On the Tradeoff Between Blocking and Dropping Probabilities in Multi-cell CDMA Networks

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Abstract— **This paper is a sequel of previous work, in which we proposed a model and computational technique to calculate the Erlang capacity of a single CDMA cell that supports elastic services. The present paper extends that base model by taking into account two important aspects of CDMA. First, we describe a simple and a refined multi-cell CDMA model that are able to capture the impact of the neighbor cells on important performance measures of the cell under study. These performance measures include the class-wise blocking probabilities and the mean time that elastic sessions spend in the system. Secondly, we model the impact of the outage by taking into account that in-progress sessions can be *dropped* with a probability that depends on the current load in the serving and neighbor cells. We then consider a system with elastic and rigid service classes and analyze the trade-off between the total (soft and hard) blocking probabilities on the one hand and the throughput and the session drop probabilities on the other.**

Index Terms— **code division multiple access (CDMA), traffic capacity, queueing theory, Markov chains**

I. Introduction

The teletraffic behavior of code division multiple access (CDMA) networks has been the topic of research ever since CDMA started to gain popularity for military and commercial applications, see for instance Chapter 6 of [1] (and the references therein) that are concerned with the Erlang capacity of CDMA networks. The paper by Evans and Everitt used an $M/G/\infty$ queue model to assess the uplink capacity of CDMA cellular networks and also presented a technique to calculate the outage probability [2]. These classical papers have focused on "rigid" traffic in the sense that elastic or best effort traffic whose bit rate can dynamically change was not part of the models. Subsequently, the seminal paper by Altman proposed a Shannon like capacity measure called the "best effort capacity" that explicitly takes into account the behavior of elastic sessions [3].

Along another line, Iversen *et al.* and Mäder *et al* proposed a CDMA model that takes account of the interference from neighbor cells by introducing the notion of *soft blocking* [7], [8]. This means that arriving sessions can be blocked in virtually any system state with a state dependent probability. These papers however have not considered the elastic traffic characteristics as described in [3] and outages are not modeled.

The importance of modeling outages and *session drops* and their impacts on the Erlang capacity in cellular networks in general and in CDMA in particular has been emphasized by several authors, see for instance [2] and more recently [9]. Session drops are primarily caused by outages, when the desired signal-to-noise ratio for a session stays under a predefined threshold during such a long time that the session gets interrupted. However, sessions can be dropped by a load control algorithm (typically located in the radio network controller in WCDMA) to preserve system stability. Session interruptions are perceived negatively by end users - more negatively than blocking a session - and therefore their probability should be minimized by suitable resource management (including admission control) techniques.

The purpose of this paper is to develop a model that can be used to analyze the trade-off between the blocking and dropping probabilities in multi-cell CDMA systems in the presence of elastic traffic. We build on the model developed for elastic traffic in previous work [5] and extend it with allowing for a state dependent soft blocking and capturing the fact that sessions are sometimes dropped. We develop two alternative models to capture the multi-cell impacts and show how these models can be used when the system supports elastic service classes. When the load is high, the interference from neighbor cells leads to outages with a higher probability than when it is low. For elastic sessions, fast rate and power control attempts to reduce the transmission rates and the required received power at the base station, as long as the transmission rates stay above the session specific so called *guaranteed bit rate* (GBR). Therefore, it seems intuitively clear that there is a trade-off between how conservative the admission control algorithm is (on the one hand) and what is the average bit rate of elastic sessions and what session drop probabilities users experience (on the other hand). The contribution of the paper is to propose a model that can be used for the analysis of this trade-off.

This paper is based on a previous work "On the Tradeoff Between Blocking and Dropping Probabilities in CDMA Networks Supporting Elastic Services,", which appeared in the Proceedings of *Networking 2006*, Coimbra, Portugal, May 2005.

