Capacity planning of electric car charging station based on discrete time observations and MAP(2)/G/c queue

Csaba Farkas, Miklós Telek

***Abstract*—The modeling of electric car charging stations is essential for determining the required number of chargers in order to ensure the required service quality. In this paper we propose a new estimation method for the stochastic modeling of electric car charging stations, based on Markov arrival process (MAP).**

The input of the proposed model is empirical data for the arrival and service process of electric cars, given as histograms: the number of arriving cars during a fixed time slot (5 minutes in our case) and the histogram of service times (in 5 minutes granularity). The fact that observations on the continuous time process of car charging is available in discrete time steps poses a modeling challenge, which was not considered before. We propose a procedure to fit the observed data with a continuous time MAP of order 2 such that three moments and a correlation parameter of the discrete time observations are matched with three moments and the correlation parameter of the continuous time MAP for the given time interval. We implemented the fitting procedure in MATLAB and verified the obtain model of car charging station against simulation. As the MAP model of the arrival processes is reasonable close to the observations the obtained MAP/G/c queue allows a more accurate dimensioning of car charging station than the previously applied ones.

***Index Terms* — electric vehicle, charging station, Markov arrival process, point process fitting, simulation.**

I. INTRODUCTION

ELECTRIC vehicles (EVs) require an adequate number of chargers at charging stations in order to have neither unwanted long queues at the station, nor a poorly utilized system due to idleness. Furthermore, without enough chargers the customers will not buy EVs out of sheer fear of range anxiety (i.e. they will not find a charging station nearby where they can recharge their cars when needed), while if there are not enough EVs, there is no point in constructing charging stations. To cope with this problem, government or industrial subsidies are required, both in promoting the dissemination of EVs and in constructing charging stations. This paper intends to suggest a modeling procedure that would help in the charging station construction by determining the required number of chargers in a fast charging station. As this topic is highly relevant today, many papers deal with it. They can be classified into two sets: some papers use optimization algorithms to dimension a charging station: for example [1] proposes a multi-objective electric vehicle charging station planning method which can ensure charging service while reducing power losses and voltage deviations of distribution systems; authors of [2] study the EV charging station placement problem by finding the best locations to construct charging stations in a city in such a way, that they minimize the construction cost with coverage extended to the whole city and they also study the complexity of the station placement problem; authors of [3] developed a mathematical model for the optimal sizing of EV charging stations with the minimization of total cost associated to these stations and solved it by a modified primal-dual interior point algorithm.

Other papers use stochastic models, with which system performance can be studied: the literature review shows that they almost exclusively use M/M/c queues for charging station modeling, i.e., they assume the arrival and service process of EVs to be Poisson-processes. In this sense [4] models the fast charging stations with the help of an M/G/S/K queue and incorporates a fast charging model (i.e. charging characteristics show that charging power during fast charging is not constant) into the queuing analysis as well as the revenue model of the charging station; the authors of [5] present a methodology of modeling the overall charging demand of plug-in hybrid electric vehicles (PHEVs) and employ queuing theory (M/M/c model) to describe the behavior of multiple PHEVs; [6] uses a discrete-state, discrete-time Markov chain to define the states of all the EVs at each time step. Four states are considered in [6], depending on whether an EV is parked or not and on the parking location.

However, as [7] clearly states, most of the M/M/c models are based on some unrealistic assumptions without validation. [4] rejects the assumption that the service process can be modeled as a Poisson-process, as charging time depends on initial battery state of charge. This statement is supported by measurements made by the authors of [8], see Fig.1. Furthermore, according to [9], sojourn times are in general not exponentially distributed. In fact, the assumption that EV charging can be modeled by M/M/c queues has to our best knowledge never really been justified in the literature.

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Our aim in this paper is thus to create a model that can be used even if the arrival and service processes are not Poisson-processes. We propose a stochastic model based on Markov arrival processes (MAP) to capture the nature of electric car charging process. A similar approach is presented in [10], where the authors use a DMAP/PH/N/R model (where DMAP stands for discrete time MAP and PH for phase type distributed service time) to investigate a battery replacement strategy to increase the efficiency of chargers and save drivers' time at charging stations.

Our proposed approach is essentially different. We assume that cars arrive according to a continuous time stochastic process to which we have observations only in equidistance discrete time instances. This assumption is motivated by the lack of available experimental data. We take the input data from [11], where the motion of a taxi fleet composed of electric cars is simulated in 5 min long time periods. That is why we assume that the EV arrivals are known in every $\Delta = 5min$ long time periods. The fact that we need to fit a continuous time MAP to a set of discrete observation instances raises a new research challenge. To the best of our knowledge this problem is considered here for the first time. We would like to emphasize, that the main contribution of our paper is the applied methodology, which has not been applied for EV modeling yet and contains the solution of previously unavailable analysis step (MAP fitting based on restricted observations), the numerical example in Section V is presented to show the capabilities of the proposed approach.

