On Providing Blocking Probability- and Throughput Guarantees in a Multi-service Environment

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Abstract

As the Internet evolves from a packet network supporting a single best effort service class towards an integrated infrastructure supporting several service classes - some with QoS guarantees - there is a growing interest in the introduction of admission control and in devising bandwidth sharing strategies, which meet the diverse needs of QoS-assured and elastic services. In this paper we show that the extension of the classical multi-rate loss model is possible in a way that makes it useful in the performance analysis of a future admission control based Internet that supports traffic with peak rate guarantee as well as elastic traffic. After introducing the model, it is applied for the analysis of a single link, where it sheds light on the trade-off between *blocking probability* and *throughput*. For the investigation of this trade-off, we introduce the throughput-threshold constraint, which bounds the probability that the throughput of a traffic flow drops below a predefined threshold. Finally, we use the model to determine the optimal parameter set of the popular partial overlap link allocation policy: we propose a computationally efficient algorithm that provides blocking probability- and throughput guarantees. We conclude that the model and the numerical results provide important insights in traffic engineering in the Internet.

Keywords

Bandwidth sharing objectives, multi-rate loss models, blocking probabilities, throughput, Markov reward models.

I. Introduction

In recent years there have been significant advances in researching and standardizing mechanisms that are capable of providing service differentiation in the Internet. While there still seems to be a wide span of the methods which aim at providing QoS differentiation between contending flows, it is widely accepted that there is a need for traffic engineering mechanisms which control the access of the different traffic classes to network bandwidth resources. In particular, there is a growing interest in devising *bandwidth sharing* algorithms which can cope with a high utilization in the network and at the same time take into account the different traffic classes' throughput and blocking probability requirements. Recent research results indicate that it is meaningful to exercise call admission control (CAC) even for elastic traffic, because CAC algorithms (and consequently the *blocking* of some arriving flows) provide a means to prevent e.g. TCP sessions from excessive throughput degradation [16], [17]. From this perspective it is important to develop models and computational techniques that make analytical studies of the behavior of such future types of networks possible.

Generally the issue of bandwidth sharing should be considered in the context of dynamically arriving and departing flows, which naturally calls for the application of the classical multi-rate loss models. These models have proved useful in the dimensioning and performance evaluation of circuit switched as well as ATM networks. Thus, they provide motivation for extending the applicability of this modeling paradigm to Internet context. Unfortunately, a direct application of the multi-rate models for traffic engineering in the Internet is non-trivial, because:

- By definition, it is not possible to associate a constant bandwidth with elastic services, like the best effort (without minimum rate guarantee) or the "better than best effort" (with minimum rate guarantee) type of services. The bandwidth occupied by the elastic flows depend on the current load on the link and on the scheduling and rate control algorithms applied in the network nodes.
- The notion of blocking, when applied to elastic flows, needs to be reconsidered because an arriving elastic flow might get into service even if at the arrival instant there is no (or very small) bandwidth available.
- For many services, we need to take account of the fact that the actual residency time of the elastic flows depend on the throughput which the flow receives. For instance, an ftp session would last longer if its throughput decreases. (Real-time

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services' holding time, on the other hand, is insensitive to the throughput, which is the case, for instance, with a flow associated with an adaptive video codec. As specified later, we will refer to these services as adaptive stream services.) Since we cannot directly use the reservation based multi-rate models, we seek the meaningful extensions so as to allow the inclusion of both QoS-assured and elastic traffic into a common framework. After putting our work into context, in Section III we consider a single link where flows belonging to three service classes arrive. *Non-adaptive stream* (also called *rigid*) calls require peak bandwidth allocation. *Adaptive* stream calls and *elastic* calls are modeled as being associated with both a peak- and a minimum bandwidth requirement, and they are allowed into service as long as their respective minimum bandwidth requirements are fulfilled. Furthermore, we allow the bandwidth given to these flows to fluctuate in time, depending on the instantaneous available capacity of the link. While the actual residency time of adaptive calls does not depend on the acquired throughput, the elastic calls' holding time is determined by the actual throughput that the flow receives. Specifically, in the case of Poisson arrivals, these assumptions lead naturally to the application of Markov reward models (MRM). We argue that the *completion time* [6] of the MRM corresponds to the flow residency times of elastic flows that depend on the throughput. Next, in Section IV we adopt the well known *partial overlap* (POL) link bandwidth sharing policy [29] to our model. Also in Section IV we introduce the notion of the throughput threshold constraint, which is a constraint on the probability that the user-perceived throughput during the transfer of a file of size x drops below a certain level. Also in this section, we consider a simple yet efficient link capacity sharing method, which allows for the tuning of the blocking probability vs. throughput trade-off for each traffic class.

In Sections V and VI we are concerned with the computation of blocking probabilities and throughputs, exploring the limitations of our modeling method in terms of the size of the state space. A recent result [30] allows us to study large state spaces. We find that in order for the dimensioning to take into account the throughput threshold constraint, the steady state analysis of the associated MRM is not sufficient, and therefore we seek methods for finding the higher moments of the completion time. Section VII gives an example of the application of the model.

We conclude by outlining some possibilities for further applications of our model in Internet-related questions.

II. Related Works

The theory of multi-rate loss models is covered by for instance [13], [24] and [26]. Application examples of this modeling paradigm include those concentrating on routing and call admission algorithms for QoS assured traffic classes in [9] and [29] and also those that are concerned with the optimal sharing of link bandwidth resources as in [8] and in [7], [18], [19]. However, none of these models addresses the issue of applying this model to cases where elastic traffic is also present in the network, as detailed by the three bullet items in the Introduction.

The notion of call admission control for elastic traffic and fairness issues are discussed by J. Roberts and L. Massoulie in a number of publications, see [15], [16], [17] and [23]. In fact, we feel that our present work here is in line with these papers, and extend them by proposing a computational model to arrive at specific performance measures on the throughputs and the blocking probabilities. The blocking probability vs. throughput tradeoff is also emphasized and directly connected to the issue of charging by Kelly and Gibbens in [12], [14].

The extension of the multi-rate model to include elastic services was proposed independently of each other by Blaabjerg et al. in [5] and by Altman et al. in [3]. The application of the MRM to compute the mean transfer time of files with exponentially distributed sizes and the blocking probabilities for the complete sharing method and assuming two traffic classes was proposed already by Blaabjerg *et al.* in [4]. Those results have been extended for the partial overlap link allocation strategy ("mixed scenario") by Nunez Queija *et al.* in [20], where the authors are concerned with the computation of the blocking probabilities and also of first moment of the transfer time of a file of size x.

The impact of pricing on the optimal bandwidth sharing strategies, again assuming two traffic classes is considered by Altman *et al.* in [3] and by Fodor *et al.* in [11].

From a more practical point of view, specifically examining the TCP traffic (which is the predominant example on the elastic traffic class in the Internet), Feng *et al.* find it beneficial to provide a minimum throughput for TCP connections, because in that case the TCP algorithm can be modified such that the "goodput" of TCP connections is much improved [10].

Our contribution to this line of works is twofold. First, by applying the multi-rate loss framework with three traffic classes and assuming the partial overlap link allocation technique we formulate the trade-off between the blocking probabilities and the throughput as an optimization task. We will find that the we need the higher moments of the file transmission time for any file size x as well. Second, we propose an efficient computational technique to derive numerical results on the single link level with large state spaces. The proposed numerical approach allows to consider models with $\sim 10^6$ states.

III. The Multi-class Model of a Single Link : Assumptions and Notations

In this section we formulate the Markovian model of a single transmission link serving peak-bandwidth assured (rigid), adaptive stream and elastic traffic classes. In the presentation we restrict ourselves to these three traffic classes, noting that the model can be extended to more general cases.

Similarly to [3] and [20], we will assume that calls of all three classes arrive at the link according to independent Poisson processes. That is, we assume that the arrival process of requests for document transfer on a given network route is Poisson. As pointed out in [16], this process results naturally when a large population of users emits requests independently, each at a relatively low intensity. Poisson statistics at the call (flow) level have been confirmed in observations of Web traffic in [2]. We note, however, that refinements of this assumption are the topic of current research, see for instance Section 3.2 of [16].

The system under consideration consists of a transmission link of capacity C, Calls arriving at the link belong to one of the following three traffic classes:

- *Non-adaptive stream* or *rigid* traffic class flows are characterized by their peak bandwidth requirement b_1, flow arrival rate λ_1 and departure rate μ_1;
- *Adaptive* stream class flows are characterized by their peak bandwidth requirement b_2, minimum bandwidth requirement b_2^{min}, flow arrival rate λ_2 and departure rate μ_2. Although the bandwidth occupied by adaptive flows may fluctuate as a function of the link load, their actual holding time is not influenced by the received throughput throughout their residency in the system. This is the case for instance with an adaptive video codec, which, in case of throughput degradation decreases the quality of the video images and thereby occupies less bandwidth.
- *Elastic* class flows are characterized by their peak bandwidth requirement b_3, minimum bandwidth requirement b_3^{min}, flow arrival rate λ_3, and their *ideal* departure rate μ_3. The ideal departure rate is experienced when the peak bandwidth is available. The actual instantaneous departure rate is proportional to the bandwidth of the flows. Note that this class can be further classified into two subclasses. If the minimum accepted bandwidth is 0, then this class is the model of the best effort traffic class. If the minimum accepted bandwidth is greater than zero, then this class corresponds to the "better-than-best-effort" traffic class. A typical example of this class is the file transfer protocol (ftp).

We denote the actual bandwidth associated with a flow of class-2 and class-3 in a given system state with b_2^r and b_3^r, both of which vary in time as flows arrive and depart. We will also use the quantity $r_{min} := b_{min}/b$ associated with elastic flows with minimum bandwidth requirements. All three types of flows arrive according to independent Poisson processes, and the (ideal) holding time for the rigid, adaptive and elastic flows are exponentially distributed. As we will see, the moments of the actual holding time of the elastic flows can be determined using the theory of Markov reward processes.

To ensure a given QoS of the different flows (that, in general, differ in their peak and minimum bandwidth, i.e. $b_2 \neq b_3$, $b_2^{min} \neq b_3^{min}$) we need to establish some policy which governs the bandwidth sharing among the adaptive stream and elastic classes. For this reason, we define the following bandwidth sharing rules between these two classes.

