that when the sessions tolerate some slowdown, the blocking probability dramatically decreases (from 7% down to 0.06% !) without much increasing the download time (from around 33s to around 34s). Secondly, we note that coordinated allocation is beneficial when the What-It-Wants of the Fifty-Fifty scheduling method is employed, and has no effect when the All-Or-Nothing scheduling is used.

4.3 Discussion of Figures 6-7

Table 2: SINR$_{bad}$ as a function of $\frac{r_0}{r1}$ when $\mu = 2$ and $\mu = 3$

X	SINR$_{bad}$	$\frac{r_0}{r_1}$; $\mu = 2$	$\frac{r_0}{r_1}$; $\mu = 3$
1	2.5	0.749	0.825
2	2.0	0.794	0.858
3	1.5	0.841	0.891
4	1.0	0.891	0.926
5	0.5	0.944	0.962
6	0	1.0	1.0

Figures 6-7 show the impact of the collisions on the download times as they happen at a mobile terminal that is gradually moved closer to the cell edge. (That is when $\frac{r_0}{r_1}$ increases and the SINR for the colliding channel decreases as shown in Table 2). When the mobile terminal is hit by an interfering downlink signal, the impact of this collision depends on the effect on the SINR. Here we let the SINR of the colliding channel decrease from 3dB (mobile terminal in the interior of the cell) down to 0 dB (equal distance from the serving and neighbor base stations). Figure 6 shows the result for the peak allocated case, while Figure 7 shows the result for the $\hat{a} = 2$ case. The download time increases in both cases, and - as expected -, this increase is greater with ”elastic” bearers, that is when slowdown is accepted. More importantly, coordinated channel allocation significantly improves the system throughput performance when the scheduler is of type What-It-Wants or All-Or-Nothing.

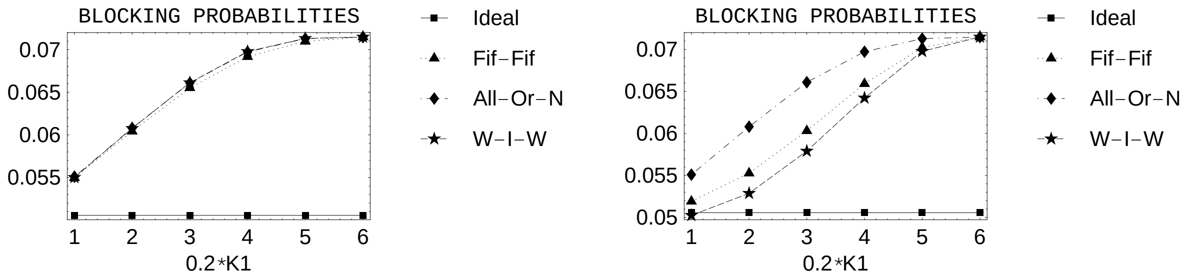

Figure 2: As the number of occupied channels in Cell-1 increases from 5 to 30 ($K1/5 = 1 \ldots 6$), the blocking probability increases both under the random (left) and the coordinated (right) allocation policies. However, the Fifty-Fifty and the What-it-Wants scheduling method performs better than the All-Or-Nothing scheduling under coordinated allocation.

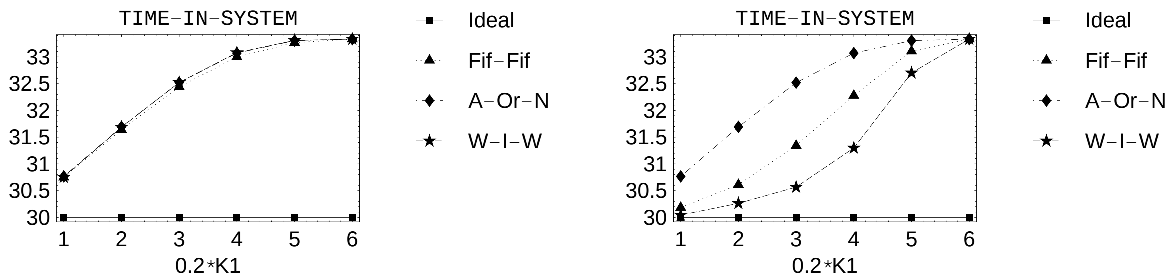

Figure 3: As the number of occupied channels in Cell-1 increases from 5 to 30 ($K1/5 = 1 \ldots 6$), the download time increases both under the random (left) and the coordinated (right) allocation policies. However, the Fifty-Fifty and the What-it-Wants scheduling method performs better than the All-Or-Nothing scheduling under coordinated allocation.