II. Modeling Elastic Traffic in CDMA (The Single Cell Case)

A. Basic CDMA Equations

Consider a single CDMA cell at which sessions belonging to one of K service classes arrive according to a Poisson arrival process of intensity λ_k ($k = 1, \ldots, K$). Each class is characterized by a peak bit-rate requirement $\hat{R}_k$ and an exponentially distributed nominal holding time with parameter μ_k. When sending with the peak rate for a session, the required target ratio of the received power from the mobile terminal to the total interference energy at the base station is calculated as follows:

$$\tilde{\Delta}_k = \frac{E_k}{WN_0} \cdot \hat{R}_k, \quad k = 1, \ldots, K, \tag{1}$$

where E_k/N_0 is the signal energy per bit divided by the noise spectral density that is required to meet a predefined QoS (e.g. bit error rate, BER); noise includes both thermal noise and interference. This required E_k/N_0 can be derived from link level simulations and from measurements. $\hat{R}_k$ is the peak bit rate of the session of class-k and W is the spread spectrum bandwidth. ($\hat{R}_k/W$ is usually referred to as the processing gain.)

Let U_k be the number of ongoing sessions of class-k and P_k the power received at the base station from the user equipment (UE) engaged in a session of class-k. We will refer to the vector $\underline{U} = \{U_k, \ k = 1, \ldots, K\}$ as the *state* of the system. When the system is in state $\underline{U}$, the total power received at the base station from within its own cell is

$$Y_{own} = \sum_{k=1}^{K} U_k P_k. \tag{2}$$

In order to determine the power required to be received from a user of class-k, we make the following considerations. The power received at the base station from a class-k session has to fulfil:

$$\frac{P_k}{P_N + Y_{own} - P_k} = \tilde{\Delta}_k, \quad k = 1, \ldots, K, \tag{3}$$

where P_N denotes the background noise power. Rewriting (3), we get:

$$P_k = \frac{\tilde{\Delta}_k}{1 + \tilde{\Delta}_k} \left(P_N + Y_{own}\right) = \Delta_k \left(P_N + Y_{own}\right), \quad k = 1, \ldots, K, \tag{4}$$

where

$$\Delta_k = \frac{\tilde{\Delta}_k}{1 + \tilde{\Delta}_k} \tag{5}$$

can be interpreted as the fraction of the system load that is generated by a user of class-k, or with less words: the *load increment* of class-k. Further, by substituting (4) into (2), we find:

$$Y_{own} = \sum_{k=1}^{K} P_k U_k = \frac{\sum_{l=1}^{K} U_l \Delta_l}{1 - \sum_{l=1}^{K} U_l \Delta_l} \cdot P_N = \frac{\Psi_{own}}{1 - \Psi_{own}} \cdot P_N, \tag{6}$$

where

$$\Psi_{own} = \Psi_{own}(\underline{U}) \triangleq \sum_{l=1}^{K} U_l \cdot \Delta_l. \tag{7}$$

Then, (4) and (6) give the power requirement of class-k as a function of the load increments:

$$P_k = \Delta_k \cdot \left(\frac{1}{1 - \Psi_{own}}\right) \cdot P_N, \quad k = 1, \ldots, K. \tag{8}$$

B. The Load Factor and the Noise Rise

Ψ_{own} is also known as the *load factor*, η_{own} [10]:

$$\eta_{own} \triangleq \sum_{i=1}^{S} \frac{P_i}{Y_{own} + P_N} = \sum_{s=1}^{S} \Delta_s = \sum_{k=1}^{K} U_k \cdot \Delta_k \equiv \Psi_{own}, \tag{9}$$

where $S = \sum_k^K n_k$ denotes the total number of in-progress sessions in state $\underline{U}$ and the index s refers to individual sessions rather than to service classes.

Closely related to the load factor is the *noise rise* in the cell:

$$T_{own} = \frac{Y_{own} + P_N}{P_N} = \frac{1}{1 - \eta_{own}}. \tag{10}$$

The noise rise describes how much the noise (i.e. the total received power as seen by a new (imaginary) session) has increased in state $\underline{U}$ compared to an empty system and is a useful measure of the current interference level in the system.

The QoS requirement of an arriving (new) class-k session is characterized by $\tilde{\Delta}_k$ according to equation (1), and if admitted, the total power increases with ΔP_{TX} which is expressed as an increase of the load factor and the noise rise in the cell:

$$\Delta\eta_{own,k} \equiv \Delta_k = \frac{P_N}{P_{TXown}} - \frac{P_N}{P_{TXown} + \Delta P_{TXown,k}};$$

$$\Delta T_{own,k} = \frac{\Delta P_{TXown,k}}{P_N}. \tag{11}$$

From (8) it is clear that if η_{own} reached $\hat{\Psi} \equiv \hat{\eta}_{own} = 1$, the required power P_k would tend to infinity. In the single class case it means that the number of admitted sessions must fulfill: $U < \lfloor \hat{\eta}_{own}/\Delta \rfloor$ (where we now let $U = U_1$ and $\Delta = \Delta_1$). In practice, the admission control procedure is often based on the noise rise value of the cell and keeps the system load under a (much) lower value. The admission control then aims to keep the noise rise value under a predefined threshold value which we denote by $\hat{T}_{own}$.