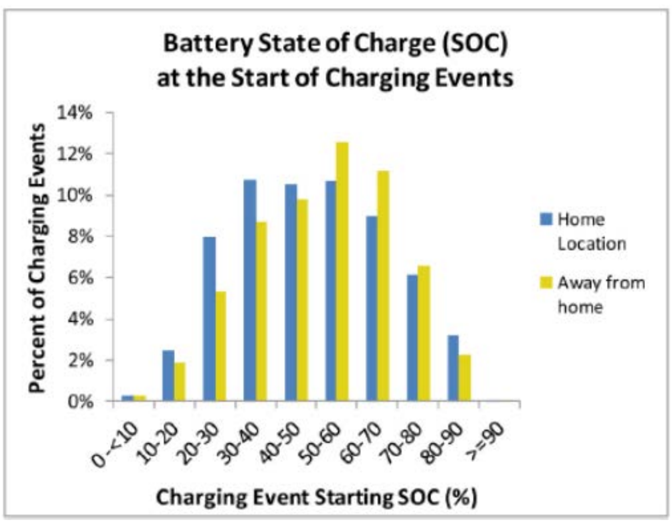

Fig. 1. Distribution of battery pack SOC at the start of charging by charging location [8]

The rest of the paper is structured as follows. Section II summarizes the basics of MAP modeling. In Section III we present the theoretical basis of restricted observation based MAP fitting and in Section IV the procedure itself. In Section V simulation results are presented and discussed. The paper is concluded in Section VI.

II. Modeling with Markov arrival process

This section introduces the basic properties of MAPs.

A. Markov arrival processes

The Markov arrival process is a point process where the arrivals are governed by a background continuous time Markov chain (CTMC). A possible interpretation of MAP is through the joint stochastic process $\{(N(t), J(t)): t \geq 0\}$ where $N(t)$ denotes the number of arrivals in the time interval $(0, t)$ and $J(t)$ denotes the state of the background CTMC (commonly referred to as phase) at time t. $J(t)$ is CTMC on the finite state space M, while $N(t)$ is a stochastic counting process depending on $J(t)$. The $(N(\cdot), J(\cdot))$ joint stochastic process is a CTMC on the state space $\{(n, j): n \geq 0, 1 \leq j \leq M\}$ with the infinitesimal generator matrix

$$Q = \begin{bmatrix} D_0 & D_1 & 0 & 0 \\ 0 & D_0 & D_1 & 0 \\ 0 & 0 & D_0 & D_1 \\ \vdots & \vdots & 0 & \ddots \end{bmatrix},$$

where

- D_0 and D_1 are $M \times M$ matrices,
- $D_1 \geq 0$ elementwise,
- $[D_0]_{i,j} \geq 0, 1 \leq i \neq j \leq M$ and $[D_0]_{i,i} < 0, 1 \leq i \leq M$,
- and $\sum_j [D_0]_{i,j} + \sum_j [D_1]_{i,j} = 0$ that is

$$(D_0 + D_1) \cdot \mathbf{1} = 0, \tag{5}$$

where $\mathbf{1}$ is the column vector of ones of the appropriate size.

This means that a Markov arrival process is defined by the matrices D_0 and D_1, where the elements of D_0 represent hidden transitions and elements of D_1 observable transitions.

We use second-order MAPs (denoted with MAP(2)) because they have significantly more modeling flexibility (e.g. correlated inter-arrival time) compared to Poisson processes while their computational complexity is still low. An important advantage of using MAP(2) is the availability of a canonical representation [12], which is a minimal unique Markovian representation for all members of the MAP(2) class. This means that $M = 2$, and both D_0 and D_1 are 2×2 matrices.

