A Recursive Formula to Calculate the Steady State of CDMA Networks

Gábor Fodor[1] and Miklós Telek[2]

[1] Ericsson Research, SE-164 80 Stockholm, Sweden,
Gabor.Fodor@ericsson.com
[2] Budapest University of Technology and Economics, H-1111 Budapest, Hungary,
telek@hit.bme.hu

Abstract. Several previous contributions have proposed calculation methods that can be used to calculate the steady state (and from it the blocking probabilities) in CDMA systems. To our best knowledge, this present work is the first one that extends the classical Kaufman-Roberts formula such that it becomes applicable in CDMA where elastic services (with state dependent instantaneous bit rate and average bit rate dependent residency time) are supported. The core of this method is to approximate the original irreversible Markov chain with a reversible one and to assign a proper blocking measure to the macro states of the system.

1 Introduction

The Kaufman-Roberts formula for a fixed transmission link carrying multi-rate traffic establishes a recursive relation between the macro states of the system. This formula has been popular to calculate the steady state of systems with large state spaces, such as the transmission links of circuit switched and asynchronous transfer mode (ATM) networks [1]. A fundamental assumption of the Kaufman-Roberts formula is that the system is Markovian and that the so called local balance equations hold, which is the case in reversible Markov chains.

The works by Stamatelos *et al.* and Rácz *et al.* extend this formula such that it includes elastic and adaptive traffic [2], [3]. The core idea in these papers is to construct a reversible Markov chain that well approximates the non-reversible system that supports elastic traffic. These papers continue to assume a fixed capacity transmission link.

Along another line, the seminal paper by Altman proposes a Shannon like capacity measure called the best effort capacity for CDMA networks supporting elastic services. This model and the calculation method makes use of some assumptions that lead to a reversible system and the steady state can be determined using product forms. Altman's calculations use the direct method for determining the system's steady state and require a matrix inversion that limits the applicability to relatively small state spaces [4]. The same is true for the extension of this model proposed by Fodor and Telek in [5]. The Kaufman-Roberts equation is attractive, because it does not suffer from the problem of state space explosion of realistic systems. Recognizing this, Iversen *et al.* and Mäder *et al.* builds on the Kaufman-Roberts equation in CDMA, but do not model elastic traffic [6], [7]. They assume so called *soft blocking* that leads to a reversible Markov model and do not face the problem of irreversibility.

In this paper we consider the uplink of a CDMA network basically as modeled in [4] and [5]. This system is conveniently modeled by a Markov chain. Next, in order to reduce the number of states and to establish a recursive relationship, we define the system macro states. The macro states are defined much the same way as in the classical Kaufman-Roberts case, that is, the set of micro states in which the overall (CDMA) resource consumption is the same. A major difference compared to the classical case is that the system is (1) irreversible and (2) the macro states are heterogeneous in the sense that there are macro states that consist of a mixture of blocking and non-blocking states. As we shall see, this second feature is due to the non-linear relation between the resource consumption of the elastic sessions and their bit rates. This structure (the combination of these two features) requires some effort in terms of establishing the balance equations on which the Kaufman-Roberts recursion can be built. Specifically, we distinguish non-blocking, *partially blocking* and fully blocking macro states.

The paper is organized as follows. Section 2 presents our basic model for elastic traffic in CDMA. In the subsequent section we develop the reversible approximation of this system and based on this approximation we propose an extension of the Kaufman-Roberts formula. We conclude the paper in Section 4.

2 CDMA Uplink Equations and State Space Structure

The CDMA uplink model is similar to the single transmission link model in that sessions belonging to different service classes share a common resource. In CDMA however, a fast explicit rate control algorithm allows the system to slow down elastic sessions and thereby reduce the required power. In this section we extend the classical multi-rate model described in the previous section and put the CDMA uplink model into a multi-rate context. This model is described in details in [9], [10] and has also been used in [4] and [5].

2.1 Basic CDMA Equations

We consider a single CDMA cell at which sessions belonging to one of I service classes arrive according to a Poisson arrival process of intensity λ_i. Each class is characterized by a peak bit-rate requirement R_i and an *exponentially distributed* nominal holding time with parameter μ_i.