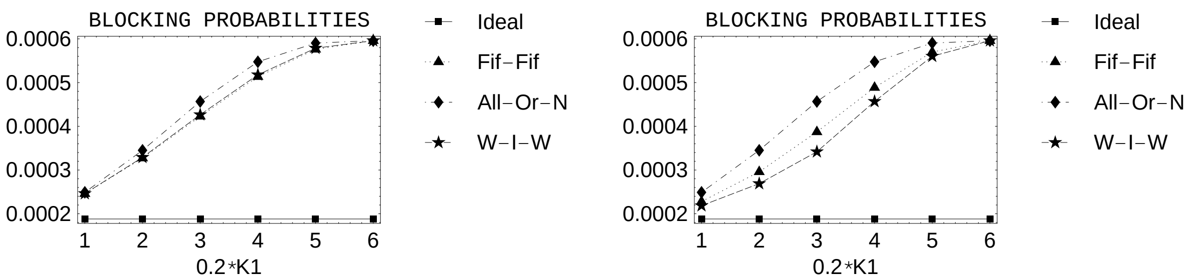

Figure 4: When the sessions tolerate some slowdown (here $\hat{a} = 2$, that is $R_{min} = \hat{R}/2$), the blocking probabilities radically decrease, (here 2 orders of magnitude), and the coordinated allocation (right) again performs somewhat better when the scheduling method is the All-Or-Nothing or What-It-Wants.

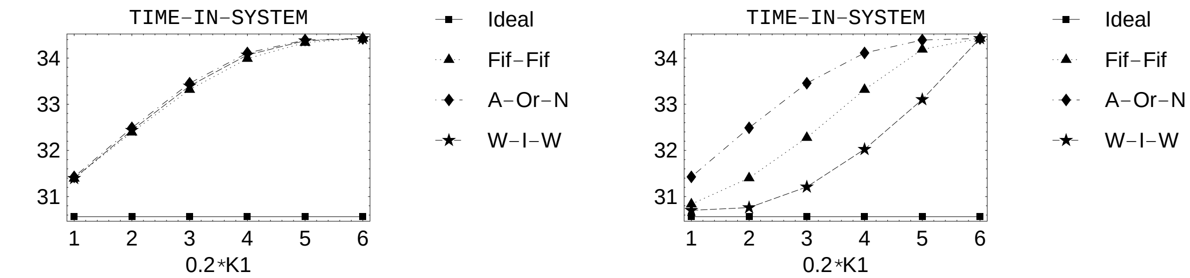

Figure 5: When the sessions tolerate some slowdown (here $\hat{a} = 2$, that is $R_{min} = \hat{R}/2$, the session holding time increases somewhat, (a few procents), and the coordinated allocation again performs somewhat better when the scheduling method is the All-Or-Nothing or What-It-Wants.