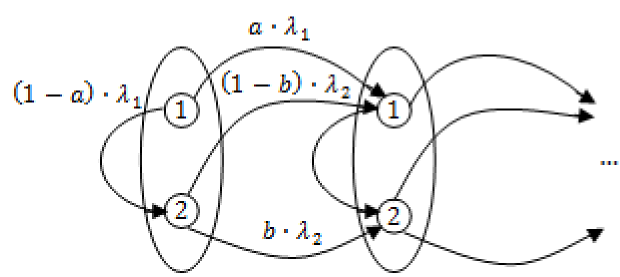

Fig. 2. Markov chain of the canonical MAP(2) with positive correlation

Depending on whether the correlation of consecutive inter-arrivals is positive or negative there are two different canonical forms [12]. Based on the properties of our data sets (which are discussed in Subsection C) we use only the canonical form with positive correlation in this paper. The D_0 and D_1 matrix representation of this canonical form is as follows:

$$D_0 = \begin{bmatrix} -\lambda_1 & (1-a)\cdot\lambda_1 \\ 0 & -\lambda_2 \end{bmatrix}, \quad (1)$$

$$D_1 = \begin{bmatrix} a\cdot\lambda_1 & 0 \\ (1-b)\cdot\lambda_2 & b\cdot\lambda_2 \end{bmatrix}, \quad (2)$$

where λ_1 and λ_2 are rate parameters, a and b are probabilities. The transition graph representation of this canonical form is depicted on Fig. 2.

The stationary distribution of the MAP arrivals in a Δ long time interval is given by the following z-transform expression:

$$p(\Delta, z) = \alpha \cdot e^{(D_0 + D_1\cdot z)\cdot\Delta} \cdot \mathbf{1}, \quad (3)$$

where $\alpha = [\alpha_0 \quad \alpha_1]$ is the time stationary phase distribution vector, and $\mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the summation vector of size 2. α is obtained from the D_0 and D_1 matrix representation as the solution of the linear system of equations

$$\alpha\cdot(D_0 + D_1) = 0, \quad \alpha \cdot \mathbf{1} = 1 \quad (4)$$

and it is

$$\alpha = \left[\frac{(1-b)\cdot\lambda_2}{(1-a)\cdot\lambda_1 + (1-b)\cdot\lambda_2} \quad \frac{(1-a)\cdot\lambda_1}{(1-a)\cdot\lambda_1 + (1-b)\cdot\lambda_2}\right].$$

B. Histogram of the empirical arrival process

The number of cars arriving to the charging station during a fixed time slot is depicted on Fig. 3 (the example obtained from [11]).

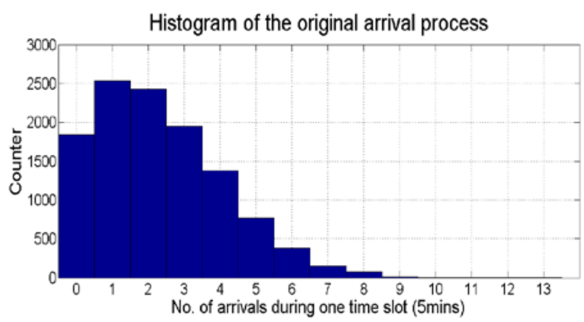

Fig. 3. Histogram showing the number of cars arriving in a Δ long time interval

We form the following z-transform polynomial from the histogram:

$$A(z) = \sum_i p_i \cdot z^i, \quad (6)$$

where p_i is the probability that i cars arrive in a Δ long time interval. The polynomial in our example (according to Fig. 3.) is the following:

$$A(z) = \frac{1}{11521}\cdot(z^{13} + z^{12} + 3z^{11} + 7z^{10} + 21z^9 + 63z^8 + \\ +149z^7 + 383z^6 + 765z^5 + 1326z^4 + 2023z^3 + 2393z^2 + \\ +2525z + 1861). \quad (7)$$

We fit the number of arrivals of a MAP(2) in a Δ long time interval, given in the form of (3), to this polynomial. More precisely we set the first three factorial moments of this data set, which can be obtained from $A(z)$ (as well as from the probabilities p_i, but we use $A(z)$ in order to exploit the similarity with the transform domain based computation of MAP(2) parameters) through its derivatives with respect to z, which are

- $\frac{d}{dz}A(z)|_{z=1} = 2.315$,
- $\frac{d^2}{dz^2}A(z)|_{z=1} = 6.2644$,
- $\frac{d^3}{dz^3}A(z)|_{z=1} = 18.2198$.

C. Correlation of the arrival data

The number of car arrivals in consecutive Δ long time intervals can be independent or dependent. We check the dependence structure of the car arrival process by computing the experimental correlation of observation intervals k lags apart. Having N samples the lag-k correlation is computed between the first $N-k$ observations: $x_1, x_2, \dots, x_{N-k}$ and the next $N-k$ observations $x_{k+1}, x_{k+2}, \dots, x_N$ according to the following expression [14]

$$\hat{\rho}_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x}_{(1)})\cdot(x_{t+1} - \bar{x}_{(k+1)})}{\sqrt{\sum_{t=1}^{N-k}(x_t - \bar{x}_{(1)})^2 \cdot \sum_{t=k+1}^{N}(x_t - \bar{x}_{(k+1)})^2}}, \quad (8)$$

where $\bar{x}_{(1)}$ is the experimental mean of the first $N-k$ observations and $\bar{x}_{(k+1)}$ is the experimental mean of the last $N-k$ observations.