When sending with the peak rate R_i for a session, the required target ratio of the received power from the mobile terminal to the total interference energy at the base station is calculated as follows:

$$\tilde{\Delta}_i = \frac{E_i}{WN_0} \cdot R_i, \quad i = 1, \ldots, I,$$

where E_i/N_0 is the signal energy per bit divided by the noise spectral density that is required to meet a predefined QoS (e.g. bit error rate, BER) for class-i sessions; noise includes both thermal noise and interference. This required E_i/N_0 can be derived from link level simulations and from measurements. W is the spread spectrum bandwidth.

Just like in the previous section, let n_i be the number of ongoing sessions of class i. We will again refer to the vector $\underline{n}$ as the *state* of the system. We now assume that arriving sessions are blocked by a suitable admission control algorithm that prevents the system to reach the state in which the power that should be received at the base station would go to infinity. In other words, a suitable admission control algorithm must prevent the system to reach its *pole capacity* (as defined by Equation (8.10) of [10] and (5) of [4]).

In order to model this, we use the standard equations (8.3)-(8.12) from [10] as follows. The power P_i that is received at the base station from the mobile terminal for class-i session must fulfill (assuming that the terminal can control the power level for each session separately):

$$\frac{P_i}{P_N + Y_{own} + Y_{other} - P_i} = \tilde{\Delta}_i; \quad Y_{own} = \sum_{i=1}^{I} n_i P_i \quad \text{and} \quad Y_{other} = \varphi \cdot Y_{own}; \quad i = 1, \ldots, I, \tag{1}$$

where Y_{own} is the total power received by the base station within its cell (or sector), and Y_{other} is the total power received from other cells (or sector). P_N is the background noise power. Rewriting (1), we get:

$$\frac{P_i}{P_N + Y_{own} + Y_{other}} = \Delta_i; \qquad \Delta_i = \frac{\tilde{\Delta}_i}{1 + \tilde{\Delta}_i}, \quad i = 1, \ldots, I. \tag{2}$$

From (2):

$$\frac{P_i}{P_N + (1+\varphi) \sum_{l=1}^{I} n_l P_l} = \Delta_i, \quad i = 1, \ldots, I. \tag{3}$$

Multiplying with n_i and summing over i:

$$\sum_{i=1}^{I} P_i n_i = \frac{P_N \sum_{i=1}^{I} n_i \Delta_i}{1 - (1+\varphi) \sum_{l=1}^{I} n_i \Delta_i}; \quad P_i = \left(P_N + (1+\varphi) \sum_{l=1}^{I} n_l P_l \right) \cdot \Delta_i, \quad i = 1, \ldots, I. \tag{4}$$

From (4) and denoting $\Psi = \sum_{i=1}^{I} n_i \cdot \Delta_i$:

$$P_i = \left(P_N + (1+\varphi) \cdot \frac{P_N \cdot \Psi}{1 - (1+\varphi) \cdot \Psi} \right) \cdot \Delta_i = \frac{P_N \cdot \Delta_i}{1 - (1+\varphi) \cdot \Psi}, \quad i = 1, \ldots, I. \tag{5}$$

2.2 The Impact of Slow Down

Recall that the class-wise required target ratio (Δ_i) depends on the required bit-rate. Explicit rate controlled elastic services tolerate a certain slow down of their peak bit-rate (R_i) as long as the actual instantaneous bit rate remains greater than the minimum required $R_i/\hat{a}_i$. When the bit rate of a class-i session is slowed down to R_i/a_i ($1 \leq a_i \leq \hat{a}_i$), its required Δ_{a_i} value becomes:

$$\Delta_{a_i} = \frac{\tilde{\Delta}_i}{a_i + \tilde{\Delta}_i} = \frac{\Delta_i}{a_i \cdot (1 - \Delta_i) + \Delta_i}, \quad i = 1, \ldots, I, \tag{6}$$

which increases the number of sessions that can be admitted into the system, since now Ψ_a must be kept below $\bar{\Psi}$, where $\Psi_a = \sum_{i=1}^{I} n_i \cdot \Delta_{a_i}$.

We use the notation $\Delta_{min,i} = \Delta_{\hat{a}_i}$ to denote the class-wise minimum target ratios (can be seen as the minimum resource requirement), that is when the session bit-rates of class-i are slowed down to that class' minimum value. It is the task of the bandwidth sharing policy to determine the $\Delta_{a_i} \geq \Delta_{min,i}$ values (and consequently the $a_i \leq \hat{a}_i$ class-wise instantaneous slow down factors) for each state of the system such that $\Psi_a < \bar{\Psi}$. Because of the admission control assumption, such a resource assignment is always possible in feasible states.