One can think of $\eta_{own}(\underline{U})$ as the overall used resource in state ($\underline{U}$) of the multi-rate CDMA system, while $\hat{\eta}_{own}$ corresponds to the "total available resource". This can be seen as an analogy between the multi-rate CDMA model and the multi-rate loss models developed in the 80's and 90's [11]. These models have been extended and used to analyze multi-rate systems with elastic traffic for fixed networks in a number of papers, see for instance [12], [15], [16] and [13]. As we shall see in the next subsection, the major difference between the classical loss models and the present CDMA model is that the relation between the *slowdown rate* a_k and the resource consumption Δ_{a_k} is not linear.

C. The Impact of Slowdown

Recall that the class-wise required target ratio (Δ_k) depends on the required bit-rate. Explicit rate controlled elastic services tolerate a certain slowdown of their peak bit-rate ($\hat{R}_k$) as long as the actual instantaneous bit rate remains greater than the minimum required $\hat{R}_k/\hat{a}_k$. When the bit rate of a class-k session is slowed down to $\hat{R}_k/a_k$, ($0 < a_k \leq \hat{a}_k$) its required Δ_{a_k} value becomes:

$$\Delta_{a_k} = \frac{\tilde{\Delta}_k}{a_k + \tilde{\Delta}_k} = \frac{\Delta_k}{a_k \cdot (1 - \Delta_k) + \Delta_k}, \quad k = 1, \ldots, K,$$

which increases the number of sessions that can be admitted into the system, since now $\eta_{own,a}$ must be kept below $\hat{\eta}_{own}$, where

$$\eta_{own,a} = \sum_{k=1}^{K} U_k \cdot \Delta_{a_k}.$$

We use the notation $\Delta_{min,k} = \Delta_{\hat{a}_k}$ to denote the class-wise minimum target ratios (can be seen as the minimum resource requirement), that is when the session bit-rates of class-k are slowed down to that class' minimum value. The smallest of these $\Delta_{min,k}$ values $\Delta = \min_k \Delta_{min,k}$ can be thought of as the finest "granularity" with which the overall CDMA resource is allocated between competing sessions.

We note that a system characterized by these parameters have been analyzed by Altman in [3], [4] and subsequently by Fodor *et al.* in [5] and [14]. One of the results from these papers is that increasing the slowdown factor for some traffic classes leads to smaller blocking probabilities at the expense of increased per-class sojourn times (throughput degradation) and sometimes also somewhat increased outage probabilities (on this latter issue see [14]). Therefore, slowing down some sessions presents some interesting trade-offs; the investigation of these trade-offs are out of the scope of the present paper.

III. The Multi-cell CDMA Models

First we note that we use the term *neighbor cell* to refer to cells which cause non-negligible interference in a cell under consideration. While all cells outside the cell under consideration can contribute to the interference situation, the interference is under practical propagation conditions dominated by contributions from a limited set of cells that are usually (but not always) located close to the cell under consideration. Which cells contribute and which do not is, however, outside the scope of this paper.

A. The Simplified Multi-cell Model

The interference contribution from neighbor cells is typically quite high (around 30-40%). In the simple multi-cell model this is taken into account as follows. We think of the CDMA system as one that has a maximum of $\hat{n} = \frac{\hat{\Psi}}{\Delta}$ number of (virtual) channels. The neighbor cell interference ξ is a random variable of log-normal distribution with the following mean and standard deviation respectively :

$$\alpha = \frac{\varphi}{\varphi + 1} \cdot \hat{n} \quad \text{and} \quad \beta = \alpha, \tag{12}$$

where φ is the factor characterizing the neighbor cell interference and is an input parameter of the model.