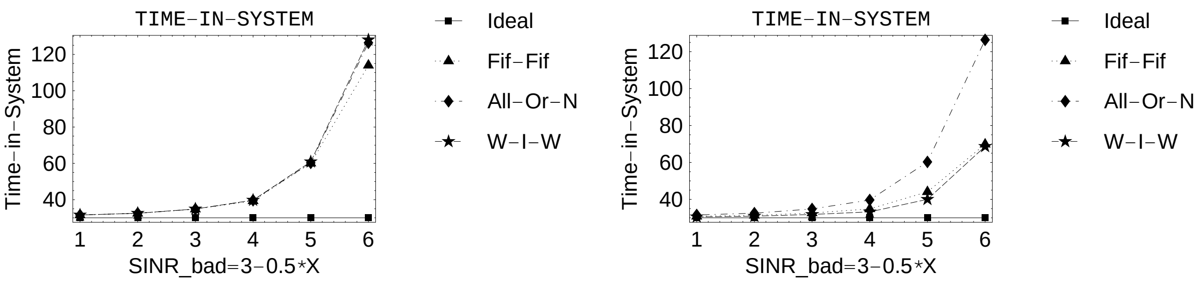

Figure 6: Along the x axis, we let the value of the $SINR$ of colliding channels ($SINR_{bad}$) decrease from 2.5dB to 0dB (see Table 2). In the downlink, this corresponds to the case the position of the mobile that is hit by the neighbor base station is moved from the interior of the cell out to the cell edge. As the system throughput decreases, the average residency time of sessions increases dramatically, but when employing the coordinated channel allocation together with the What-It-Wants or Fifty-Fifty scheduling method, the system can be kept under normal operational conditions "closer to the cell edge".

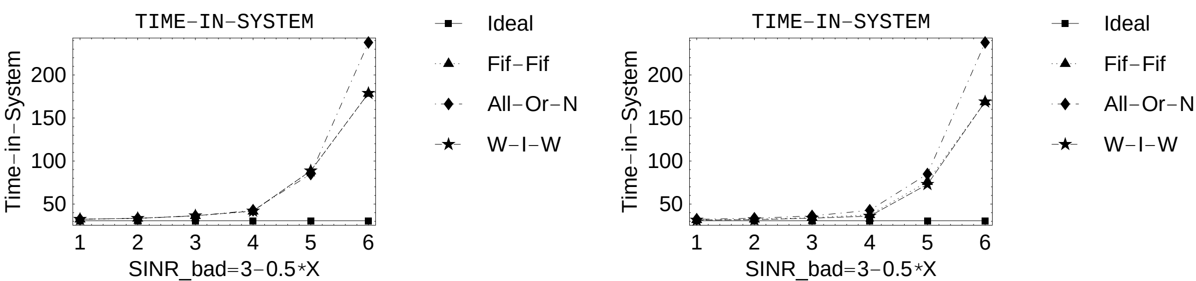

Figure 7: This figure is similar to the previous figure, but now $\hat{a} = 2$. We notice the increase of the average session holding time and again the usefulness of the coordinated allocation policy when used together with the What-It-Wants or Fifty-Fifty scheduling.

4.4 The Impact of ICIC in Multi-cell Systems

Parameter	Value
Cellular layout	7 sites, 1 sector each
Cell radius	500 meters
Carrier frequency	2 GHz
System Bandwidth	20 MHz
Subcarrier bandwidth	15 kHz
Number of subcarriers per RB	16
Number of RBs (N)	80
Frame length	0.56 msec
Number of frames per super-frame	22
Average number of users per cell	15
Users' mean speed	10 m/s
Handover	Hard handover
Shadow fading	Lognormal distribution $\mu_{dB} = 0\,dB,\ \sigma_{dB} = 8\,dB$ Correlation distance 110 m
Multipath fading	3GPP SCM SuburbanMacro model
Link adaptation	ACM Modulations: BPSK, QPSK, 16QAM, 64QAM Code rates: 0.01-0.99

Figure 8: Simulation parameters in the multicell scenario that we use to study the impact of ICIC and scheduling in a realistic network setting. The system consists of 7 sites and operates in 20 MHz, the number of resource blocks (channels) being 80. We implemented the 3GPP spatial channel model (SCM) and allow for adaptive coding and modulation (ACM) and with various schemes as recommended by the 3GPP [31].