2.3 Determining the System State Space

The maximum number of sessions from each class can be calculated as follows:

$$n_{max,i} = \lfloor (\Delta_{\hat{a}_i})^{-1} \rfloor, \quad i = 1, \ldots, I.$$

Recall that in each $\underline{n}$ state of the system, the inequality $\sum_i n_i \cdot \Delta_{a_i} < \bar{\Psi}$ must hold. The states that satisfies this inequality are the *feasible states* and constitute the state space of the system (Θ). The feasible states, in which the acceptance of an additional class-i session would result in a state outside of the state space are the class-i *blocking states*. The set of the class-i blocking states is denoted by Θ_i. Due to the "Poisson Arrivals See Time Averages" (PASTA) property, the sum of the class-i blocking state probabilities gives the (overall) class-i blocking probability [1].

In each feasible state, it is the task of the bandwidth sharing policy to determine the $\Delta_{a_i}(\underline{n}) = f_i(\underline{n})$ class-wise target ratios for each class. The functions $f_i(\cdot)$ reflect the fairness criterion that is implemented in the resource sharing policy and is out of the scope of this paper. From these, the class-wise slow down factors and the instantaneous bit-rates of the individual sessions can be calculated as follows:

$$a_i(\underline{n}) = \frac{\Delta_i \cdot (1 - \Delta_{a_i}(\underline{n}))}{\Delta_{a_i}(\underline{n}) \cdot (1 - \Delta_i)}; \quad R_{a_i}(\underline{n}) = R_i / a_i(\underline{n}) \tag{7}$$

For ease of presentation, in the rest of the paper we will not indicate the dependence of a_i, Δ_{a_i} and R_{a_i} on the micro state $\underline{n}$.

2.4 An Example of the State Space

Figure 1 depicts an example of the state space. In this example we neglect the other-cell interference ($\varphi = 0$), that is we consider the overall CDMA resource to be 1 ($\bar{\Psi} = 1$), which corresponds to $C = 1$ in the classical multi-rate model. There are two traffic classes with peak resource requirements $\Delta_1 = 2\Delta$ and $\Delta_2 = \Delta$ respectively (where $\Delta = 0.299$). Both traffic classes are elastic and tolerate a slow down of their bit rates to one third of their respective peak bit rates (i.e. $\hat{a}_1 = \hat{a}_2 = 3$), which corresponds to $\Delta_{min,1} = 0.3315$ and $\Delta_{min,2} = 0.1245$.