The mean value of the interference α is equal to the average capacity loss in the cell due to the neighbor cell interference and β is chosen to be equal to α as proposed by [8] and also adopted by [7]. (When $\varphi = 0$, the neighbor cell interference is ignored in the model.)

Recall that we think of $\Psi(\underline{U})$ as the used resource in state $\underline{U}$. Then in a given state $\underline{U}$ let $b_\Psi(\underline{U})$ denote the probability that the neighbor cell interference is greater than the available capacity in the current cell that is ($\hat{\Psi} - \Psi$):

$$b_\Psi(\underline{U}) = Pr\{\xi > \hat{\Psi} - \Psi\} = 1 - Pr\{\xi < \hat{\Psi} - \Psi\} =$$

$$= 1 - D(\hat{\Psi} - \Psi),$$

where $D(x)$ is the cumulative distribution function of the log-normal distribution:

$$D(x) = \frac{1}{2}\left(1 + \text{erf}\Big(\frac{\ln(x) - N}{S\sqrt{2}}\Big)\right); N = \ln\left(\frac{\alpha^2}{\sqrt{\alpha^2 + \sigma^2}}\right);$$

$$S^2 = \ln\left(1 + \frac{\sigma^2}{\alpha^2}\right).$$

The impact of state dependent soft blocking caused by the neighbor cell interference, can conveniently be taken into account by modifying the λ_i arrival rates in each state by the (state dependent) so called passage factor: $\sigma_k(\underline{U}) = g_k(1 - b_\Psi(\underline{U})) = g_k(D(\hat{\Psi} - \Psi(\underline{U})))$. The passage factor is the probability that a class-k session is not blocked by the admission control algorithm when such a session arrives in system state $\underline{U}$ [7]. Obviously, the passage factor of the hard blocking states is zero. [1] When $g_k(x) = x \ \forall i$, the passage factor only depends on the state of the system through the total number of occupied virtual channels (the "macro state" of the system) and is the same for all classes. This is the assumption of the current paper. We note that the notion of the passage factor is needed in the simple model that builds on the probability that the neighbor cell interference exceeds a certain value. As we shall see next, when the state of the neighbor cells is explicitly modeled, the passage factor becomes 1 in non-blocking states and 0 in the blocking states.

[1] From this point we somewhat casually use the term *blocking* to refer to *hard* blocking, while we explicitly spell out *soft blocking* when this casual usage is not confusing.

B. The Refined Multi-cell Model

In the multi-cell case, we need to distinguish between MSs that belong to the same service class but cause different interference to neighbor cells. Therefore, in the refined multi-cell model, the index k refers to a *group* of mobile stations (MS) rather than to a set of MSs belonging to the same service class. Likewise K is the number of groups rather than the number of service classes. We say that a set of MSs that are served by the same cell (base station) and belong to the same service class and cause (approximately) the same neighbor cell interference to neighbor cells belong to the same group G_k. The indexing of these groups will be useful, which we do as follows. Let C be the number of cells (and base stations, BS), K_c the number of groups served by Cell-c and U_k the number of MSs that belong to Group-k in the system. Also, define $s_m \triangleq \sum_{i=1}^{m} K_i + 1$. Then the groups belonging to Cell-c are labeled by $s_{c-1}, \ldots s_{c-1} + K_c - 1$, the groups belonging to cells in neighbor cells with lower (less than c) and higher (greater than c) indexes are labeled by $s_{c-2}, \ldots s_{c-2} + K_{c-1} - 1, c \geq 2$ and $s_c, \ldots s_c + K_{c+1} - 1, c \leq C - 1$, respectively. We will use the notation $\mathcal{K}_c$ to refer to the set of indices of the groups that belong to Cell-c, $\bar{\mathcal{K}}_c$ to refer to the (set of indices of the) "neighbor groups", while $\mathcal{C}(k)$ denotes the index of the cell that accommodates Group-k.