The correlation of the data sample for arrivals can also be obtained by using MATLAB's *autocorr* function. The experimental lag-k correlation parameters are depicted in Fig. 4. The lag-1 correlation is $\hat{\rho}_1 = 0.2443$.

To emphasize the applicability of the proposed method we note that the MAP(2) class can also represent the case when the correlation is zero. In that case parameter b equals to zero, thus the canonical form of MAP(2) simplifies to

$$D_1 = \begin{bmatrix} a\cdot\lambda_1 & 0 \\ \lambda_2 & 0 \end{bmatrix}, \quad (9)$$

while D_0 remains the same.

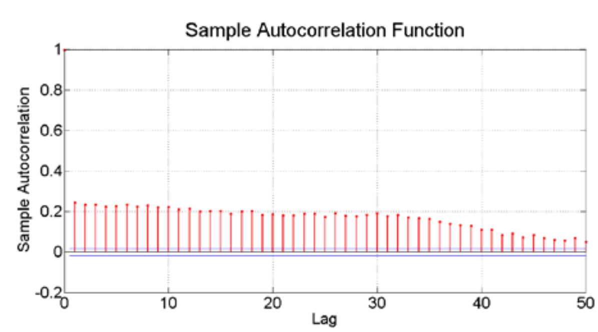

Fig. 4. Correlation of the original dataset

D. Moment matching procedure

The MAP(2) canonical form has four unknown parameters (a, b, λ_1, λ_2). We set these parameters such that the first three moments of the inter-arrival time distribution ($\widehat{m}_k, k=1,2,3$), and the lag-1 correlation of the experimental data ($\hat{\rho}_1$) is matched. This procedure requires calculating these four parameters from both the empirical data and the MAP(2) canonical from.

For the later ones we need the first 3 derivatives of $p(\Delta, z)$ with respect to z at $z=1$ and the lag-1 correlation from the double transform description of the number of car arrivals in consecutive intervals given in (23).

Finally, we have to solve the obtained system of equation for the variables a, b, λ_1 and λ_2.

III. Derivatives of $p(\Delta, z)$ and the Correlation

Although theoretically the symbolic derivation of the $p(\Delta, z)$ polynomial is possible, but it is computationally challenging. Instead of the direct, brute force solution, we apply some algebraic manipulations to make computations faster (and feasible).

A. First moment

Only the part $e^{(D_0+D_1\cdot z)\cdot\Delta}$ contains the parameter z in (3), thus we have to calculate $\frac{d}{dz}e^{(D_0+D_1\cdot z)\cdot\Delta}\Big|_{z=1}$. The Taylor-series expansion of the matrix exponential function is

$$\frac{d}{dz}e^{(D_0+D_1\cdot z)\cdot\Delta} = \tag{10}$$
$$=\frac{d}{dz}\sum_{i=0}^{\infty}\frac{\Delta^i}{i!}\cdot(D_0+D_1\cdot z)^i=\sum_{i=0}^{\infty}\frac{\Delta^i}{i!}\cdot\frac{d}{dz}(D_0+D_1\cdot z)^i.$$

Due to the matrices in the series, the order of the parts matter this time. The calculation yields

$$\frac{d}{dz}e^{(D_0+D_1\cdot z)\cdot\Delta}\Big|_{z=1} = \tag{11}$$
$$=\sum_{i=1}^{\infty}\frac{\Delta^i}{i!}\cdot\sum_{k=0}^{i-1}(D_0+D_1\cdot z)^k\,D_1\cdot(D_0+D_1\cdot z)^{i-k-1}\Big|_{z=1},$$

so

$$p(\Delta,z)=\alpha\cdot e^{(D_0+D_1\cdot z)\cdot\Delta}\cdot\mathbf{1}= \tag{12}$$
$$=\alpha\cdot\sum_{i=1}^{\infty}\frac{\Delta^i}{i!}\cdot\sum_{k=0}^{i-1}(D_0+D_1\cdot z)^k\,D_1\cdot(D_0+D_1\cdot z)^{i-k-1}\Big|_{z=1}\cdot$$
$$\cdot\mathbf{1}=\alpha\cdot\sum_{i=1}^{\infty}\frac{\Delta^i}{i!}\cdot\sum_{k=0}^{i-1}(D_0+D_1)^k\,D_1\cdot(D_0+D_1)^{i-k-1}\cdot\mathbf{1}.$$

This means that we can simplify equation (12) further as follows:

$$\frac{d}{dz}p(\Delta,z)|_{z=1}=\alpha\cdot\Delta\cdot D_1\cdot\mathbf{1}. \tag{13}$$

This formula gives the first moment.