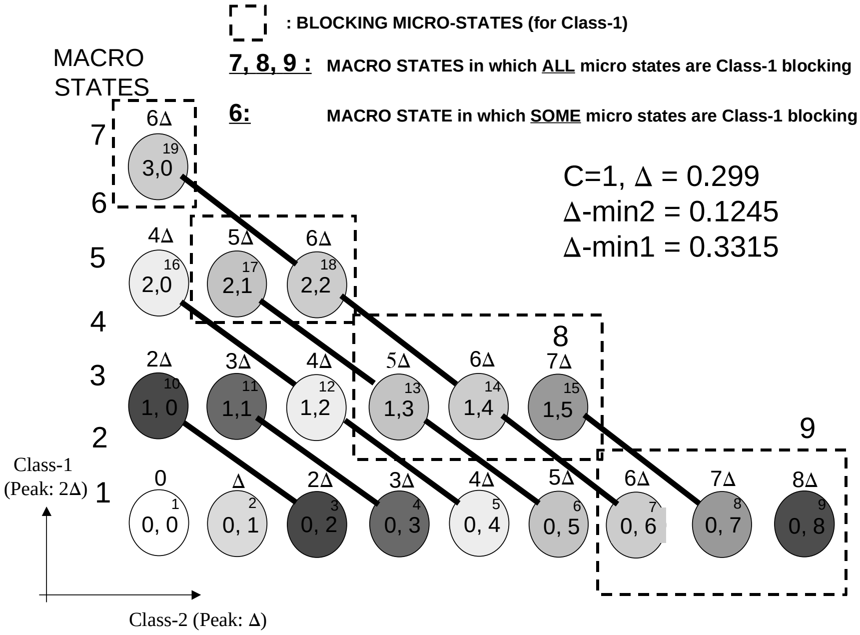

Fig. 1. An example of a state transition diagram

This system can be in one of the 19 feasible micro states as illustrated in Figure 1. The 9 class-1 blocking micro states are indicated in dotted squares. There are 7 macro states, out of which macro state 6 (consisting of micro states: (2,1), (1,3) and (0,5)) deserves attention. In this macro state the overall peak resource consumption is $5\Delta > \bar{\Psi}$ and therefore there is a need for slowing down some of the sessions. On the other hand, all three micro states are feasible states, because if all sessions are slowed down to their minimum rates, the resource consumptions become $2\Delta_{min,1} + \Delta_{min,2} = 0.7875$, $\Delta_{min,1} + 3\Delta_{min,2} = 0.705$ and $5\Delta_{min,2} = 0.6225$. What is noteworthy is that micro states (2,1) and (1,3) are class-1 blocking states (micro states (3,1) and (2,3) are certainly not feasible), but micro state (0,5) is non-blocking, since micro state (1,5) is part of the state space ($\Delta_{min,1} + 5\Delta_{min,2} = 0.954 < \bar{\Psi}$).

2.5 Assigning a Blocking Measure to the Macro States

In contrast to the classical Kaufman-Roberts case, a CDMA macro state $\Omega(j)$ may consist of non-blocking micro states only, a "mixture" of non-blocking and blocking micro states or blocking micro states only. In order to define the recursive relationship between macro states, there is a need to assign a blocking measure to each macro state.

In order to define such a macro state blocking measure, we first introduce the following micro state blocking measure. In each micro state $\underline{n}$:

$$\beta_i(\underline{n}) = \begin{cases} 0 & \text{if } \underline{n} \text{ is not a class-}i \text{ blocking state} \\ \Pi(\underline{n}) & \text{if } \underline{n} \text{ is a class-}i \text{ blocking state.} \end{cases} \quad (8)$$

Consider now the micro states of the system in which the macro state is j and $|\Omega(j)| > 0$. Then:

– Macro state j is a non-blocking macro state with respect to class-i if $|\Omega_i(j)| = 0$. In these states: $\beta_i(j) = 0$.
– Macro state j is a fully blocking macro state with respect to class-i if $|\Omega_i(j)| > 0$ and $\Omega_i(j) = \Omega(j)$. That is, all micro states of this macro state are blocking with respect to class-i. In these states: $\beta_i(j) = \sum_{\Omega(j)} \Pi(\underline{n})$.
– Macro state j is a partially blocking macro state with respect to class-i if $|\Omega_i(j)| > 0$ and $\Omega_i(j) \subset \Omega(j)$. In these states: $\beta_i(j) = \sum_{\Omega_i(j)} \Pi(\underline{n})$. Clearly, the calculation of $\beta_i(j)$ in this case requires that the blocking micro states within macro state j (that is the set $\Omega_i(j)$) are known.

3 Recursive Equation for Elastic Traffic in CDMA

3.1 Constructing the Reversible System

In general the rate controlled system is not reversible, as it was discussed for two specific work conserving $f(\cdot)$ resource sharing functions in [5]. As a first step, we devise a method that can be used to construct a reversible Markov chain for virtually all work conserving such $f(\cdot)$ functions.

Recall that in system state $\underline{n}$ all class-i sessions are slowed down by a factor of $a_i(\underline{n})$, which implies that the class-i session departure (death) rates become $\mu_i/a_i(\underline{n})$. Since the system is not reversible, the local balance equations in general do not apply:

$$n_i \cdot \frac{\mu_i}{a_i(\underline{n})} \cdot \Pi(\underline{n}) \neq \lambda_i \Pi(\underline{n} - \underline{e_i}) \qquad \forall i \in I. \tag{9}$$

Similarly to what had been done in [2] and [3], we look for an approximating Markov chain with the same Ω state space in the form that is defined by (10).

$$\Phi_i(\underline{n}) n_i \mu_i \Pi(\underline{n}) = \lambda_i \Pi(\underline{n} - \underline{e_i}) \qquad \forall i \in I. \tag{10}$$