C. The per-Mobile Station Power Coupling Factor

We need to calculate the power received at the base station of Cell-c from within its own cell and also from the neighboring cells. Let $h_{k,u_k}^{\mathcal{C}(k)}$ denote the path gain from the u_k-th MS of Group-k to the BS of Cell-$\mathcal{C}(k)$ and let p_{k,u_k}^{c} ($k \in \bar{\mathcal{K}}_c$) denote the coupling factor of groups that belong to the neighbor cells. Also, let P_{TX,k,u_k} denote the transmit power of this MS. Then, the power received at BS from its own groups and from the neighbor groups respectively, can be expressed as follows:

$$\begin{aligned} Y_{c,own} &= \sum_{k \in \mathcal{K}_c} \sum_{u_k=1}^{U_k} h_{k,u_k}^{c} \cdot P_{TX,k,u_k}; \\ Y_{c,neigh} &= \sum_{k \in \bar{\mathcal{K}}_c} \sum_{u_k=1}^{U_k} p_{k,u_k}^{c} h_{k,u_k}^{\mathcal{C}(k)} \cdot P_{TX,k,u_k} \\ & \qquad c = 1, \ldots, C. \end{aligned} \tag{13}$$

We will continue to assume that the power received at the BS from all MSs belonging to Group-k are equal (which we denoted by P_k) and make use of the definition of a group by noting that $h_{k,u_k}^{\mathcal{C}(k)} = h_k^{\mathcal{C}(k)}$ and $p_{k,u_k}^{\mathcal{C}(k)} = p_k^{\mathcal{C}(k)} \quad \forall u_k \in G_k$, which leads to

$$Y_{c,own} = \sum_{k \in \mathcal{K}_c} U_k h_k^c \cdot P_{TX,k} = \sum_{k \in \mathcal{K}_c} U_k P_k; \tag{14}$$

$$Y_{c,neigh} = \sum_{k \in \bar{\mathcal{K}}_c} U_k p_k^c h_k^{\mathcal{C}(k)} \cdot P_{TX,k} = \sum_{k \in \bar{\mathcal{K}}_c} p_k^c U_k P_k \qquad c = 1, \ldots, C. \tag{15}$$

Then, similarly to (3), but now taking into account the interference from neighbor cells and the impact of the slowdown as described in Subsection II-C we have:

$$\begin{aligned} \frac{P_k}{P_N + Y_{\mathcal{C}(k),own} + Y_{\mathcal{C}(k),neigh} - P_k} &= \tilde{\Delta}_{a,k}; \\ \frac{P_k}{P_N + \sum_{\mathcal{K}_c} U_k P_k + \sum_{\bar{\mathcal{K}}_c} U_k P_k p_k^c - P_k} &= \tilde{\Delta}_{a,k}, \end{aligned} \tag{16}$$

for all $k = 1, \ldots, K; \quad c = \mathcal{C}(k)$. The equation system (16) consists of K equations, where the unknowns are the group-wise power values received at the BSs of the own cells (P_k). The "inputs" to this equation system are the system state ($\underline{U}$), the per-group power coupling factors to each neighbor cell p_k^c and the group-wise target ratios $\tilde{\Delta}_{a,k}$. We note that the coupling factors can be obtained from pilot measurement reports used also for handover decisions in operating CDMA networks (see for instance Section 9.3.1.2 of [10]).

From the solution vector $\underline{P}$ of (16), Equations (14) - (15) and the definition of the noise rise (the multi-cell version of (10)) the calculation of the noise rise value in each cell is straightforward:

$$T_c \triangleq \frac{P_{TX,c}}{P_N} = \frac{Y_c + P_N}{P_N} = \frac{Y_{c,own} + Y_{c,neigh} + P_N}{P_N} \tag{17}$$

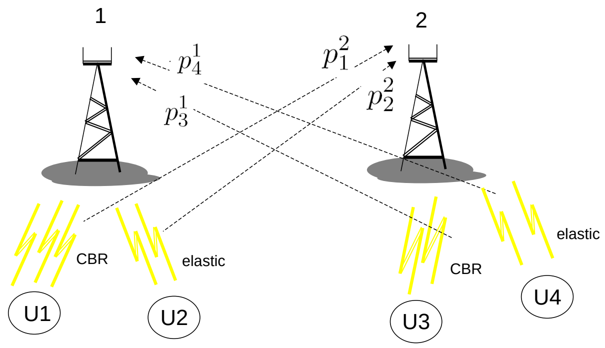
In the refined multi-cell model, equation (17) forms the basis for admission control. An arriving session is admitted into the system if T_c remains under the pre-defined threshold $\hat{T}$ for each cell. In practice, the issue becomes estimating the coupling factors for the newly arriving session - this is usually solved by (pilot signal) path loss measurements by the MSs and by base stations.