- If there is enough bandwidth for all flows to get their respective peak bandwidth demands, then class-2 and class-3 flows occupy b_2 and b_3 bandwidth units respectively.
- If there is a need for bandwidth compression, i.e. $n_1 \cdot b_1 + n_2 \cdot b_2 + n_3 \cdot b_3 > C$, then the bandwidth compression of the flows is such that $r_2 = r_3$, where $r_2 = b_2^r/b_2$ and $r_3 = b_3^r/b_3$, as long as the minimum rate constraint is met for both classes (i.e. $b_2^{min}/b_2 \leq r_2 \leq 1$ and $b_3^{min}/b_3 \leq r_3 \leq 1$).
- If there is still need for further bandwidth compression, but either one of the two classes does not tolerate further bandwidth decrease, (i.e. r_i is already b_i^{min}/b_i for either $i = 2$ or $i = 3$) at the time of the arrival of a new flow, then the service class which tolerates further compression decreases equally the bandwidth occupied by its flows, as long as the minimum bandwidth constraint is kept for this traffic class.

Three underlying assumptions of the above model are noteworthy. First, we assume that both the adaptive and the elastic flows are greedy, in the sense that they always occupy the maximum possible bandwidth on the link, which is the smaller of their peak bandwidth requirement (b_2 and b_3 respectively) and the equal share (in the above sense) of the bandwidth left for them by the rigid flows (which will depend on the link allocation policy). Second, we assume that all adaptive and elastic flows in progress share proportionally equally the available bandwidth among themselves, i.e. the newly arrived flow and the in-progress flows will be squeezed to the same r_i value. (This assumption actually corresponds to a weighted max-min fair allocation the weights being determined by the peak rates of the flows. By associating a minimum and maximum bandwidth requirements with the flows we in this paper focus on the throughput and blocking probability performance measures.) If a newly arriving flow decreased the flow bandwidth below b_2^{min} and b_3^{min} (i.e. both the adaptive and the elastic classes were compressed to their respective minima), that flow is not admitted into the system, but it is blocked and lost. Note that all arriving flows are allowed to "compress" the in-service adaptive and elastic flows, as long as the minimum bandwidth constraints are kept. Third, the model assumes that the rate control of the adaptive and elastic flows in progress is ideal, in the sense that an infinitesimal amount of time after any system state change (i.e. flow arrival and departure) these sources readjust their current bandwidth on the link. We realize that the connection between packet level mechanisms and the call level model we consider here needs further research, but it is not the topic of this paper.

It is intuitively clear that the residency time of the elastic flows in this system depends not only on the amount of data they want to transmit (which is a random variable), but also on the bandwidth they receive during their holding times. Similarly, the amount of data transmitted through an adaptive elastic flow depends on the received bandwidth. In order to specify this relationship we define the following quantities:

- $\theta_2(t)$ and $\theta_3(t)$ defines the instantaneous *throughput* of adaptive and elastic flows of at time t, respectively, (e.g., if there are n_1, n_2, n_3 rigid, adaptive, and elastic flows in the system at time t, respectively, the instantaneous throughput are $min(b_2, (C - n_1 \cdot b_1 - n_3 \cdot r_3(n_1, n_2, n_3) \cdot b_3)/n_2)$ and $min(b_3, (C - n_1 \cdot b_1 - n_2 \cdot r_2(n_1, n_2, n_3) \cdot b_2)/n_3))$ for adaptive and elastic flows, respectively. Note that $\theta_2(t)$, and $\theta_3(t)$ are discrete random variables (r.v.) for any $t \geq 0$.
- $\tilde{\theta}_t = \frac{1}{t}\int_0^t \theta_2(\tau)d\tau$ defines the *throughput* of the adaptive flow whose holding time is t.
- $\tilde{\theta} = \int_0^\infty \tilde{\theta}_\tau \; dF(\tau) = \mu_2 \int_0^\infty \tilde{\theta}_\tau \; e^{-\mu_2 \tau} \; d\tau$ (r.v.) defines the *throughput* of the adaptive flow, where $F(t)$ is the exponentially distributed flow holding time.
- $T_x = \inf\{t \mid \int_0^t \theta_3(\tau)d\tau \geq x\}$ (r.v.) gives the time it takes for the system to transmit x amount of data through an elastic flow,
- $\hat{\theta}_x = x/T_x$ defines the *throughput* of the elastic flow during the transmission of x data unit. Note that θ_x is a continuous r.v.
- $\hat{\theta} = \int_0^\infty \hat{\theta}_x \; dG(x) = \mu_3/b_3 \int_0^\infty \hat{\theta}_x \; e^{-x \; \mu_3/b_3} \; dx$ (r.v.) defines the *throughput* of the elastic flow, where the amount of transmitted data is exponentially distributed with parameter μ_3/b_3.

In addition, we associate the maximum accepted blocking probability with all three traffic classes i.e., B_1^{max}, B_2^{max} and B_3^{max}, respectively, and the minimum accepted throughput $\tilde{\theta}^{min}$, $\hat{\theta}^{min}$ with the adaptive and elastic classes respectively. (The meaning of the minimum accepted throughput and their relation with the random variables, $\tilde{\theta}, \hat{\theta}$ are discussed later.)

We refer to the set of the arrival $(\lambda_1, \lambda_2, \lambda_3)$ and departure rates (μ_1, μ_2, μ_3)[1], the bandwidths (b_1, b_2, b_3) and minimum bandwidth demands (b_2^{min}, b_3^{min}), the blocking probabilities (B_1^{max},B_2^{max},B_3^{max}) and throughput constraints ($\tilde{\theta}^{min}$, $\hat{\theta}^{min}$) as the *input parameters* of the system.

IV. The Partial Overlap Link Allocation Strategy

A. System Description

The system under investigation (with the above assumptions regarding the arrival processes and holding times/transmission requirements) is a Continuous Time Markov Chain (CTMC) whose state is uniquely characterized by the triple (n_1, n_2, n_3), where n_1 is the number of rigid flows, n_2 and n_3 are the number of adaptive and elastic flows in the system, respectively.

It is clear that in order to obtain the performance measure of this system we need to determine the CTMC's generator matrix $\mathbf{Q}$ and its steady state solution, $\underline{P} = \{P_{(n_1,n_2,n_3)}\}$.

We would like to define the link allocation policy such that it is able to provide predefined call blocking probability for the adaptive and for the elastic flows, while it is able to take into account the GoS (blocking probability) constraints for the rigid flows and the minimum throughput constraint for the adaptive and elastic flows. Because of its flexibility (in that it is able to take into account the above constraints) and simplicity (in that the performance measures of interest

[1] μ_3 is the maximum departure rate of the elastic class assuming that the bandwidth of the elastic flow equals to b_3.

can be determined even for large systems) we in this paper adopt the *partial overlap, POL* link allocation policy from the multi-rate circuit switched modeling paradigm [29].

According to the POL policy, the link capacity C is divided into two parts, the C_{COM} common part and the C_{ELA} part, which is reserved for the adaptive and elastic flows only, such that $C = C_{COM} + C_{ELA}$. Under the considered POL policy the number of flows in progress on the link is subject to the following constraints:

$$n_1 \cdot b_1 \leq C_{COM} \quad (1)$$
$$N_2 \cdot b_2^{min} + N_3 \cdot b_3^{min} \leq C_{ELA} \quad (2)$$
$$n_2 \leq N_2 \quad (3)$$
$$n_3 \leq N_3 \quad (4)$$

where N_2 and N_3 stand for the maximum number of adaptive and elastic flows in the system (sometimes referred to as the *cut-off* parameter [27]) and will be determined later. Note that this policy has only three free parameters, (C_{COM}, N_2 and N_3) which allows for the easy dimensioning of a system with blocking and throughput constraints.

The set of such (n_1, n_2, n_3) triples that satisfies these constraints constitutes the set of *feasible states* of the system which we denote by $\mathcal{S}$. The cardinality of the state space can be determined as:

$$\#\mathcal{S} \leq \left(\frac{C_{COM}}{b_1} + 1 \right) \cdot (N_2 + 1) \cdot (N_3 + 1) \quad (5)$$

In (1) the adaptive and elastic flows are protected from rigid flows. In (2-4) the maximum number of adaptive and elastic flows is limited by three constraints. Eq. (2) protects the rigid flows, while (3-4) protect the in-progress adaptive and elastic flows from the arriving new flows, because if too many such flows were admitted into the system then either $\tilde{\theta}$ or $\hat{\theta}$ could decrease below $\tilde{\theta}^{min}$ or $\hat{\theta}^{min}$ respectively. Clearly, the θ of the i-th class can be modified by changing the value of the N_{ELi}'s.

The generator matrix, $\mathbf{Q}$, possesses a nice structure, because only transitions between "neighbouring states" are allowed in the following sense. Let $q(n_1, n_2, n_3 \rightarrow n_1', n_2', n_3')$ denote the transition rate from state (n_1, n_2, n_3) to state (n_1', n_2', n_3'). Then, taking into account the above constraints associated with the proposed POL policy, the non-zero transition rates between the feasible states are:

$$\begin{aligned}
q(n_1, n_2, n_3 \rightarrow n_1 + 1, n_2, n_3) &= \lambda_1 \\
q(n_1, n_2, n_3 \rightarrow n_1, n_2 + 1, n_3) &= \lambda_2 \\
q(n_1, n_2, n_3 \rightarrow n_1, n_2, n_3 + 1) &= \lambda_3 \\
q(n_1, n_2, n_3 \rightarrow n_1 - 1, n_2, n_3) &= n_1 \cdot \mu_1 \\
q(n_1, n_2, n_3 \rightarrow n_1, n_2 - 1, n_3) &= n_2 \cdot \mu_2 \\
q(n_1, n_2, n_3 \rightarrow n_1, n_2, n_3 - 1) &= n_3 \cdot r_3(n_1, n_2, n_3) \cdot \mu_3
\end{aligned}$$

The first three equations represent the state transitions due to call arrivals, while the second three equations represent the transitions due to call departures. The $n_3 \cdot r_3(n_1, n_2, n_3) \cdot b_3$ quantity denotes the total bandwidth of the elastic flows when the system is in state (n_1, n_2, n_3). The $\mathbf{Q}$ generator matrix of the CTMC is constructed automatically based on the above set of equations using the MRMSolve tool [21].