	Uncoordinated Resource Allocation	**Coordinated Resource Allocation**
Non-opportunistic Allocation	**("Random")**	Superframe level coordination (**"RNC"**)
Opportunistic Allocation (Multi-user Frequency Diversity)	No superframe level coordination, but frame level optimization (**"BSunc"**)	Superframe level coordination + frame level optimization (**"RNC+BS"**)

Figure 9: Simulation scenarios. Random allocation is a simple scheme requiring no channel state information. In contrast, opportunistic scheduling requires requires channel state information at the base station but no coordination between cells ("BSunc"). Non-opportunistic resource block allocation means that any of the available resource blocks gets assigned to a session with equal probability. Opportunistic scheduling utilizes channel quality information (CQI) reporting in every frame and assigns the best (respective) channel to each user. Inter-cell coordinated resource allocation employs the RNC algorithm in order to avoid inter-cell collisions.

Although a two-cell model allows analytical studies, it is useful to evaluate the performance of ICIC in multi-cell systems as well. To this end, we developed a multi-cell OFDMA simulator based on the 3GPP E-UTRA specifications [31].

The core of this tool is based on a system level radio network simulator called the *Rudimentary Network Simulator* (RUNE) [30]. This simulator has been successively extended to include numerous features of an OFDMA environment, including channel models, scheduling and link adaptation algorithms and also various modulation and coding schemes [29]. Most recently, it has been extended to include a centralized ICIC algorithm (effectively implementing a radio network controller, RNC) [29]. With this simulator in hand, we now consider a 20 MHz OFDM cellular system with seven 1-sector sites. In line with the 3GPP specifications, the subcarrier bandwidth is 15 kHz and the number of subcarriers per resource block is 16 yielding 80 resource blocks per cell [31]. The shadow fading is modeled as a lognormally distributed random variable, whereas multipath fading is modeled as the 3GPP spatial channel model (SCM). The system uses adaptive modulation and coding (ACM) allowing for BPSK, QPSK, 16-QAM and 64-QAM schemes. Figure 8 summarize the input parameters of our simulation study.

4.5 Numerical Results for Multi-cell Systems

We fix the number of required resource blocks per user to $\mathcal{S}(R_{min}) = n_{min} = 2$ (from now own we omit subscript i since all users have the same service requirements). We will first consider a case in which the total number of users is fixed (7x15) such that the average load per cell in terms of the used resource blocks is 37.5% (Figures 10-11).

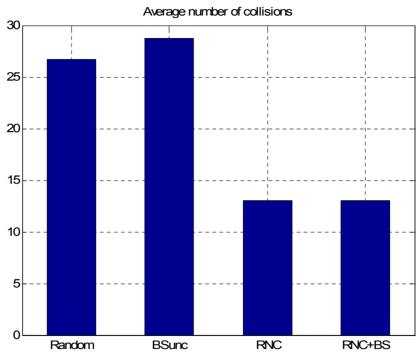

Figure 10: Average number of collisions. The BSunc allocation method uses opportunistic scheduling without inter-cell coordination. Due to the correlation in the instantaneous SINR values it tends to select the same resource blocks in neighbor cells. In contrast, random allocation has no such correlation and therefore the average number of collisions of BSunc is slightly higher than that of Random. Inter-cell coordination (RNC) drastically reduces the number of collisions.

Figure 10 shows the number of collisions for the four cases of Figure 9. This figure clearly illustrates the benefit of inter-cell coordination achieved by the RNC algorithm in terms of the number of collisions. Specifically for the 37.5% load case, the number of collisions is drastically reduced by employing the RNC algorithm. We can also see that, as expected, the BS algorithm does not further decrease the number of collisions.

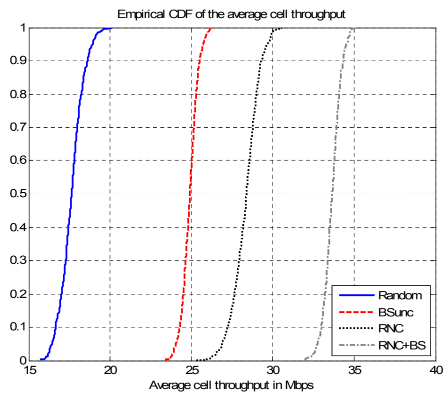

Figure 11: The distribution function of the average cell throughput. Opportunistic scheduling together with inter-cell coordination is superior not only to random allocation but also to the allocations that employ opportunistic scheduling without coordination.