D. A Comment on Equation System (16)

Equation system (16) constitutes the core of the model, since it determines the group-wise received power value at the BS in each system state. The power vector is necessary to compute the noise rise. Therefore, from a computational perspective, an efficient solution of this equation system in each state is required in order to generate numerical results. To solve Equation system (16), one needs to assume that the $\tilde{\Delta}_{a,k}$ group-wise target ratios are known. The target ratios however depend on the slow down factors, which need to be determined in each system state assuming some fairness policy that decides on how much resource should be assigned to each group when some of the groups need to be slowed down. The formulation of such fairness policies is outside the scope of this paper, but we will illustrate the problem more in detail in the 2-cell example.

E. Example: The Two-Cell Case

In this subsection we consider a two-cell system as illustrated in Figure 1. Both cells support two groups, one with constant bit rate sessions and one with elastic



$$P_{RX,1} = U_1 P_1 + U_2 P_2 + p_3^1 U_3 P_3 + p_4^1 U_4 P_4 + P_N$$

$$P_{RX,2} = U_3 P_3 + U_4 P_4 + p_1^2 U_1 P_1 + p_2^2 U_2 P_2 + P_N$$

Figure 1. An example of a 2-cell CDMA system. Cell-1 supports two groups (here with U_1 constant bit rate and U_2 elastic in-progress sessions), Cell-2 also supports two groups (with U_3 constant bit rate and U_4 elastic in-progress sessions). These groups can comprise sessions that belong to the same service class, but they need to be distinguished because they belong to different cells and within a cell they can be at different geographical positions causing different inter-cell interference. The figure illustrates the (power) coupling factors p_1^2, p_2^2 and p_3^1, p_4^1 between the groups and their respective neighbor BSs.

sessions. In total, there are four groups. We describe the state space structure and present the generator matrix.

In order to determine the set of feasible states, we need to calculate the minimum noise rise value in an arbitrary system state. If this minimum noise rise value (when all sessions are slowed down to their minimum transmission rates) is under the noise rise threshold, then that state is part of the feasible state space. The minimum noise rise can be calculated from the state dependent power vector (obtained from Equation system (16)) by means of Equation (22). For the two-cell example (see Figure 1) we rewrite Equation system (16).

Considering the groups of Cell-1, we get:

$$\frac{P_k}{P_N + \sum_{i=1}^{2} U_i P_i + \sum_{i=3}^{4} U_i P_i p_i^1 - P_k} = \tilde{\Delta}_{a,k}, \quad k = 1, \ldots, 2; \qquad (18)$$

likewise the groups in Cell-2:

$$\frac{P_k}{P_N + \sum_{i=3}^{4} U_i P_i + \sum_{i=1}^{2} U_i P_i p_i^2 - P_k} = \tilde{\Delta}_{a,k}, \quad k = 3, \ldots, 4, \qquad (19)$$

where now $\tilde{\Delta}_{a,1} = \tilde{\Delta}_1$, $\tilde{\Delta}_{a,3} = \tilde{\Delta}_3$ are kept fixed (peak allocated sessions).

IV. System Behavior

A. Modeling Session Drop

When the system is in state $\underline{U}$, a class-k session leaves the system with intensity $\gamma_k(\underline{U}) \cdot \frac{\mu_k}{a_k(\underline{U})}$, where $\gamma_k(\underline{U})$ is the state dependent session drop factor. The session drop factor is such that for all k: $\gamma_k(\underline{U})\,|_{U_k=0} = 1$; and $\gamma_k(\underline{U})\,|_{U_k \neq 0} \geq 1$. Furthermore, we can assume that the drop probability for a given session does not depend on the instantaneous slowdown of that session. This is because whether a session gets out of coverage or whether it gets dropped by the radio network does not depend on the slowdown. The session drop probabilities, however, depend on the actual level of the noise rise, because higher noise rise level at the base station makes decoding of signals more difficult. We will thus assume that the session drop factor is a function of the macro state only and is the same for all classes : $\gamma_k(x) = f(x) = f(\Psi) \quad \forall k \in K$. That is, we assume that the session drop probability is determined by the load in the system and is equal for all service classes.