B. Second moment

The calculation is similar to the first moment: we want to obtain $\frac{d^2}{dz^2}e^{(D_0+D_1\cdot z)\cdot\Delta}\Big|_{z=1}$, so we take the derivation of equation (11) with respect to z, and we obtain

$$\frac{d^2}{dz^2}e^{(D_0+D_1\cdot z)\cdot\Delta} = \tag{14}$$
$$=\frac{d}{dz}\left[\sum_{i=1}^{\infty}\frac{\Delta^i}{i!}\cdot\sum_{k=0}^{i-1}(D_0+D_1\cdot z)^k\cdot D_1\cdot(D_0+D_1\cdot z)^{i-k-1}\right].$$

After the Taylor-series expansion and the derivation of the first few parts we can see that the solution is

$$\frac{d^2}{dz^2}e^{(D_0+D_1\cdot z)\cdot\Delta}=\sum_{i=2}^{\infty}\frac{\Delta^i}{i!}\cdot(\sum_{k=1}^{i-1}\sum_{l=0}^{k-1}(D_0+D_1\cdot z)^l\cdot D_1\cdot$$
$$\cdot(D_0+D_1\cdot z)^{k-l-1}\cdot D_1\cdot(D_0+D_1\cdot z)^{i-k-1}+$$
$$+\sum_{k=0}^{i-2}\sum_{l=0}^{i-k-2}(D_0+D_1\cdot z)^k\cdot D_1\cdot(D_0+D_1\cdot z)^l\cdot D_1\cdot$$
$$\cdot(D_0+D_1\cdot z)^{i-k-l-2}). \tag{15}$$

After further simplifications using (4) and (5) we obtain

$$\alpha\cdot\frac{d^2}{dz^2}e^{(D_0+D_1\cdot z)\cdot\Delta}\Big|_{z=1}\cdot\mathbf{1}= \tag{16}$$
$$=2!\cdot\alpha\cdot\sum_{i=2}^{\infty}\frac{\Delta^i}{i!}\cdot D_1\cdot(D_0+D_1)^{i-2}\cdot D_1\cdot\mathbf{1}.$$

Let's denote D_0+D_1 with D. This means that we can reformulate (16) as

$$2!\cdot\alpha\cdot D_1\cdot\sum_{i=2}^{\infty}\frac{\Delta^i}{i!}\cdot D^{i-2}\cdot D_1\cdot\mathbf{1}, \tag{17}$$

where we can see that $\sum_{i=2}^{\infty}\frac{\Delta^i}{i!}\cdot D^{i-2}$ resembles to the Taylor-series expansion of the matrix exponential function $e^{D\Delta}$. To obtain that formula, we have to alter (17) a little, but we cannot extend the formula by multiplying simply with $D^2\cdot D^{-2}$, because D^{-2} does not exist as D is singular (see (5)). Instead, we have to do the extension using $(D-\mathbf{1}\cdot\alpha)^2$ $\cdot(D-\mathbf{1}\cdot\alpha)^{-2}$. If D is an irreducible Markov-chain, then $D-\mathbf{1}\cdot\alpha$ is not singular [15]. With further calculations, utilizing (4) and (5), we can reformulate (17) as

$$\alpha\cdot\frac{d^2}{dz^2}e^{(D_0+D_1\cdot z)\cdot\Delta}\Big|_{z=1}\cdot\mathbf{1}= \tag{18}$$
$$=2!\cdot[\alpha\cdot D_1\cdot\frac{\Delta^2}{2!}\cdot D_1\cdot\mathbf{1}+\alpha\cdot D_1\cdot$$
$$\cdot\left(e^{D\cdot\Delta}-I-D\cdot\Delta-\frac{(D\cdot\Delta)^2}{2!}\right)\cdot(D-\mathbf{1}\cdot\alpha)^{-2}\cdot D_1\cdot\mathbf{1}].$$

This is the second momentum of the number of arrivals in $(0,\Delta)$, where I denotes the 2×2 identity matrix.