Next, Figure 11 illustrates the overall gain (that is the "sum" of the collision avoidance *and* the gain of the opportunistic scheduling) when employing both ICIC and channel state dependent scheduling ("RNC+BS"). Indeed, the RNC+BS allocation yields roughly 20% higher throughput than the RNC algorithm without the opportunistic scheduling of the BS.

5 Conclusions

Inter-cell interference coordination is an important radio resource management function for OFDMA based cellular systems in general [5] and for the evolving Universal Terrestrial Radio Access Network (E-UTRA) in particular [2], [3]. In a previous work we have showed that coordinated channel allocation (in [5] also called *channel segregation*) helps to improve the SINR and throughput performance of the system [18]. In this paper we built on the base model of that paper and investigated the performance of three scheduling disciplines with/without coordinated channel allocation. The All-Or-Nothing scheduling method is a pure time domain scheduling technique, according to which a single session takes all available channels into use at any one time. The What-It-Wants scheduling method is a combined time and frequency domain technique: it allows the simultaneous transmission of different sessions. At every point in time, it allocates s channels to in-progress sessions such that $\mathcal{S}(R_{min}) \leq s \leq (\hat{R})$. The Fifty-Fifty scheduler can be seen as a scheduler that is "in between" these two scheduling methods.

We proposed the notion of the (scheduling) policy vector to model the behavior of the packet scheduler. Using the policy vector, we were able to derive the conditional distribution of the number of colliding and collision free channels for all three cases. This in turn allowed us to determine the distribution of the number of colliding and collision free (i.e. co-channel interference free) channels in each scheduled packet. We used this knowledge to calculate the *effective SINR* and from it the packet error rate and thereby the useful packet throughput of the system. This useful throughput determines the session wise blocking probabilities and the time it takes for elastic sessions to complete a file transfer.

Our major finding is that the performance of the ICIC function (its impact on the system throughput) somewhat (but not drastically) depends on the employed scheduler when the traffic load is medium. Specifically, when frequency domain scheduling is used in combination with time domain scheduling, it is useful to employ coordinated channel allocation in neighbor cells. Coordinated ICIC has little impact when the scheduler is pure time domain based. In general, the gain of ICIC is small when the traffic load is low (the collision probabilities are low even without ICIC) or when the load is too high (ICIC cannot avoid collisions).

One interesting outstanding issue is the modeling of the scheduling gain that can be different for the different schedulers investigated in this paper. Time domain scheduling has been found beneficial in fast fading environments by allowing avoiding the fading dips. Studying the impact of such scheduling gains is left for future work.

Appendix I: Deriving the Distribution of K_0

```
j = 1;
iNoOfSchedulingPositions = 0;
For[i = 1, i ≤ CH + 1, i++,
  If[tPolicyVector[[i]] ≠ 0,
    iNoOfSchedulingPositions = iNoOfSchedulingPositions + 1;
   ];
 ];
tSchedulingPositions = Table[0, {t1, 1, iNoOfSchedulingPositions}];
For[i = 1, i ≤ CH + 1, i++,
  If[tPolicyVector[[i]] ≠ 0,
    tSchedulingPositions[[j++]] = i;
   ];
 ];
For[i = 1, i ≤ iNoOfSessions, i++,
  tDenom[[i]] = (iNoOfSchedulingPositions^iNoOfSessions) / (iNoOfSchedulingPositions^i);
 ];
iRaw = iNoOfSchedulingPositions^iNoOfSessions;
mChannel = Table[0, {t1, 1, iRaw}, {t2, 1, iNoOfSessions}];
mChannel2 = Table[0, {t1, 1, iRaw}, {t2, 1, iNoOfSessions}];
For[i = 1, i ≤ iRaw, i++,
  For[j = 1, j ≤ iNoOfSessions, j++,
    mChannel[[i, j]] = Mod[Floor[(i - 1) / tDenom[[j]]], iNoOfSchedulingPositions] + 1;
    mChannel2[[i, j]] = tSchedulingPositions[[mChannel[[i, j]]]];
   ];
 ];
```