A Markov chain constructed like this is reversible if and only if the values of $\Phi_i(\underline{n})$ for every state is derived from another positive state dependent variable $x(\underline{n})$ as described by the following equation (see also [1]):

$$\Phi_i(\underline{n}) = \frac{x(\underline{n_i^-})}{x(\underline{n})}; \qquad x(\underline{0}) = 1. \tag{11}$$

We select the class dependent function $\Phi_i(\underline{n})$ in such a way that the modified model possesses the following properties (without further motivating these properties):

– The resource sharing discipline keeps its work conserving property, i.e. either all flows get their respective required peak resource (and are served with their peak rate R_i) or the system is serving at its full capacity.
– Any flow may transfer the same amount of data in state $\underline{n}$ of the modified Markov chain as it would do in state $\underline{n}$ of the original Markov chain.

Table 1. Comparison of the Original and the Modified Markov Chains

Properties in state $\underline{n}$		
	Original Markov chain	Reversible Markov Chain
Departure rate	$\mu_i / a_i(\underline{n})$	$\Phi_i(\underline{n})\mu_i$
Traffic demand	Δ_i / μ_i	$\delta_i(\underline{n})/(\Phi_i(\underline{n})\mu_i)$
Used resource	Δ_{a_i}	$\delta_i(\underline{n}) = \Phi_i(\underline{n})\Delta_i$

Table 1 compares the properties of the original (irreversible) and the modified (reversible) Markov chains. The first row corresponds to the holding times and show that they needed to be changed to get a reversible Markov chain. The second row shows the *traffic demand* of the flows. We may think of the traffic demand of a class-i session as its peak resource requirement multiplied by its mean nominal holding time, (that is its mean holding time when served at its peak rate). This product corresponds to the number of bits that the flow would transfer were it served at its peak bit-rate. The key observation is that according to the second property and the reasoning above, these values need to be the same in state $\underline{n}$ of both Markov chains. The third raw shows the actual resource consumed by class-i flows in state $\underline{n}$, which we may think of as a "bandwidth like" quantity in the multi-rate model. Thus, as shown in the last row of Table 1, the second property determines the resource shares ($\delta_i(\underline{n})$) of the flows in the modified Markov chain.

We now focus on the work conserving property. This consists of two parts. If the system is not fully loaded, that is $j = \underline{n} \cdot \underline{\Delta} \leq \bar{\Psi}$, then the sum of the bandwidth shares must exactly match the total amount of resource occupied. This relation is shown in (12). We note here that this part is handled the same way as it was done in [2] and [3].

$$\sum_{i=1}^{I} n_i \, \delta_i = \sum_{i=1}^{I} n_i \, \Delta_i \, \Phi_i(\underline{n}) = j \qquad j \leq \bar{\Psi} \tag{12}$$

Equation (12) implies that when $j \leq \bar{\Psi}$, then $\Phi(\underline{n}) = 1$. On the other hand, when the resource is over-booked, the bandwidth shares of the flows must sum up to the link capacity. Equation (13) quantifies this:

$$\sum_{i=1}^{I} n_i \, \delta_i = \sum_{i=1}^{I} n_i \, \Delta_i \, \Phi_i(\underline{n}) = \bar{\Psi} \qquad j > \bar{\Psi} \tag{13}$$

Equations (12) and (13) are a system of equations for $\Phi_i(\underline{n})$ which assure that the modified model remains work conserving and describes the system load well.

Substituting (11) into (12) and (13) results in (14), which with the condition that $x(\underline{0})$ equals to 1, determines the values of $x(\underline{n})$ recursively:

$$x(\underline{n}) = \frac{\sum_{i=1}^{I} n_i \, \Delta_i \, x(\underline{n}_i^-)}{\text{Min}(\bar{\Psi}, j)}. \tag{14}$$

3.2 Steady State Analysis

The state space of the reversible Markov chain is identical with that of the original system, but the state transitions are modified. We can therefore immediately establish the local balance equations in this modified system (recall the blocking measure β introduced in Subsection 2.5):

$$n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right) . \tag{15}$$

Subtracting $\beta_i(\underline{n}_i^-)$ on the right hand side of this equation reflects that there is no class-i state transition from state $(\underline{n}_i^-)$ to state $\underline{n}$ if the former is a class-i blocking state.