C. Third moment

Based on our calculations regarding the first and the second moment, we can determine the third one. We have seen that there was a single summation in the case of the first moment, a double summation in the second and here in the case of the third moment, a triple summation would come, with the argument being something like

$$(D_0 + D_1 \cdot z)^k \cdot D_1 \cdot (D_0 + D_1 \cdot z)^l \cdot D_1 \cdot (D_0 + D_1 \cdot z)^m \cdot D_1 \cdot \cdot (D_0 + D_1 \cdot z)^n.$$

This one, however is hard to deal with, as not all of the factors disappear when we multiply with the vectors α and $\mathbf{1}$, so convolutions would appear. To make calculations easier, we can trace the summation back to matrix products: if we raise to powers the

$$\mathbb{D}_1 = \begin{bmatrix} D & D_1 \\ 0 & D \end{bmatrix} \tag{19}$$

hyper matrix, we can obtain the factors in the aforementioned sums; they are given by the upper right block of the $\mathbb{D}_1$ hyper matrix, so we have to multiply $\mathbb{D}_1$ with $[I_2 \quad 0_2]$ from the left and with $\begin{bmatrix} 0_2 \\ I_2 \end{bmatrix}$ from the right, where I_2 is the 2×2 identity matrix and 0_2 is the 2×2 zero matrix. Using the hyper matrix we can obtain the third momentum as follows:

$$\alpha \cdot \frac{d^3}{dz^3} e^{(D_0 + D_1 \cdot z) \cdot \Delta}\bigg|_{z=1} \cdot \mathbf{1} = 3! \cdot \alpha \cdot D_1 \cdot [I_2 \quad 0_2] \cdot \sum_{n=3}^{\infty} \frac{\Delta^n}{n!} \cdot \cdot \mathbb{D}_1^{n-2} \cdot \mathbb{D}_e^2 \cdot \mathbb{D}_e^{-2} \cdot \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} \cdot D_1 \cdot \mathbf{1}, \tag{20}$$

where

$$\mathbb{D}_e = \begin{bmatrix} D - \mathbf{1} \cdot \alpha & D_1 \\ 0 & D - \mathbf{1} \cdot \alpha \end{bmatrix}. \tag{21}$$

If we calculate the powers of the $\mathbb{D}_e$ hyper matrix and substitute the obtained results into (20), we can see that the third moment is simplified:

$$\alpha \cdot \frac{d^3}{dz^3} e^{(D_0 + D_1 \cdot z) \cdot \Delta}\bigg|_{z=1} \cdot \mathbf{1} = 3! \cdot \alpha \cdot D_1 \cdot [I_2 \quad 0_2] \cdot \sum_{n=3}^{\infty} \frac{\Delta^n}{n!} \cdot \cdot \left(\mathbb{D}_1^n + \begin{bmatrix} 0 & D^{n-3} \cdot (-D \cdot D_1 \cdot \mathbf{1} \cdot \alpha + D_1 \cdot \mathbf{1} \cdot \alpha) \\ 0 & 0 \end{bmatrix} \right) \cdot \mathbb{D}_e^{-2} \cdot \cdot \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} \cdot D_1 \cdot \mathbf{1}. \tag{22}$$

The inner hyper matrix can be rewritten into matrix exponential form utilizing the summation, so we can obtain the formula for the third moment:

- $\alpha \cdot D_1 \cdot [I_2 \quad 0_2] \cdot \sum_{n=3}^{\infty} \frac{\Delta^n}{n!} \cdot \mathbb{D}_1^n \cdot \mathbb{D}_e^{-2} \cdot \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} \cdot D_1 \cdot \mathbf{1} = \alpha \cdot \cdot D_1 \cdot [I_2 \quad 0_2] \cdot \left(e^{\mathbb{D}_1 \cdot \Delta} - I_4 - \mathbb{D}_1 \cdot \Delta - \frac{(\mathbb{D}_1 \cdot \Delta)^2}{2!} \right) \cdot \mathbb{D}_e^{-2} \cdot \cdot \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} \cdot D_1 \cdot \mathbf{1}$ is the matrix exponential form of (22) without the inner hyper matrix and
- $-\left(e^{D\Delta} - I - D\Delta - \frac{(D\Delta)^2}{2!} \right) \cdot (D - \mathbf{1} \cdot \alpha)^{-2} \cdot D_1 \cdot \mathbf{1} \cdot \alpha$,
- $\frac{\Delta^3}{3!} \cdot D_1 \cdot \mathbf{1} \cdot \alpha + \left(e^{D\Delta} - I - D\Delta - \frac{(D\Delta)^2}{2!} - \frac{(D\Delta)^3}{3!} \right) \cdot \cdot (D - \mathbf{1} \cdot \alpha)^{-3} \cdot D_1 \cdot \mathbf{1} \cdot \alpha$

are the additional terms obtained from the inner hyper matrix. With this, the third moment is also given.