Note that the derivation of the last equation of the generator matrix relies on the fact that the system is Markovian. This interesting and non-trivial system phenomenon with elastic flows was independently of each other observed and formally proven by Altman, Artiges and Traore in [3], by Andersen, Blaabjerg, Fodor and Telek in [1] and also by Nunez Queija, van der Berg and Mandjes in [20]. It is also used by Massoulie and Roberts in e.g. [16], where the death rates of the birth-death process are modulated by the actual instantaneous bandwidth of the elastic traffic.

The POL policy as described above is fully determined by specifying the following parameters: the capacity of the common part, C_{COM}, and the maximum number of adaptive and elastic flows, N_2 and the N_3. We refer to the C_{COM}, the N_2 and the N_3 parameters of the POL policy as the *output parameters* of the system.

For illustration purposes we consider a small system with a link of capacity $C = 7$ and for ease of presentation $n_1 = 1$ is kept fixed, i.e. the available bandwidth for the adaptive and the elastic flows is 6 bandwidth unit. Further more $b_2 = 3$ and $b_3 = 2$. The adaptive and the elastic flows are further characterized by their *minimum* accepted bandwidth, which we set to $b_2^{min} = 1.8$ and $b_3^{min} = 0.8$. The *cut-off* parameters are $N_2 = 2$ and $N_3 = 3$. This setting gives rise to 12 feasible states, out of which there are 5 (gray) states where at least one of the flows is compressed below the peak bandwidth specified by b_2 and b_3. The Markov chain that describes the system behaviour is depicted in Figure 1. The states are identified by the number of active flows (n_1, n_2, n_3). The number below the state identifier indicates the bandwidth compression of the adaptive and elastic traffic (r_2, r_3). The last state $(1, 2, 3)$ is the only one where the bandwidth compression of the adaptive and elastic class differs due to the different minimum bandwidth requirement ($r_2^{min} = 0.6$, $r_3^{min} = 0.4$).

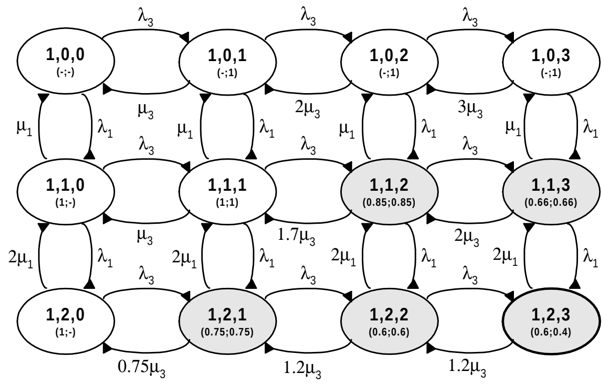

Fig. 1. Part of the state space where $n_1 = 1$ is kept fixed

B. Constraints for Determining the Output Parameters of the POL Policy

The POL policy is easy to dimension, since its performance can be tuned by its three output parameters. At the same time it guarantees call level GoS for rigid, adaptive and elastic flows and throughput level for adaptive and elastic services. The GoS of rigid flows is guaranteed by the proper setting of C_{COM}. In the case of a change in the adaptive and/or elastic traffic load (i.e. the call arrival intensity or the lengths of the flows), the N_2 and N_3 parameters have to be adjusted to keep the required throughput and blocking probabilities. We divide the problem of determining the output parameters of the POL policy into two steps. In the first step we determine the minimum required capacity (C_{COM}) for rigid flows, that guarantees the required blocking probability:

$$\mathbf{min}\Big\{C_{COM} \ : \ B_1 \leq B_1^{max}\Big\} \tag{6}$$

where B_1 is the blocking probability of the rigid flows. The Erlang-B formula can be applied for solving this problem. In the second step we determine the maximum number of adaptive and elastic flows (N_2,N_3) simultaneously present in the system.

In fact, we determine the pairs of maximum number of adaptive/elastic flows (i.e. (N_2;N_3)) where the system can provide the required throughput and blocking probabilities. It is intuitively clear that if we increase the maximum number of adaptive flows (N_2) the blocking probability of adaptive flows (B_2) decreases and its throughput decreases. Moreover, unfortunately, changing N_2 might affect both the blocking probability (B_3) and the throughput of the elastic flows and vice-versa.

The following two constraints are considered:

- constraint on the average throughput:

The (N_2;N_3) pair fulfills the blocking probability and the throughput constraints if

$$B_2 \leq B_2^{max}, \ B_3 \leq B_3^{max}, \ E(\tilde{\theta}) \geq \tilde{\theta}^{min}, \ E(\hat{\theta}) \geq \hat{\theta}^{min}$$

To make a plausible interpretation of this constraint let us assume that the distribution of θ is fairly symmetric around $E(\theta)$, i.e. the median of θ is close to $E(\theta)$. In this case the probability that an adaptive or elastic flow obtains less bandwidth than θ^{min} is around 50%. Users (even with adaptive or elastic traffic) often prefer more informative throughput constraints like the next one.

- constraint on throughput threshold:

The (N_2;N_3) pair fulfills the blocking probability and the throughput constraints if

$$B_2 \leq B_2^{max}\ , \quad B_3 \leq B_3^{max}$$
$$Pr(\tilde{\theta}_t \geq \tilde{\theta}^{min}) \geq \varepsilon_2,\ \forall t;\ Pr(\hat{\theta}_x \geq \hat{\theta}^{min}) \geq \varepsilon_3,\ \forall x;$$

This throughput threshold constraint requires that the throughput of adaptive and elastic flows be greater than $\tilde{\theta}^{min}$ and $\hat{\theta}^{min}$ with predefined probabilities ε_2 and ε_3 independent of the associated service requirements (x) or holding times (t). Hence, if the (input) parameter θ^{min} is much less than $E(\theta)$ then this second constraint is much more informative for the user about the expected minimum level of the elastic flow throughput.

In the case of applying the throughput threshold constraint ε_2 and ε_3 are also input parameters of the model [2].

In general, N_2 and N_3 have to increase to fulfill the blocking probability constraints and N_2 and N_3 have to decrease to fulfill the throughput constraints. Depending on the model parameters and the bounds it can occur that the constraints cannot be satisfied at the same time, which means that the link is overloaded with respect to the GoS and QoS requirements.

V. Analysis of Call Blocking Probabilities

The call blocking probabilities are obtained from the steady state distribution ($\underline{P}$) of the CTMC specified by its generator matrix $\mathbf{Q}$. Considering the model size of practically interesting cases iterative analysis methods are applicable for steady state analysis [28]. Iterative methods begin from an initial guess and produce a sequence of intermediate results, which converge to the solution. The number of required iteration steps to achieve a given precision depends on the model properties, the applied iterative scheme and the initial guess. We applied the Gauss-Seidel algorithm for the iteration and an initial guess that is fairly close to the solution. The initial guess is computed utilizing the fact that the system with only non-adaptive and adaptive traffic classes can be closely approximated by a product form solution [22].

Based on the steady state distribution of the CTMC, the call blocking probabilities of the different classes are obtained as the sum of the steady state probability of blocking states [3].

VI. Analysis of Throughput Measures of Elastic Flows

A. Average Throughput Constraint

The calculation of the average throughput of the adaptive and the elastic flows is straightforward based on the steady state distribution of the CTMC, since

$$E(\tilde{\theta}) = b_2 \cdot \sum_{(n_1,n_2,n_3)\in\mathcal{S}} P_2(n_1,n_2,n_3) \cdot r_2(n_1,n_2,n_3)$$

where the probability that an adaptive flow is compressed to $r_2(n_1,n_2,n_3)$ is

$$P_2(n_1,n_2,n_3) = \frac{n_2 \cdot p_{(n_1,n_2,n_3)}}{\sum\limits_{(n_1',n_2',n_3')\in\mathcal{S}} n_2' \cdot p_{(n_1',n_2',n_3')}}$$

Similarly,

$$E(\hat{\theta}) = \frac{\sum\limits_{(n_1,n_2,n_3)\in\mathcal{S}} n_3 \cdot p_{(n_1,n_2,n_3)} \cdot b_3 \cdot r_3(n_1,n_2,n_3)}{\sum\limits_{(n_1',n_2',n_3')\in\mathcal{S}} n_3 \cdot p_{(n_1',n_2',n_3')}}\ .$$

[2]In practical cases, the value of ε_2 (ε_3) is between 50% and 100%. Setting ε_2 (ε_3) to 50% provides approximately the average throughput constraint. Higher value of ε_2 (ε_3) gives tighter throughput guarantee. The case when $\varepsilon_2 = 100\%$ ($\varepsilon_3 = 100\%$) is equivalent with setting b_2^{min} equal to $\tilde{\theta}^{min}$ (b_3^{min} to $\hat{\theta}^{min}$).

[3]Blocking states of a given traffic class are those states in which a new arrival of that class would result in an infeasible state.

B. Throughput Threshold Constraint

Unfortunately, it is harder to check the throughput threshold constraint, since neither the distribution nor the higher moments of $\tilde{\theta}_t$ and $\hat{\theta}_x$ can be analyzed based on the steady state distribution of the above studied Markov chain. Hence, a new analysis approach is applied to analyze the system with the throughput threshold constraint.
The throughput threshold constraint on adaptive flows can be check based on the distribution of $\tilde{\theta}_t$ and on the elastic flows based on the distribution of T_x, because:

$$Pr\left(\hat{\theta}_x \geq \hat{\theta}^{min}\right) = Pr\left(\frac{x}{T_x} \geq \hat{\theta}^{min}\right) = Pr\left(T_x \leq \frac{x}{\hat{\theta}^{min}}\right) \ .$$

Since it is computationally too hard to evaluate the distribution of T_x and $\tilde{\theta}_t$ for realistic models, but there are effective numerical methods to obtain their moments, as discussed later, we check the throughput threshold constraint applying moment based distribution estimation methods. The applied estimation method uses the first three moments of $\tilde{\theta}_t$ and T_x and provides upper and lower bounds of their distribution.