Summing over i, we get for micro states $\underline{n}$ in which $j \leq \bar{\Psi}$:

$$\sum_{i=1}^{I} n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right)$$

$$j \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right) ; \qquad j \leq \bar{\Psi}$$

And for micro states, in which $j > \bar{\Psi}$:

$$\sum_{i=1}^{I} n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right)$$

$$\bar{\Psi} \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right) ; \qquad j > \bar{\Psi}$$

Summarizing:

$$\text{Min}[j, \bar{\Psi}] \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right) \tag{16}$$

Summing over $\Omega(j)$:

$$\text{Min}[j, \bar{\Psi}] \cdot \sum_{\underline{n}:\underline{n}\cdot\underline{\Delta}=j} \Pi(\underline{n}) = \sum_{\underline{n}:\underline{n}\cdot\underline{\Delta}=j} \sum_{i=1}^{I} \rho_i \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right) \tag{17}$$

Combining (16) and (17) and making use of the macro state blocking measures defined in Subsection 2.5 leads to our first result.

Theorem 1. *The $q(j)$ macro state probabilities satisfy the following set of recursive equations:*

$$\textit{Min}[j, \bar{\Psi}] \cdot q(j) = \sum_{i=1}^{I} \rho_i \Delta_i \left(q(j - \Delta_i) - \beta_i(j - \Delta_i) \right) . \tag{18}$$

Equation (18) establishes a recursive relationship between macro states $\Omega(j)$ and $\Omega(j - \Delta_i)$ that can be seen as an analogy to the relationship between macro states $\Omega(j)$ and $\Omega(j - b_i)$ expressed by the classical Kaufman-Roberts formula. Note that (18) yields the relative (unnormalized) macro state probabilities when setting $q(0)$ to some convenient value (typically to 1). In order to arrive to the macro state probabilities, a normalization is necessary. In order to arrive at performance measures of interest and specifically the class-wise blocking probabilities, the issue is to determine the $\beta_i(j - \Delta_i)$ class-wise blocking measures for each macro state.

3.3 Revisiting the Example of Subsection 2.4

Macro State	Value of j	Micro State	Micro state probabilities	Macro state probabilities	Is it a Class-1/2 blocking micro state ?	β_1(j)	β_2(j)	q(j-Δ)	q(j-2Δ)
1	0	1	0,28261	0,28261	NO/NO	0,00000	0,00000	-	-
2	**Δ**	2	0,17127	0,17127	NO/NO	0,00000	0,00000	q(0)	-
3	**2Δ**	3	0,05180	0,22280	NO/NO	0,00000	0,00000	q(Δ)	q(0)
		10	0,17100		NO/NO				
4	**3Δ**	4	0,01040	0,11400	NO/NO	0,00000	0,00000	q(2Δ)	q(Δ)
		11	0,10360		NO/NO				
5	**4Δ**	5	0,00190	0,10190	NO/NO	0,00000	0,00000	q(3Δ)	q(2Δ)
		12	0,03800		NO/NO				
		16	0,06200		NO/NO				
6	**5Δ**	6	0,00034	0,05934	NO/NO	0,05900	0,00000	q(4Δ)	q(3Δ)
		13	0,01000		YES/NO				
		17	0,04900		YES/NO				
7	**6Δ**	7	0,00006	0,04746	YES/NO	0,04746	0,04480	q(5Δ)	q(4Δ)
		14	0,00260		YES/NO				
		18	0,02240		YES/YES				
		19	0,02240		YES/YES				
8	**7Δ**	8	0,00001	0,00060	YES/NO	0,00060	0,00059	q(6Δ)	q(5Δ)
		15	0,00059		YES/YES				
9	**8Δ**	9	2,030E-06	2,030E-06	YES/YES	2,030E-06	2,030E-06	q(7Δ)	q(6Δ)

Fig. 2. Micro and macro state probabilities of the system of Figure 1

To illustrate the usefulness of equation (18), we briefly revisit the state space example of Subsection 2.4. Recall that in this example there are 19 micro states and 9 macro states, where the macro state values are $j = 0, \Delta, \ldots, 8\Delta$. Figure 2 shows which micro states belong to which macro states and also the micro and macro state probabilities. The micro state probabilities were calculated by the direct method described in [5]; the macro state probabilities are simply the sum of the associated micro state probabilities. We note in Figure 2 that macro state 6 is a partially blocking Class-1 macro state and macro states 7 and 8 are are partially blocking Class-2 macro states.