Note that the session drop model as described above is applicable in both the simplified and the refined multi-cell models. In the simple model, the system state $\underline{U}$ describes the state of the cell under study (and captures the impact of the neighbor cells by means of the ξ random variable. In the refined model, the impact of the neighbor cells is explicitly taken into account by allowing the number of in-progress sessions vary and by introducing the per-MS coupling factors. The session drop model is common to both these cases, since it relies only on the state dependent session drop factor $\gamma(\underline{U})$.

B. State Space Structure

For the simple multi-cell model, the maximum number of sessions from each class can be calculated as follows:

$$\hat{U}_k = \lfloor (\Delta_{min,k})^{-1} \rfloor, \quad k = 1, \ldots, K. \qquad (20)$$

Recall that in each $\underline{U}$ state of the system, the inequality $\sum_k U_k \cdot \Delta_{a_k} < \hat{\Psi}$ must hold. The states that satisfy this inequality are the *feasible states* and constitute the state space of the system (Θ). The feasible states, in which the acceptance of an additional class-k session would result in a state outside of the state space are the class-k *blocking states*. The set of the class-k blocking states is denoted by Θ_i. Due to the "Poisson Arrivals See Time Averages" (PASTA) property, the sum of the class-k blocking state probabilities gives the (overall) class-k blocking probability [11].

In each feasible state, it is the task of the bandwidth sharing policy to determine the $\Delta_{a_k}(\underline{U})$ class-wise target ratios for each class. $\Delta_{a_k}(\underline{U})$ reflect the fairness criterion that is implemented in the resource sharing policy mentioned above. From these, the class-wise slowdown factors and the instantaneous bit-rates of the individual sessions can be calculated as follows:

$$a_k(\underline{U}) = \frac{\Delta_k \cdot (1 - \Delta_{a_k}(\underline{U}))}{\Delta_{a_k}(\underline{U}) \cdot (1 - \Delta_k)}; \quad R_{a_k}(\underline{U}) = R_k / a_k(\underline{U}) \tag{21}$$

For ease of presentation, in the rest of the paper we will not indicate the dependence of a_k, Δ_{a_k} and R_{a_k} in state $\underline{U}$.

For the refined model, the states in which (16) has a solution ($\underline{P}$) whose elements are non-negative and that power setting causes less noise rise than the noise rise threshold in each cell (that is $T_c \leq \hat{T} \quad \forall c$) are the *feasible states*, the set of which we denote by Γ. From this definition and Equation system (16) it follows that the feasibility of a state depends on the coupling factors.

To determine the feasible state space, we need to calculate the minimum noise rise value in each state. Since the noise rise is monotonously increasing with respect to the rates of individual sessions, that is $\frac{\partial T_c}{\partial R_k}(R_1, R_2, \ldots, R_K) > 0$, the minimum noise rise value can be calculated from the state dependent power vector obtained from equation system (16) setting $\tilde{\Delta}_{a,k} = \tilde{\Delta}_{\hat{a},k}$ (that is assuming maximum slowdown):

$$T_{i,\hat{a}}(\underline{U}) = \frac{Y_{i,own} + Y_{i,neigh} + P_N}{P_N} = \\ = \frac{\sum_{\mathcal{K}_i} U_k P_k + \sum_{\bar{\mathcal{K}_i}} p_k^i U_k P_k + P_N}{P_n}. \tag{22}$$

The set of feasible states (Γ) consists of $\underline{U}$:s for which $T_{i,\hat{a}}(\underline{U}) < \hat{T}_i$. That is: $\Gamma = \{\underline{U} : T_{i,\hat{a}}(\underline{U}) < \hat{T}_i\}$.

C. The Markovian Property

We now make use of the assumptions that the arrival processes are Poisson and the nominal holding times are exponentially distributed. In both the simple and the refined models, the transitions between states are due to an arrival or a departure of a session of class-k. The arrival rates are given by the intensity of the Poisson arrival processes. Due to the memoryless property of the exponential distribution, the departure rates from each state depend on the nominal holding time of the in-progress sessions and on the slowdown factor in that state. Specifically, when the slowdown factor of a session of class-k is $a_k(\underline{U})$, its departure rate is $\gamma_k(\underline{U})\mu_k / a_k(\underline{U})$. Thus, the system under these assumptions is a continuous time Markov chain (CTMC) whose state is uniquely characterized by the state vector $\underline{U}$.