C. Customer Tagging and System Behavior During Adaptive/Elastic Traffic Service

The method we follow to evaluate the moments of $\tilde{\theta}_t$ and T_x is based on tagging an adaptive or an elastic flow arriving to the system, and carefully examining the possible transitions from the moment this tagged call enters the system until it leaves the system. The system behaviour during the service of the tagged flow can be described by a slightly modified Markov chain. To analyze $\tilde{\theta}_t$ a tagged adaptive flow is considered while to analyze T_x a tagged elastic flow is used.

Here we detail the analysis of a tagged adaptive flow and at the end of this section we consider the analysis of a tagged elastic flow. The system introduced in Section III, is specified by a CTMC over the state space $\mathcal{S}$ with generator matrix $\mathbf{Q}$. The *modified system* used to evaluate $\tilde{\theta}_t$ has the following properties:

- Since we assume that at least the tagged adaptive flow is in the system we exclude states where $n_2 = 0$.
- With each state of the state space there is an associated entrance probability, which is the probability of the event that the modified CTMC starts from that state. When the tagged adaptive flow finds the system in state (n_1, n_2, n_3) it will bring the system into state (n_1, n_2+1, n_3) unless (n_1, n_2, n_3) happens to be a blocking state of the tagged adaptive flow.

Let $\{\mathcal{Z}^{2+}(t),\ t \geq 0\}$ be the modified CTMC assuming that the tagged adaptive flow never leaves the system over the finite state space $\mathcal{S}^{2+}$ with generator $\mathbf{Q}^{2+}$. The state space $\mathcal{S}^{2+}$ can be defined as:

$$0 \leq \quad n_1 \cdot b_1 \quad \leq C_{COM} \tag{7}$$
$$1 \leq \quad n_2 \quad \leq N_2 \tag{8}$$
$$0 \leq \quad n_3 \quad \leq N_3 \ . \tag{9}$$

Indeed, $\mathcal{S}^{2+} = \mathcal{S} \backslash \mathcal{S}_0^{2+}$ where $\mathcal{S}_0^{2+}$ is the states in $\mathcal{S}$ where $n_2 = 0$. The transition rates in $\mathbf{Q}^{2+}$ are closely related to the appropriate rates in $\mathbf{Q}$ and they differ only in (10):

$$\begin{aligned}
q^{2+}(n_1, n_2, n_3 \rightarrow n_1+1, n_2, n_3) &= \lambda_1 \\
q^{2+}(n_1, n_2, n_3 \rightarrow n_1, n_2+1, n_3) &= \lambda_2 \\
q^{2+}(n_1, n_2, n_3 \rightarrow n_1, n_2, n_3+1) &= \lambda_3 \\
q^{2+}(n_1, n_2, n_3 \rightarrow n_1-1, n_2, n_3) &= n_1 \cdot \mu_1 \\
q^{2+}(n_1, n_2, n_3 \rightarrow n_1, n_2-1, n_3) &= (n_2 - 1) \cdot \mu_2 \\
q^{2+}(n_1, n_2, n_3 \rightarrow n_1, n_2, n_3-1) &= n_3 \cdot r_3(n_1, n_2, n_3) \cdot \mu_3
\end{aligned} \tag{10}$$

The initial probability of the modified Markov chain, $p^{2+}_{(n_1,n_2,n_3)}$, is obtained by considering the system state immediately after the tagged adaptive flow joints the system in steady state. i.e. the probability that the system is in state (n_1, n_2, n_3) after the tagged adaptive flow arrival is proportional to the steady state probability of state $(n_1, n_2 - 1, n_3)$. Hence

$$p^{2+}_{(n_1,n_2,n_3)} = \frac{p_{(n_1,n_2-1,n_3)}}{\displaystyle\sum_{(n_1',n_2',n_3') \in \mathcal{S}^{2+}} p_{(n_1',n_2',n_3')}}$$

Figure 2 depicts the modified Markov chain that describes the system behaviour during the service of a tagged adaptive flow assuming the same system as in Figure 1. The numbers in brackets under the state identifier indicate the bandwidth of the tagged adaptive flow in the given state. The initial probabilities of the states are evaluated based on the steady state probability of the "original" states that are related with the states with dashed arrows.

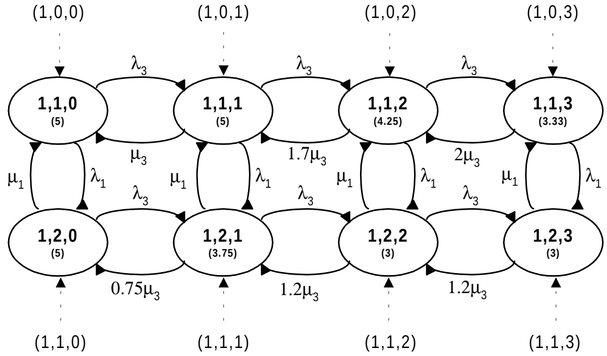

Fig. 2. Tagging an adaptive flow

To obtain the moments of $\tilde{\theta}_t$ a Markov Reward Model [30] is defined over $\{\mathcal{Z}^{2+}(t),\ t \geq 0\}$. $\tilde{\theta}_t$ is a random variable which depends on the (random) arrival and departure of the rigid, adaptive and elastic flows as described by $\mathbf{Q}^{2+}$. The reward rate associated with the states of the modified Markov chain represents the bandwidth of the tagged adaptive flow in that state. Let $r^{2+}(n_1, n_2, n_3)$ be the reward rate (the bandwidth of the tagged adaptive flow) in state (n_1, n_2, n_3) and $\mathbf{R}^{2+}$ the diagonal matrix composed by the $r^{2+}(n_1, n_2, n_3)$ entries. $r^{2+}(n_1, n_2, n_3) = r_2(n_1, n_2, n_3) \cdot b_2$, where $r_2(n_1, n_2, n_3)$ is the bandwidth compression in state (n_1, n_2, n_3). This way the dynamics of the number of flows in the system during the service of the tagged adaptive flow is described by the Modified Markov chain and the instantaneous bandwidth of the tagged flow by the instantaneous reward rate. If there are more flows in the system the bandwidth of the tagged adaptive flow decreases toward b_2^{min} and if there are less flows it increases to b_2. The generator matrix $\mathbf{Q}^{2+}$ and the reward matrix $\mathbf{R}^{2+}$ define the Markov Reward Model that accumulates $t \cdot \tilde{\theta}_t$ amount of reward in the $(0, t)$ interval. It can be interpreted as the reward accumulated in the $(0, t)$ interval represents the amount of data transmitted through the tagged adaptive flow in this interval, i.e. $\tilde{\theta}_t =$ amount of transferred data / t.

The tagging of an elastic flow follows the same pattern as the tagging of an adaptive one. The appropriate measures are denoted by $p^{3+}_{(n_1,n_2,n_3)}$, $\mathbf{Q}^{3+}$ and $\mathbf{R}^{3+}$. T_x is the (random) amount of time it takes to transmit x unit of data through the tagged elastic flow. Defining a Markov Reward Model as before the reward accumulated in $(0, t)$ represents the (random) amount of data transmitted through the tagged flow, hence T_x is the (random) time the Markov Reward Model takes to accumulate x amount of reward. This measure is commonly referred to as *completion time*.

Having the initial probability distributions $p^{2+}_{(n_1,n_2,n_3)}$, and $p^{3+}_{(n_1,n_2,n_3)}$, the generator matrices $\mathbf{Q}^{2+}$, and $\mathbf{Q}^{3+}$ and the reward matrices $\mathbf{R}^{2+}$ and $\mathbf{R}^{3+}$, we applied the numerical analysis method proposed in [30] to evaluate the moments of $\tilde{\theta}_t$ and T_x, which is applicable for Markov reward models with large state spaces ($\sim 10^6$ states).

D. The Complete Link Allocation Procedure

Finally, the steps of the link allocation procedure is summarized (Figure 3). Here we discuss the procedure which maximize the throughput of adaptive and elastic flows.

1) $\underline{C_{COM}}$: C_{COM} is calculated using Erlang's loss formulae from $\lambda_1, \mu_1, b_1, B_1^{max}$ such that the blocking probability of the rigid flows, B_1, is less than B_1^{max}. If the obtained C_{COM} is larger than the link capacity, C, the link is overloaded by the rigid flows and the requirements can not be satisfied.

2) Initial value of N_2: The number of adaptive flows in the system is independent of the other flows as long as the system stays in the non-blocking region, i.e., the bandwidth of the adaptive flows is greater than b_2^{min} such that a new adaptive flow can enter the system. Assuming this independent property is dominant, we calculate the initial value of N_2 independent of N_3. Actually, we calculate the initial value of N_2, when $N_3 = 0$. In this case a reversible, "two dimensional" Markov chain characterize the blocking probabilities, hence it is fast and easy to calculate N_2 [22]. Due to