D.Correlation

The correlation is calculated from the joint probability distribution of the number of cars arrived in the first and the second 5 minute time step. Let $P(z_1 z_2)$ denote the z-transform of the joint probability, then

$$P(z_1 z_2) = \alpha \cdot e^{(D_0 + D_1 \cdot z_1) \cdot \Delta} \cdot e^{(D_0 + D_1 \cdot z_2) \cdot \Delta} \cdot 1. \tag{23}$$

From this probability we can calculate the expected value of these variables as follows:

$$E(x_1 x_2) = \frac{\partial}{\partial z_1} \frac{\partial}{\partial z_2} P(z_1 z_2)|_{z_1 = z_2 = 1}. \tag{24}$$

The correlation is obtained from the expected value as follows:

$$corr = \frac{E(x_1 x_2) - E(x_1) \cdot E(x_2)}{\sigma_{x_1} \cdot \sigma_{x_2}}, \tag{25}$$

where σ_{x_1} and σ_{x_2} are the variances of the random values x_1 and x_2. As x_1 and x_2 represent the number of arriving cars in the first and the second time slot, respectively, and the investigated process is assumed to be stationary, the variances are equal to each other and can be calculated from the moments as shown in (26):

$$\sigma^2 = E(x_1^2) - \left(E(x_1) \right)^2. \tag{26}$$

We have already calculated all the required parameters before (see (13), (18) and (22)), so the correlation is

$$corr = \frac{\alpha \cdot D_1 \cdot \left\{ \left[\left(e^{D \cdot \Delta} - I - \Delta \cdot \mathbf{1} \cdot \alpha \right) \cdot (D - \mathbf{1} \cdot \alpha)^{-1} \right]^2 - \Delta^2 \cdot \mathbf{1} \cdot \alpha \right\} \cdot D_1 \cdot \mathbf{1}}{\alpha \cdot D_1 \cdot \left[\Delta^2 \cdot (I - \mathbf{1} \cdot \alpha) + 2! \cdot \left(e^{D \cdot \Delta} - I - D \cdot \Delta - \frac{(D \cdot \Delta)^2}{2!} \right) \cdot (D - \mathbf{1} \cdot \alpha)^{-2} \right] \cdot D_1 \cdot \mathbf{1}}. \tag{27}$$

IV. The Matching Procedure

We have obtained the symbolic forms of the first three moments of the $p(\Delta, z)$ polynomial and the $corr$ parameter. Now, we have to solve the system of non-linear equations

$$\begin{cases} \frac{d}{dz} p(\Delta, z)|_{z=1} = \frac{d}{dz} A(z)|_{z=1} = 2.315 \\ \frac{d^2}{dz^2} p(\Delta, z)|_{z=1} = \frac{d^2}{dz^2} A(z)|_{z=1} = 6.2644 \\ \frac{d^3}{dz^3} p(\Delta, z)|_{z=1} = \frac{d^3}{dz^3} A(z)|_{z=1} = 18.2198 \\ corr = \hat{\rho}_1 = 0.2443 \end{cases}$$

for the variables a, b, λ_1, λ_2. Fortunately, the *fsolve* function of MATLAB managed to obtain the results thanks to the algebraic manipulations summarized in the previous section. Without those manipulations all of our attempts failed. For our data set the obtained solution was:

- $a = 0.2971$,
- $b = 0.6762$,
- $\lambda_1 = 0.3196$,
- $\lambda_2 = 0.9861$,

which gives a proper MAP(2) canonical form [12] with valid probability and rate values.

A. The service process

The service process can as well be modeled by a MAP(2) process: the fitting is done similarly, like before. In our example, however we constructed a simpler model for the service process as the histogram of the service time is much simpler, as depicted on Fig. 5.

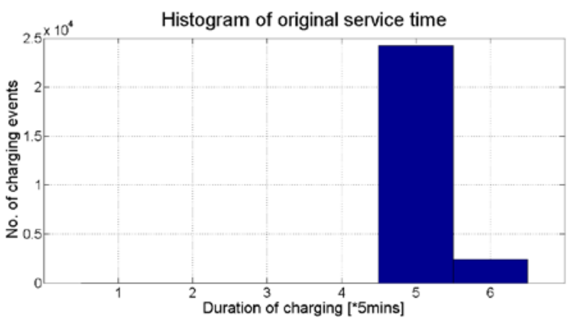

Fig. 5. Histogram of service time duration obtained from [11]

It is clear from Fig. 5. that in this case the MAP(2) modeling would be preposterous: all we have to do is to determine the probabilities of each option (i.e. charging lasts for 5 or 6 time intervals) and raffle one of these numbers randomly, using the obtained probabilities for weighting. This is why the service process is considered to be G (general) instead of MAP(2) in our example. We have to note that this service process obtained from [11] implicitly incorporates the initial battery state of charge (SOC) of cars. For further applications data regarding battery SOC is also needed to be able to model the service process properly.