Figure 12: Pseudo Code, Part I. This algorithm takes the `PolicyVector` and the number of currently served sessions (the system state, `iNoOfSessions`) as its input and generates the `mChannel2` matrix as the output. The `mChannel2` matrix has as many rows as there are combinations of the number of scheduled channels for each session. For instance, in system state 3, a row may contain `0, 5, 6` which corresponds to the case that Session-1 is given 0 channels, Session-2 is given 5 and Session-3 is given 6 channels.

```
For[i = 1, i ≤ iRaw, i++,
  dSum = 0;
  For[j = 1, j ≤ iNoOfSessions, j++,
   dSum = dSum + (mChannel2[[i, j]] - 1);
  ];
  dOccupationProb = 1.0;
  For[k = 1, k ≤ iNoOfSessions, k++,
   dOccupationProb = dOccupationProb * tPolicyVector[[mChannel2[[i, k]]]];
  ];
  If[dSum + iBase > CH,
   kk = CH - iBase;
   tNumberOfOccupiedChannels[[kk + 1]] = tNumberOfOccupiedChannels[[kk + 1]] + 1;
   tOccupationProbs[[kk + 1]] = tOccupationProbs[[kk + 1]] + dOccupationProb,
   (* Else *)
   tNumberOfOccupiedChannels[[dSum + 1]]++;
   tOccupationProbs[[dSum + 1]] = tOccupationProbs[[dSum + 1]] + dOccupationProb;
  ];
 ];
j = 1;
For[i = 1, i ≤ iNoOfSessions * CH + 1, i++,
  If[tOccupationProbs[[i]] ≠ 0,
    tFinal[[j]] = tOccupationProbs[[i]];
    tShiftedFinalOccupiedChannels[[j]] = iBase + i - 1;
    j = j + 1;
   ];
 ];
```

Figure 13: Pseudo Code, Part II. This algorithm takes `mChannel2` and the number of occupied channels by the session under study (`iBase`) as its input and generates the possible values of K_0 (`tShiftedFinalOccupiedChannels`) and the associated probabilities (`tFinal`). j gives the number of possible values of K_0.

Appendix II: Deriving the Probability Distribution Function of $X \triangleq \frac{r_0}{r_1}$

Consider 2 circles (Cell-0 and Cell-1) of the same radius r, such the center of the two circles, C_0 and C_1, are $2r$ apart (as in Figure 1). For a given point P in Cell-0, let r_0 and r_1 denote the distances from the center of Cell-0 and Cell-1, respectively, and γ the angle of the line from the given point, P, to C_0 with the line connecting C_0 and C_1. We are interested in the distribution of r_0/r_1, i.e., $Pr(r_0/r_1 < x)$, if P is uniformly distributed over the surface of Cell-0.

When P is uniformly distributed on Cell-0, γ is uniformly distributed in $(0, 2\pi)$ and r_0 is linearly increasingly distributed in $(0, r)$, with density $f_{r_0}(x) = \frac{2x}{r^2}$. Exploiting the symmetry of the model we assume that γ is uniformly distributed in $(0, \pi)$. Having this assumption

$$F_\gamma(x) = Pr(\gamma < x) = x/\pi,$$

for $x \in (0, \pi)$, from which

$$F_{\cos(\gamma)}(x) = Pr(\cos(\gamma) < x) = 1 - Pr(\cos(\gamma) > x) = 1 - Pr(\gamma < \arccos(x)) = 1 - \frac{\arccos(x)}{\pi},$$

for $x \in (-1, 1)$. More precisely

$$F_{\cos(\gamma)}(x) = \begin{cases} 0, & \text{if } x < -1, \\ 1 - \dfrac{\arccos(x)}{\pi}, & \text{if } -1 < x < 1, \\ 1, & \text{if } 1 < x. \end{cases}$$