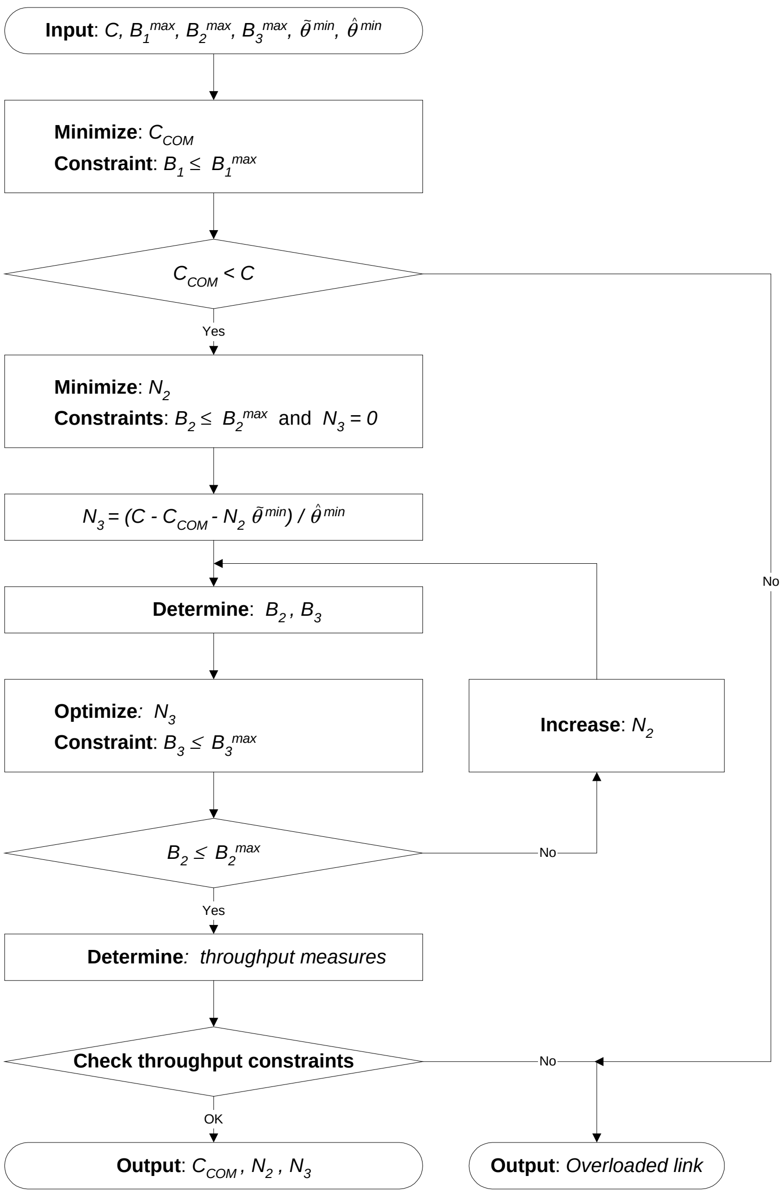

Fig. 3. The block diagram of the link allocation procedure

the above mentioned independence of the adaptive and elastic load, N_2 hardly changes during the consecutive iterative procedure when B_2 and B_3 take practically interesting values ($< 15\%$).

3) Initial value of N_3: We apply the following heuristic relation for the initial value of N_3:

$$N_3 = \frac{C - C_{COM} - N_2\tilde{\theta}^{min}}{\hat{\theta}^{min}} \tag{11}$$

4) Iterative refinement of N_2 and N_3:

4A) Calculation of B_2 and B_3: The calculation of B_2 and B_3 with the given N_2, N_3 pair ($N_2 > 0, N_3 > 0$) is rather expensive, since it requires the analysis of a (non-reversible) "3 dimensional" Markov chain (Section V).

4B) Optimization of N_3: The blocking probability of elastic flows, B_3, is a monotone function of N_3 for a given fixed N_2. In this step we search for the minimal N_3 which results in a blocking probability B_3 less than B_3^{max}.

4C) Refinement of N_2: If $B_2 > B_2^{max}$ with the given N_2, N_3 pair then we increase N_2 and go back to step A).

5) Throughput check: The above iterative procedure obtains the minimal N_2, N_3 pair which fulfills the blocking constraints. This minimal N_2, N_3 pair results in the maximal throughput for the adaptive and elastic flows with the given blocking constraints. Hence a final check of the throughput constraints (Sec. VI) decides if the obtained set of output parameters fulfills all requirements or the link is overloaded and the requirements cannot be fulfilled.

E. The Computational Complexity of the Link Allocation Procedure

The only computationally intensive step of the link allocation procedure are the analysis of the "3 dimensional" Markov chains. Unfortunately, this computationally intensive step is repeated in the iterative loop for the analysis of the blocking probabilities as long as a proper N_2, N_3 pair is obtained. Using the notation $N_{RIG} = C_{COM}/b_1$, in each cycle of the iteration a Markov chain of size $N_{RIG} \times N_2 \times N_3$ has to be analyzed. The blocking probabilities and the average throughput of adaptive and elastic flows are evaluated based on the steady state behaviour of this Markov chain. Instead, in case of applying the throughput threshold constraint the reward analysis provided in [30] has to be completed. Calculating the first n moments of reward measures, the complexity of this procedure (regarding both, the number of matrix-vector multiplication and the memory requirement) is n times more than the transient analysis of the underlying Markov chain. Fortunately, the throughput analysis step is performed only once when we optimize for the throughput of adaptive and elastic flows, as it is discussed in section VI-D.

VII. Numerical Examples on the Application of the POL Link Allocation Policy

In this Section we present and discuss a numerical example which demonstrates the use of the average throughput- and the throughput threshold constraints.

A. Input Parameters

System Input Parameter	Interpretation	Value	Unit
C	Link capacity	100	[BU]
b_1	Rigid traffic class bandwidth demand	1	[BU]
b_2	Adaptive traffic class maximum bandwidth demand	3	[BU]
b_2^{min}	Adaptive traffic class minimum bandwidth demand	0.6	[BU]
b_3	Elastic traffic class maximum bandwidth demand	3	[BU]
b_3^{min}	Elastic traffic class minimum bandwidth demand	0.6	[BU]
$\lambda_1 = \lambda_2 = \lambda_3$	Rigid/Adaptive/Elastic traffic flows' arrival intensity	15	1/[TU]
$1/\mu_1 = 1/\mu_2 = 1/\mu_3$	Rigid/Adaptive/Elastic (ideal) mean holding time	1	[TU]
B_1	Rigid traffic class blocking probability	2	%
B_2	Adaptive traffic class blocking probability	10 (10)	%
B_3	Elastic traffic class blocking probability	10 (10)	%
$\tilde{\theta}^{min}$	Adaptive traffic class throughput constraint	2.5 (2.0)	[BU]
$\hat{\theta}^{min}$	Elastic traffic class throughput constraint	2.5 (2.0)	[BU]
ϵ_2	Adaptive traffic class throughput threshold constraint	(90)	%
ϵ_3	Elastic traffic class throughput threshold constraint	(90)	%

TABLE I

THE INPUT PARAMETERS OF THE EXAMPLE SYSTEM, CONSISTING OF A SINGLE TRANSMISSION LINK AND THREE TRAFFIC CLASSES (RIGID, ADAPTIVE AND ELASTIC). THE LINK CAPACITY IS SPECIFIED IN BANDWIDTH UNITS [BU], THE TIME UNIT [TU] IS EXPLICITLY CHOSEN TO s.

In the following we will refer to the quantity $S_i = b_i \cdot \lambda_i/\mu_1$ as the class-i offered load to the system, $i = 1 \dots 3$. The class-1 traffic may represent a voice or a fax over IP application that requires peak allocation. We choose this bandwidth requirement to be our bandwidth unit [BU]. Class-2 traffic may correspond to an adaptive video codec (requiring three times the bandwidth of the voice application). We assume that the application tolerates a temporary throughput degradation down to 20% of the peak bandwidth requirement ($r_2^{min} = 0.2$). Finally, class-3 represents a wide band (three times the bandwidth of the voice application) file transfer (ftp) application where a minimum bandwidth demand is associated with the application, which, also in this case is 20% of the peak data rate demand ($r_3^{min} = 0.2$). Note that for the sake of this example we have chosen the parameters such that the class-wise offered traffic is the same for all three classes ($\lambda_i/\mu_i = 15, i = 1 \cdots 3$), which makes an intuitive interpretation of the results more straightforward. For our time unit

[TU], we have chosen the equal holding times of the three traffic classes. Note that we set the throughput constraints of the adaptive and the elastic traffic classes to 2.5 [BU], which is somewhat lower than the maximum required 3 [BU], but higher than the minimum required 0.6 [BU]. Recall that the minimum required bandwidth is the one, which must be ensured for all in-progress class flows at all times, whereas the interpretation of the $\tilde{\theta}^{min}$, $\hat{\theta}^{min}$, ϵ_2 and ϵ_3 values is throughput definition dependent, as discussed in Section VI. We will return to these definitions in the subsequent subsections.

B. Determining the C_{COM} Parameter

Recall from Section VI and from the Figure 3 that both under the average throughput constraint and under the throughput threshold constraint the first step is to determine the common part of the link, C_{COM}. From the offered traffic load of the rigid traffic class, applying the Erlang-B formula, it directly follows that C_{COM} must at least be 23 [BU] in order to meet the $B_1 \leq 2\%$ blocking probability constraint. ($B_1 = 1.35\%$ when $C_{COM} = 23$ and $B_1 = 2.1\%$ when $C_{COM} = 22$).

C. Determining the N_2 and N_3 Parameters with Average Throughput Constraints

The feasible N_2, N_3 pairs (cut-off parameters) are limited by the following constraints:

- $B_2 \leq B_2^{max}$, $B_3 \leq B_3^{max}$
- throughput constraints for adaptive and elastic flows
- Eq. (2)

The blocking probability constraints define the lower bounds of the feasible N_2 and N_3 values. An upper bound of the feasible region is defined by Eq. (2). If the throughput constraints are too loose, e.g., $\tilde{\theta}^{min} \leq b_2^{min}$ and $\hat{\theta}^{min} \leq b_3^{min}$, Eq. (2) limits the feasible N_2, N_3 pairs. (In our example, Eq. (2) yields that $N_2 + N_3 \leq 128$.) In case of meaningful throughput constraints the feasible N_2, N_3 region is further restricted.

The impact of the average throughput constraint and of the maximum blocking probabilities of the adaptive and the elastic classes on the feasible cut-off parameters is shown in Figures 4 - 7.