V. Simulation of the Electric Car Charging Station

We simulated the whole process using MATLAB. Cars arrive to charge according to the MAP(2) process with the calculated parameters.

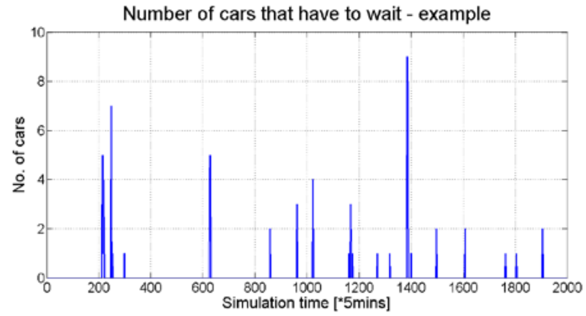

Fig. 6. Number of cars that have to wait - example

If there is any available charger, they connect to it and begin charging and the charger becomes occupied. Charging time is raffled according to the service process as presented in Section IVA. In every time step, the charging time left for a given car decreases and if it reaches 0, the car is recharged, leaves the station and the charger becomes available again. If there is no available charger, the incoming cars have to wait, hence a waiting queue forms. The waiting queue has an FCFS discipline. For a given number of chargers we can determine the number of cars that have to wait (see Fig. 6. as an example). The aim is to have enough chargers in the charging station so that the probability of waiting is below a pre-defined threshold.

Running the simulation for 100 times we can determine the number of waiting cars for a given number of chargers (see Fig. 7.).

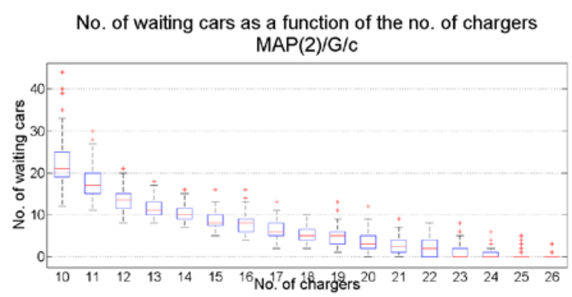

Fig. 7. Number of cars that have to wait for given no. of chargers

To indicate the variance of the simulation we used MATLAB's *boxplot* function, where on each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually. We can compare the obtained results with the results from the original simulation from [11] (see Fig. 8.). If we set the threshold of the permitted number of waiting cars to be 5, we can see that both the original model and the MAP model also results in a required number of 19 chargers. Fig. 8. also depicts that although the MAP model underestimates the number of waiting cars when there are too few (in fact, an inadequate number of) chargers, the result of the MAP model converges to the original dataset, thus predicts the required number of chargers well. This means that with the proposed model one can determine the required number of fast chargers in a charging station, if traffic data is known.

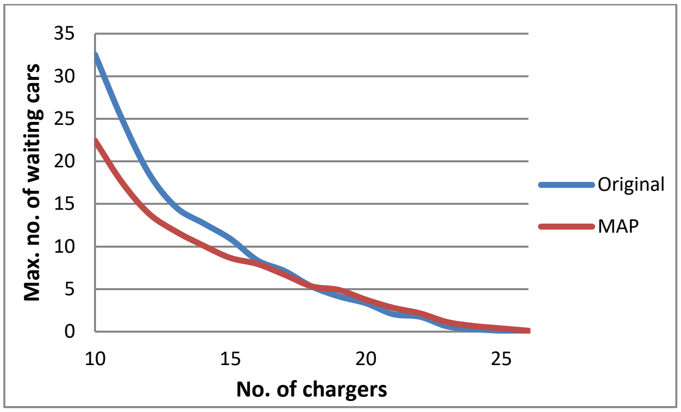

Fig. 8. Comparison of the obtained results (MAP vs. original)

VI. Conclusion

The problem of appropriately dimensioned recharging units for electric vehicles is as important as appropriately dimensioned fueling units for cars with internal combustion engines. In this paper, exceeding the modeling restrictions of previously applied Poisson process based analyses, we addressed this dimensioning problem using continuous time MAPs assuming that only aggregate experimental data is available for time intervals of the same length. This limitation on the available data arises new modeling challenges for parameter matching of MAPs. We proposed a solution method using the canonical representation of MAP(2) processes. The proposed method and its results are verified against simulation and suggests appropriately accurate prediction for the required number of charging stations.

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