Figure 2 also shows the class-wise $\beta_i(j)$ values which are simply the sum of the blocking micro state probabilities within macro state j. Using the terminology introduced in Subsection 2.5, this $\beta_i(j)$ value is 0 in the case of non-blocking macro states, equals to the macro state probability in the case of fully blocking macro states and equals to the sum of blocking micro states in the case of partially blocking macro states. (See columns 7-8 in Figure 2.)

In this example we have selected $\rho_1 = \rho_2 = \rho$ and $\Delta_1 = 2 \cdot \Delta_2 = 2 \cdot \Delta$ and so (18) becomes:

$$q(j) = \frac{1}{\text{Min}[j, \bar{\Psi}]} \cdot \Big(2\rho\Delta\big(q(j-2\Delta) - \beta_1(j-2\Delta)\big) + \rho\Delta\big(q(j-\Delta) - \beta_2(j-\Delta)\big)\Big) \quad (19)$$

Since Figure 2 lists all the input data that is required at the right hand side of (19), the $q(j)$'s can be easily verified for this example.

4 Conclusions

In this paper we considered a CDMA cell that supports elastic services. Sessions belonging to such service classes can dynamically adjust their transfer rates depending on the interference situation in the system. The dynamic adjustment of the session bit rates makes the calculation of the session-wise blocking probabilities non-trivial, because (1) the session holding times depend on the received throughput and (2) the CDMA resource consumption and the instantaneous bit rate is non linear. In this fairly general setting the paper develops an extension of the Kaufman-Roberts recursion that has been the de facto standard to calculate the steady state of fixed multi-rate systems. In order to establish this recursion, we approximated the original irreversible Markovian system with a reversible one. It turns out that the system macro states are such that a single macro state may contain both blocking and non-blocking micro states. Because of this, the issue is to associate appropriate blocking measures with the macro states. We have shown that if such blocking measures are available then the extended Kaufman-Roberts formula is useful to establish the steady probability distribution of the system. Determining these blocking measures is left for future work.

References

1. K. W. Ross, ”Multiservice Loss Models for Broadband Telecommunication Networks”, ISBN 3-540-19918-8, *Springer Verlag*, 1995.
2. G. M. Stamatelos and V. N. Koukoulidis, ”Reservation-Based Bandwidth Allocation in a Radio ATM Network”, *IEEE Transactions on Networking*, Vol. 5, No. 3, pp. 420-428, June 1997.
3. S. Racz, B. P. Gero and G. Fodor, ”Flow Level Performance Analysis of a Multi-service System Supporting Elastic and Adaptive Services”, *Performance Evaluation 49*, Elsevier, pp. 451-469, 2002.
4. E. Altman, ”Capacity of Multi-service Cellular Networks with Transmission-Rate Control: A Queueing Analysis”, *ACM Mobicom ’02*, Atlanta, GA, September 23-28, 2002.
5. G. Fodor and M. Telek, ”Performance Anlysis of the Uplink of a CDMA Cell Supporting Elastic Services”, in the Proc. of *IFIP Networking 2005*, Springer LNCS 3462, pp. 205-216, Waterloo, Canada, May 2005.
6. V. B. Iversen, V. Benetis, N. T. Ha and S. Stepanov, ”Evaluation of Multi-service CDMA Networks with Soft Blocking”, *Proc. ITC Specialist Seminar*, pp. 223-227, Antwerp, Belgium, August/September 2004.
7. A. Mäder and D. Staehle, ”Uplink Blocking Probabilities in Heterogeneous WCDMA Networks Considering Other-cell Interference”, *Southern African Telecommunication Networks and Applications Conference*, South Western Cape, South Africa, September 2004.
8. A. Mäder and D. Staehle, ”An Analytic Approximation of the Uplink Capacity in a UMTS Network with Heterogenous Traffic”, 18^{th} *International Teletraffic Congress (ITC 18)*, Berlin, September 2003.
9. A. J. Viterbi, ”CDMA - Principles of Spread Spectrum Communication”, *Addison-Wesley*, ISBN 0-201-63374-4, 1995.
10. H. Holma and A. Toskala, ”WCDMA for UMTS - Radio Access for Third Generation Mobile Communications”, Wiley, ISBN 0 471 72051 8, **First Edition, 2000**.