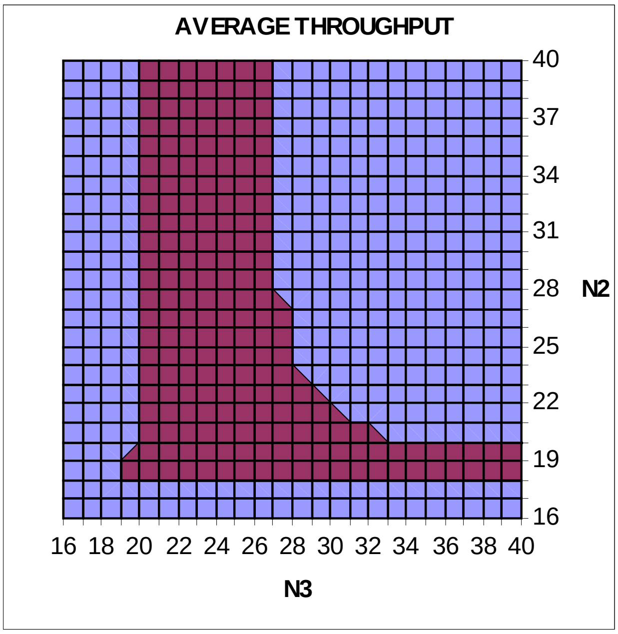

Fig. 4. Feasible N_2, N_3 region with average throughput constraints ($B_2^{max} = B_3^{max} = 10\%$, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2.5$)

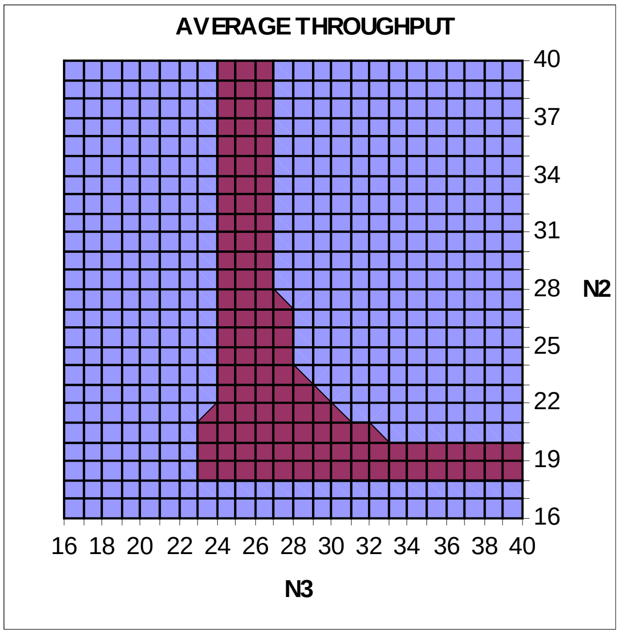

Fig. 5. Feasible N_2, N_3 region with average throughput constraints ($B_2^{max} = 10\%, B_3^{max} = 5\%$, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2.5$)

In Figure 4 the minimum average throughput for the adaptive and elastic classes is set as in Table I, $E(\tilde{\theta}) \geq 2.5$ and $E(\hat{\theta}) \geq 2.5$. The set of N_2 and N_3 pairs that fulfill this throughput constraint and at the same time meet the blocking probability constraints ($B_2 \leq 10\%$ and $B_3 \leq 10\%$) is shown as the "interior" (dark part) of the framed area.

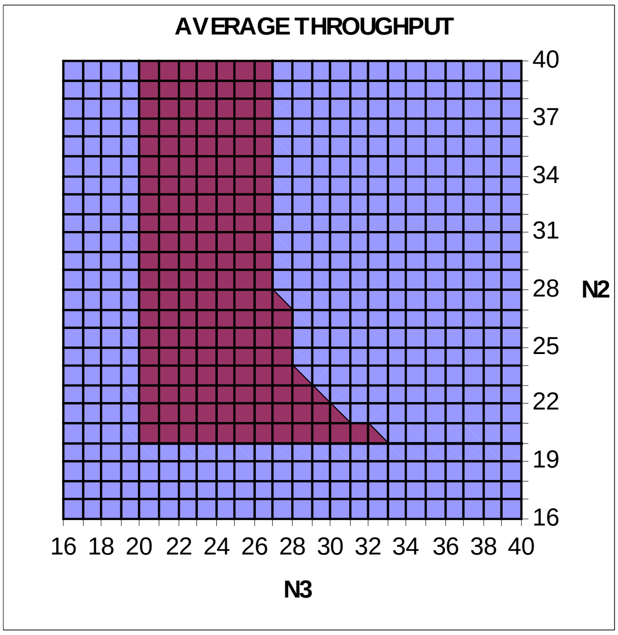

Fig. 6. Feasible N_2, N_3 region with average throughput constraints ($B_2^{max} = 5\%, B_3^{max} = 10\%$, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2.5$)

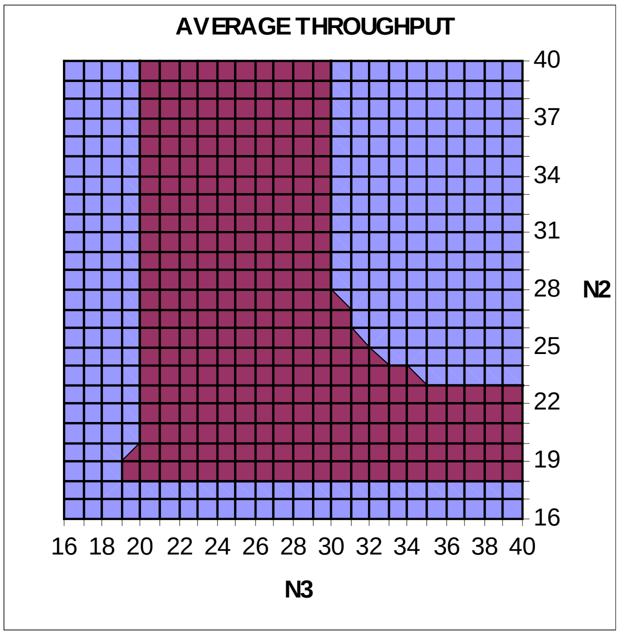

Fig. 7. Feasible N_2, N_3 region with average throughput constraints ($B_2^{max} = B_3^{max} = 10\%$, $\tilde{\theta}^{min} = 2.5, \hat{\theta}^{min} = 0.6$)

For instance, the pairs ($N_2 = 18$, $N_3 = 19$) and ($N_2 = 28$, $N_3 = 26$) both meet the throughput constraint, but (not surprisingly), the blocking probabilities are minimized under the highest possible cut-off parameters (i.e. under the second cut-off parameter pair). On the other hand, when the blocking probability constraints ($B_2 \leq 10\%$ and $B_3 \leq 10\%$ are "just kept", the throughput values are maximized when keeping the admitted calls to the minimum, in this example under the $N_2 = 18$ and $N_3 = 19$ cut-off parameter pair.

Figure 5 and 6 investigate the effect of required minimal blocking probabilities. In Figure 5 the same area is depicted when reducing the maximum blocking probabilities of the elastic class to $B_3^{max} = 5\%$ and leaving B_2^{max} and the minimum average throughput, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2.5$, unchanged. Figure 6 shows the case when the blocking probability of the adaptive class is reduced to $B_2^{max} = 5\%$ and all the other parameters are as in Figure 4. It can be seen that the independent behaviour of the lower bounds of N_2 and N_3 (which we utilize in our link allocation procedure in section VI-D) is verified in this example. The maximum blocking probability of the adaptive (elastic) class affects only the lower bound of N_2 (N_3) and leaves the other boundaries of the feasible cut-off parameter set unchanged.

Figure 7 illustrates the effect of throughput constraints. Starting from the parameters of Figure 4, we relaxed the throughput constraint on the adaptive class first. We obtained the same feasible region as in Figure 4 even for meaningless low throughput constraint (i.e., $\tilde{\theta}^{min} = b_2^{min}$). It means that, in this example, the upper bound of the feasible cut-off parameter region is determined by the throughput constraint of the elastic class. Starting again from the parameters of Figure 4, we relaxed the throughput constraint on the elastic class ($\hat{\theta}^{min} = b_3^{min}$) second. The obtained enlarged feasible region is depicted in Figure 7. Thus, the upper boundary in Figure 4 comes from the $E(\hat{\theta}) \geq \hat{\theta}^{min} = 2.5$ throughput constraints (independent of the $E(\tilde{\theta}) \geq \tilde{\theta}^{min}$ constraint as long as $\tilde{\theta}^{min} \leq 2.5$), and the higher upper boundary in Figure 7 comes from the $E(\tilde{\theta}) \geq \tilde{\theta}^{min} = 2.5$ throughput constraints. The lower one of these two upper boundaries limits the feasible cut-off parameter region, as it is in Figure 4.

Our algorithm is capable of determining the "framed" area, that is the set of feasible (N_2;N_3) pairs including the case when the set is empty. Once this finite set is determined, it is straightforward to select the one pair which is desirable (i.e. maximizing the throughput or minimizing the blocking probabilities). In subsection VII-H we consider an example when we are interested in maximizing the average throughput of adaptive and elastic flows under the blocking probability constraints.

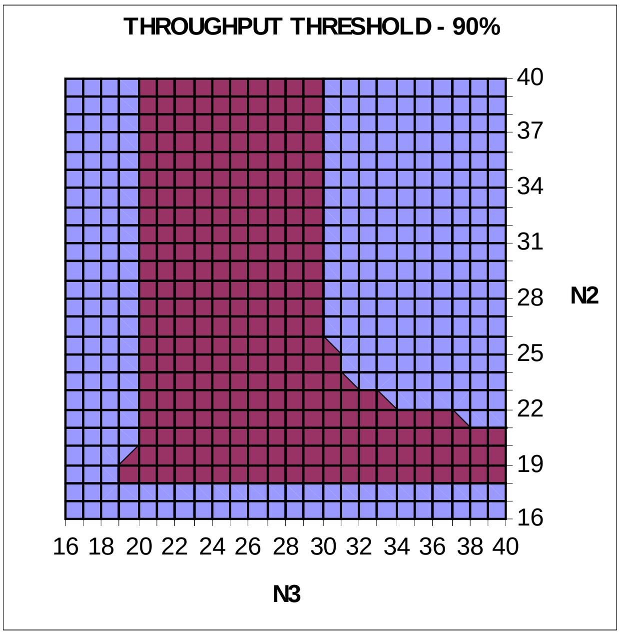

Fig. 8. Feasible N_2, N_3 region with throughput threshold constraints ($B_2^{max} = B_3^{max} = 10\%$, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2$, $\epsilon_2 = \epsilon_3 = 90\%$)