According to the *law of cosines* we have

$$r_1^2 = r_0^2 + 4r^2 - 4r_0 r \cos(\gamma) .$$

Applying this relation we can write

$$F_{r_0/r_1}(x) = Pr\left(\frac{r_0}{r_1} < x\right) = Pr\left(\frac{r_0^2}{r_0^2 + 4r^2 - 4r_0 r\cos(\gamma)} < x^2\right) =$$

$$= \int_{y=0}^{r} f_{r_0}(y)\, Pr\left(\frac{y^2 + 4r^2 - 4yr\cos(\gamma)}{y^2} > \frac{1}{x^2}\right) dy = \int_{y=0}^{r} f_{r_0}(y)\, F_{\cos(\gamma)}\left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2}\right) dy.$$

To substitute $F_{\cos(\gamma)}(x)$ into the last expression we need to study the behavior of

$$g(x, y) = \frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2},$$

at $x \in (0, 1)$ and $y \in (0, r)$. $g(x, y)$ is a monotone decreasing function of y in the $(0, r)$ interval and it tends to infinity as $y \to 0^+$.

The relevant roots of $g(x, y) = 1$ is $r_1 = \dfrac{2rx}{x+1}$ and the one of $g(x, y) = -1$ is $r_{-1} = \dfrac{2rx}{1-x}$. r_1 is always in $(0, r)$, and r_{-1} is in $(0, r)$ when $x \le 1/3$.

At $y = r$ we have

$$g(x, r) = \frac{5}{4} - \frac{1}{4x^2},$$

which is

$$g(x, r) < -1, \quad \text{if } 0 < x < \frac{1}{3},$$

$$-1 < g(x, r) < 1, \quad \text{if } \frac{1}{3} < x < 1.$$

Using these, we can substitute $f_{r_0}(x)$ and $F_{\cos(\gamma)}(x)$ as follows.
If $0 < x < \frac{1}{3}$, i.e., $r_{-1} < r$:

$$F_{r_0/r_1}(x) = \int_{y=0}^{r} \frac{2y}{r^2} F_{\cos(\gamma)} \left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2} \right) dy =$$

$$= \int_{y=0}^{r_1} \frac{2y}{r^2} dy + \int_{y=r_1}^{r_{-1}} \frac{2y}{r^2} \frac{\pi - \arccos\left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2} \right)}{\pi} dy =$$

$$= \int_{y=r_1}^{r_{-1}} \frac{2y}{r^2} \frac{\pi - \arccos\left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2} \right)}{\pi} dy + \frac{r_1^2}{r^2} \ .$$

If $\frac{1}{3} < x < 1$, i.e., $r_{-1} > r$:

$$F_{r_0/r_1}(x) = \int_{y=0}^{r} \frac{2y}{r^2} F_{\cos(\gamma)} \left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2} \right) dy = \int_{y=r_1}^{r} \frac{2y}{r^2} \frac{\pi - \arccos\left(\frac{y^2 + 4r^2}{4ry} - \frac{y}{4rx^2} \right)}{\pi} dy + \frac{r_1^2}{r^2}.$$

Note that the limit of the integral is different in the two cases.

Figure 14 illustrates $F_{r_0/r_1}(x)$.

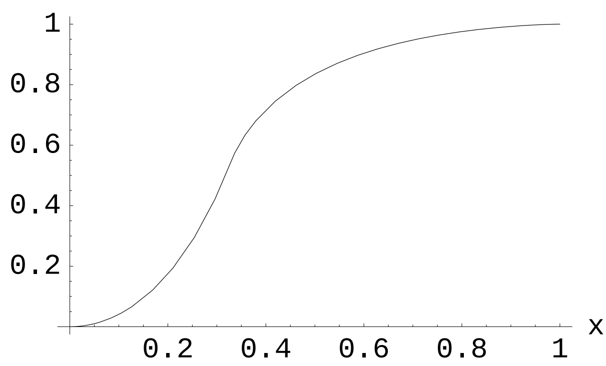

Figure 14: Probability distribution function of $X \triangleq \frac{r_0}{r_1}$.

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