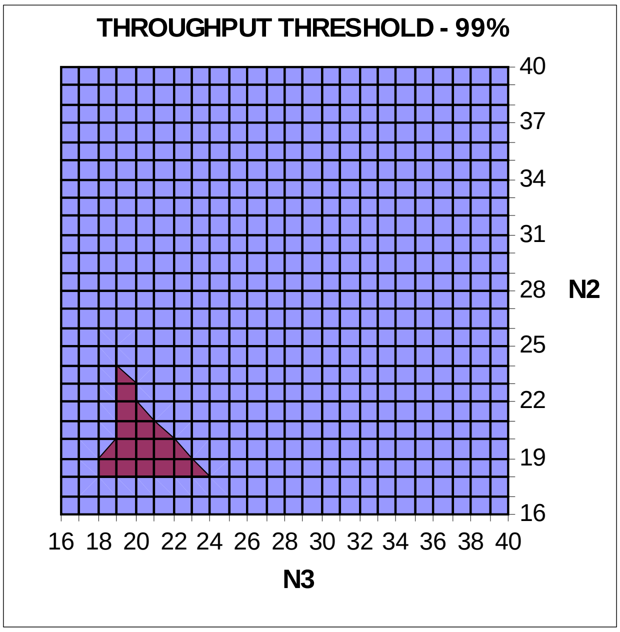

Fig. 9. Feasible N_2, N_3 region with throughput threshold constraints ($B_2^{max} = B_3^{max} = 10\%$, $\tilde{\theta}^{min} = \hat{\theta}^{min} = 2$, $\epsilon_2 = \epsilon_3 = 99\%$)

D. Determining the N_2 and N_3 Parameters with Throughput Threshold Constraints

In our example the adaptive and elastic throughput fluctuate between 0.6 and 3 [BU]. The mean throughput parameter indicates only the average behaviour of this fluctuation. For example, the average throughput is 2.5 when the adaptive and elastic flows always receive 2.5 [BU] throughput, and also when they receive the maximum throughput with probability ~ 0.8 and the minimum throughput with probability ~ 0.2. The real throughput fluctuation is always between these two extreme cases. For some applications it could be important to limit the probability that the flow receives "low" throughput. For example, we may want to require that the throughput of adaptive and elastic flows are higher than 2 [BU] with probability 0.9.

Figure 8 specifies the set of $(N_2; N_3)$ pairs which satisfy the same blocking probability constraints for B_2 and B_3 as above ($B_2 \leq 10\%$, $B_3 \leq 10\%$) and the "90%-threshold constraints" ($Pr(\tilde{\theta} \geq 2) \geq 0.9$ and $Pr(\hat{\theta} \geq 2) \geq 0.9$). With the cut-off parameters inside the framed area the throughput for the adaptive and elastic connections are higher than 2 [BU] with probability 0.9 and the blocking probabilities are less than 10%. Figure 9 shows the same $(N_2; N_3)$ set under the "99%-threshold constraint", where we observe that the set of such cut-off parameter pairs that satisfy this constraint "shrinks" significantly as compared to the "90%-threshold" constraint.

E. Blocking Probabilities

In this subsection we study the dependency of the class-wise blocking probabilities on the system output parameters N_2, N_3. Figure 10 shows how the adaptive traffic class blocking probability depends on the cut-off parameters. As expected, as the maximum number of simultaneously admitted adaptive calls increases, the blocking probability decreases, at the expense of decreasing this class' throughput (Figure 12). For instance, at $N_2 = 28; N_3 = 26$, the average throughput constraints are kept (both average throughput values are above 2.5), and the blocking probabilities are minimized ($B_2 < 0.29\%$ and $B_3 < 2\%$).

Two observations are noteworthy. First, we note that B_2 is strongly dependent on N_2, but basically independent from N_3 (Figure 10). On the other hand, B_3 does depend on both cut-off parameters when $N_2 \leq 22$), as it is in Figure 11. This observation verifies the assumptions used in the proposed link allocation procedure for this example.

Secondly, under the given input parameters (i.e., $b_2^{min} = b_3^{min}$) it is clear that *without* the cut-off parameters, B_2 would be equal B_3, since the blocking states of the underlying Markov chain for the two class are the same. However, as we observe, with the cut-off parameters, the elastic class blocking probability is significantly higher. This significant

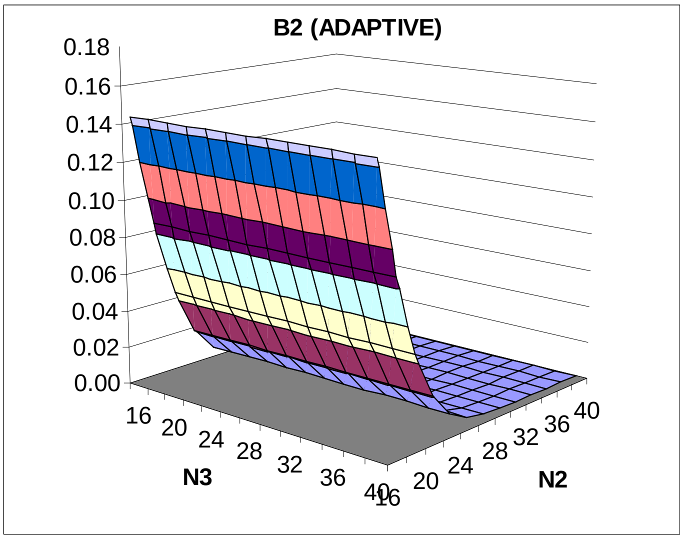

Fig. 10. Adaptive class blocking probability as the function of the cut-off parameters

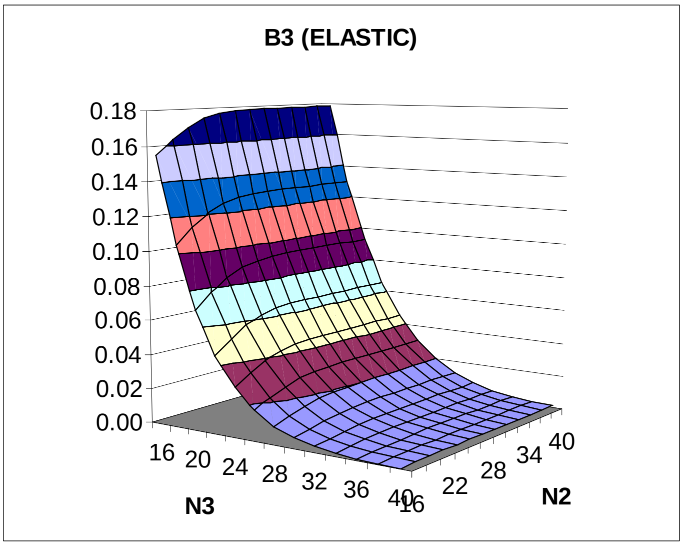

Fig. 11. Elastic class blocking probability as the function of the cut-off parameters

difference of the blocking probabilities comes from the different nature of the adaptive and elastic flows. The adaptive flows “reduce” their load, i.e., they transmit less data, in case of link saturation and depart from the system at the same rate as the link is not saturated. In contrast, the elastic flows transmit the same amount of data independently of the actual link load (since they tend to increase their holding time if the received throughput decreases). Thus, in the case of link saturation elastic the flows stay much longer in the system, which results in a higher blocking probability for the incoming flows.

F. Average Throughput

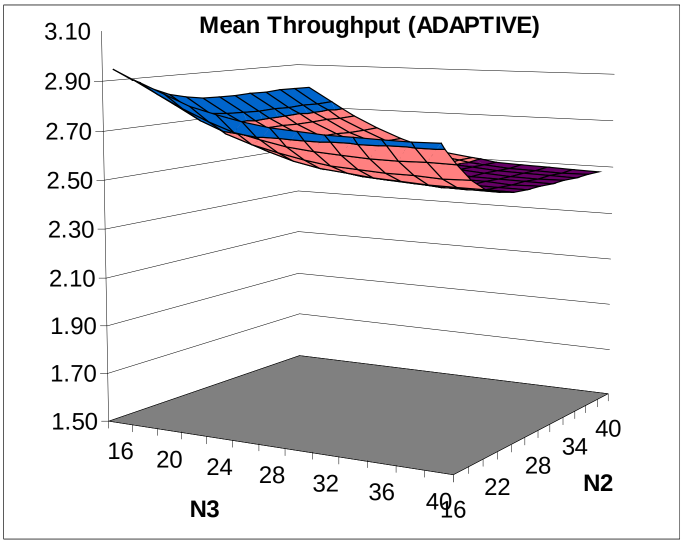

Fig. 12. Average throughput of the adaptive class

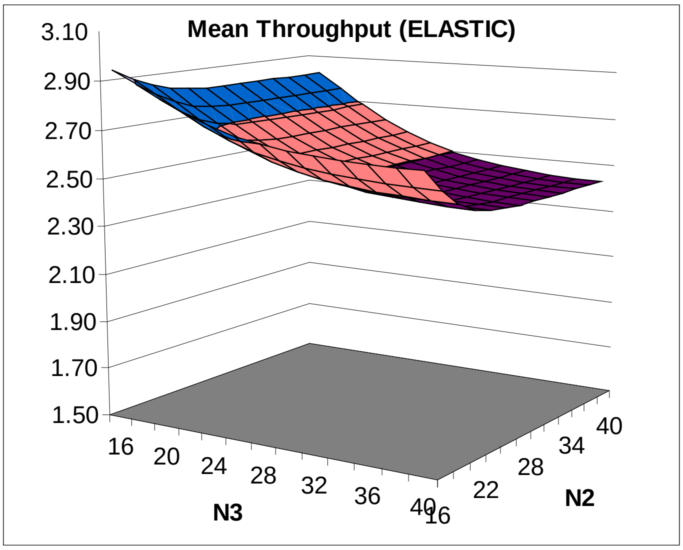

Fig. 13. Average throughput of the elastic class

With respect to the proposed link allocation procedure the most important feature is the *monotonicity* of the average throughput of adaptive and elastic flows as a function of the cut-off parameters. In figure 12 and 13 we can see that both average throughput surfaces are monotone with respect to both cut-off parameters. The mean throughput of adaptive and elastic flows are very close, because the same throughput is assigned with the adaptive and elastic flows in each state of the Markov process. The slice difference is due to the different distribution of the number of adaptive and elastic flows in the system.

G. Throughput Thresholds

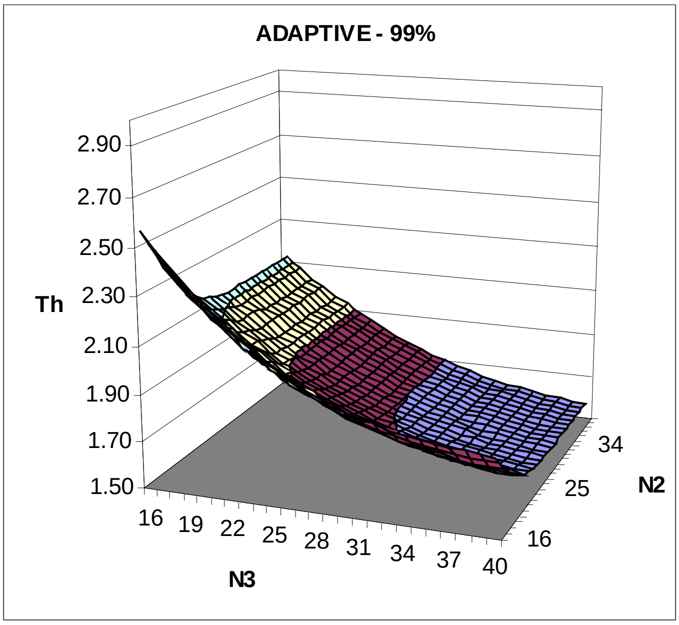

Fig. 14. The 99%-threshold plane of the adaptive class

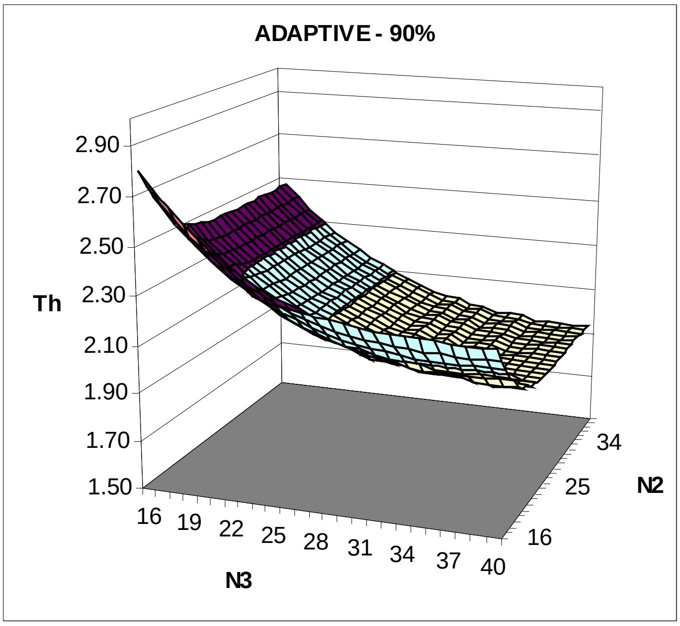

Fig. 15. The 90%-threshold plane of the adaptive class

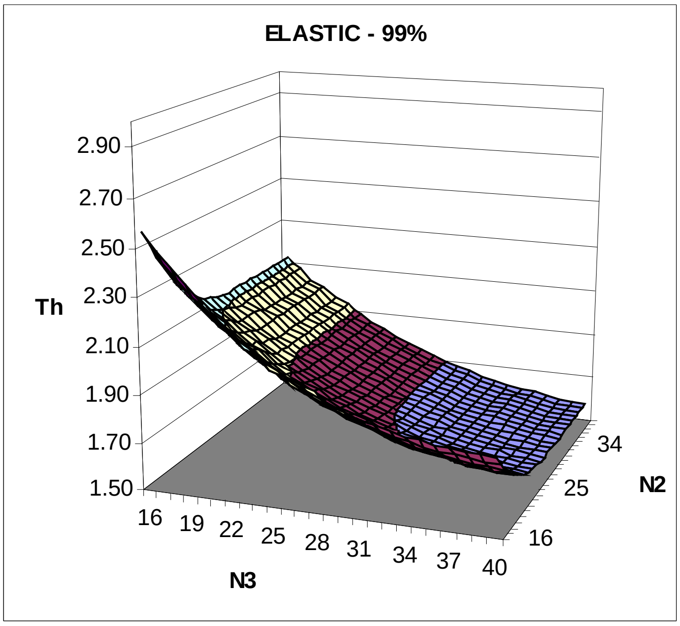

Fig. 16. The 99%-threshold plane of the elastic class

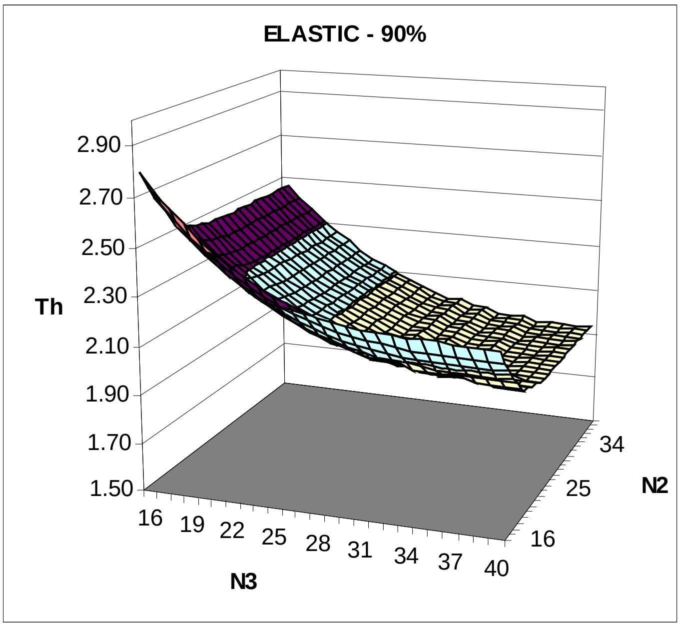

Fig. 17. The 90%-threshold plane of the elastic class

Figures 14 depicts the 99%-threshold plane of adaptive flows' throughput, i.e., the probability that the throughput of adaptive flows, $\tilde{\theta}$, is greater than the given surface point is 99%. Similarly, the 90%-threshold plane of adaptive flows' throughput is shown in Figures 15. The significant difference of the two surfaces indicates that a small reduction of the GoS requirements results in significant gain with respect to acceptable system load. The nature of the same threshold planes of elastic flows' throughput is the same, as it is in Figure 16 and 17. Similarly to the mean throughput surfaces (Figure 12 and 13) the threshold planes show monotone behaviour.

H. The Link Allocation Procedure

We applied the proposed link allocation procedure (section VI-D) for the considered example. The first step provided $C_{COM} = 23$ (as it is already mentioned above). In the second step, the analysis of the "two dimensional" Markov chain (with $N_3 = 0$) resulted $N_2 = 18$ and $B_2 = 8.61\%$. Using (11), 13 is the initial value of N_3. The steps of the iterative refinement of N_2 and N_3 are provided in Table II. We applied the interval bisection method to find the minimal N_3 parameter which still fulfills the blocking constraint. Due to the independence of B_2 and N_3 the N_2 parameter remained unchanged during the iterative refinement. The optimal cut-off parameter pair is $N_2 = 18$ and $N_3 = 19$. The very last step of the iterative analysis checks if N_3 can be reduced below 19. Finally, the mean throughput of adaptive and elastic flows with the optimal cut-off parameters are $E(\tilde{\theta}) = 2.82$ and $E(\hat{\theta}) = 2.82$, respectively. The evaluation of this complete link allocation procedure with 6 iteration steps (i.e., analysis of 6 "3 dimensional" Markov chains) required 6 min running time on a 500 Mhz Pentium PC.

(N_2, N_3)	(B_2, B_3)
$(18, 13)$	$(8.61\%; 26.86\%)$
$(18, 26)$	$(8.61\%; 2.18\%)$
$(18, 20)$	$(8.61\%; 7.78\%)$
$(18, 17)$	$(8.61\%; 13.65\%)$
$\boxed{(18, 19)}$	$\boxed{(8.61\%; 9.43\%)}$
$(18, 18)$	$(8.61\%; 11.37\%)$

TABLE II
THE STEPS OF THE ITERATIVE REFINEMENT OF THE CUT-OFF PARAMETERS

VIII. CONCLUSION

In this paper we argued that providing QoS in the Internet necessitates the use of models that allow us to quantitatively study the impact of admission control on flow throughput and blocking. Indeed, there seems to be a growing consensus regarding the necessity of traffic engineering [31] and the use of analytical models for quantifying the relationship between demand, capacity and performance for both streaming and elastic flows [25]. Therefore, we proposed the extension of the classical loss model - successfully applied for the dimensioning of circuit and ATM networks - such that it takes into account the properties of peak allocated stream (*rigid*), adaptive *stream* and *elastic* flows. Our flow differentiation at this abstraction level attempts to capture the fundamental distinguishing characteristics of three broad categories of applications. Under not too limiting assumptions at the call level, we have also shown that a relatively simple *partial overlap* link allocation scheme used in concert with appropriate *cut-off parameters* can be very flexible in the sense that by determining a few parameters (link division and the two cut-off parameters), the system can meet different performance objectives. Specifically, this link capacity division method can take into account blocking probability constraints and maximize the stream and elastic throughputs. Alternatively, it can meet the stream and elastic throughput constraints and minimize the blocking probabilities. During the construction of the model and analyzing the numerical results of a specific example we also found that it is important to realize the (often overlooked) differences between the streaming and the elastic flows in terms of how their actual carried traffic is impacted by the system load. Streaming flows will suffer from quality degradation (but their holding time remains unchanged), while elastic flows will remain for a longer time in the system in case of overload. These distinguishing characteristics of these two classes of flows in turn lead to different blocking probability behavior when admission control (by enforcing cut-off parameters) is exercised in the system. (As discussed in details in Section VII-E.) Furthermore, we have also argued and shown that the throughput threshold constraint can be a more informative performance measure than the simple average throughput. From the user perspective, it provides more information on the expected quality of e.g. a video session, while from the network engineer's point of view it may result in different admission control parameters. Thus, we believe that our extension of the multi-rate model and the associated capacity sharing algorithm together with the numerical examples provide some insights and provide arguments for applying traffic engineering methods in IP networks. It is an interesting future research topic how the proposed high level classification can be used for various future applications, including adaptive applications. Also, simulation could be used to evaluate the behavior of this link sharing method at the network level. Finally, the impact of the fact that real rate-control methods (e.g. TCP) and scheduling algorithms can only provide an approximation of our ideal model requires further studies.

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