D. Determining the Generator Matrix

The generator matrix for the simple model has been derived in [6]. For the refined model, the derivation is similar and exemplified below for the 2-cell system. In the feasible states of the 2-cell system, the noise rise values in both cells must remain under the predefined threshold. In other words, the feasible states are given by the $\underline{U}$ vectors for which the noise rise values calculated from (22) remain under $\hat{T}$.

Based on the considerations of the preceding subsections we see that the generator matrix $\mathbf{Q}$ possesses a nice structure, because only transitions between "neighboring states" are allowed in the following sense. Let $q(U_1, U_2, U_3, U_4 \rightarrow U_1', U_2', U_3', U_4')$ denote the transition rate from state (U_1, U_2, U_3, U_4) to state (U_1', U_2', U_3', U_4'). Then the non-zero transition rates between the feasible states are (taking into account the impact of the slowdown factors):

$$\begin{aligned} q(U_1, U_2, U_3, U_4 \rightarrow U_1 + 1, U_2, U_3, U_4) &= \lambda_1 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2 + 1, U_3, U_4) &= \lambda_2 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2, U_3 + 1, U_4) &= \lambda_3 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2, U_3, U_4 + 1) &= \lambda_4 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1 - 1, U_2, U_3, U_4) &= U_1 \cdot \mu_1 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2 - 1, U_3, U_4) &= \\ &= U_2 \cdot \mu_2 / a_2(\underline{U}) \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2, U_3 - 1, U_4) &= U_3 \cdot \mu_3 \\ q(U_1, U_2, U_3, U_4 \rightarrow U_1, U_2, U_3, U_4 - 1) &= \\ &= U_4 \cdot \mu_4 / a_4(\underline{U}) \end{aligned} \tag{23}$$

Note that the derivation of the generator matrix relies on the fact that the system is Markovian. This is not trivial because one could intuitively argue that since the elastic flows bring with themselves a certain amount of workload (a file to transmit), the memoryless property does not hold, even if this workload is exponentially distributed. However, the Markovian property for such systems was independently of one another observed and formally proven by Altman *et al.* [12] and Nunez Queija *et al.* [15]. It is also used by Massoulie and Roberts in [16], where the death rates of the birth-death process are modulated by the actual instantaneous bandwidth of elastic traffic.

E. Determining the Blocking Probabilities and Session Drop Probabilities

From the steady state analysis, the blocking and dropping probabilities directly follow. The hard blocking probabilities can be easily calculated, because we assume that the sessions from each class arrive according to a Poisson process: $P_{hard,k} = \sum_{\underline{U} \in \Theta_k} \pi(\underline{U})$. In the simplified model, the total blocking probabilities include the soft blocking probabilities in each state and the hard blocking probabilities: $P_{total,k} = 1 - \sum_{\underline{U}\in\Theta} \pi(\underline{U})\sigma_k(\underline{U})$. Finally, the class-wise dropping probabilities can be calculated using the following observation. Since the dropping related departure rate from state $\underline{U}$ is $(\gamma_k(\underline{U}) - 1) \cdot \frac{U_i \mu_k}{a_k(\underline{U})}$, the long-term fraction of the dropped sessions must be proportional to $\frac{\gamma_k(\underline{U})-1}{\gamma_k(\underline{U})} \cdot \frac{U_k \mu_k}{a_k(\underline{U})}$. Weighing this quantity with the stationary probability distribution of the system and normalizing yields:

$$P_{drop,k} = \frac{\sum_{\underline{U}\in\Theta} \pi(\underline{U}) \cdot \frac{\gamma_k(\underline{U})-1}{\gamma_k(\underline{U})} \cdot \frac{U_k \mu_k}{a_k(\underline{U})}}{\sum_{\underline{U}\in\Theta} \pi(\underline{U}) \cdot \frac{U_k \mu_k}{a_k(\underline{U})}}. \quad (24)$$

In the next section we will show how this intuitively clear formula can be verified by defining a trapping state in this system.

V. Solution Based on the Tagged Customer Approach

The calculation of the (mean and the distribution of the) time to completion of successful sessions requires some additional effort. As we shall see, the method we follow here can also be used to verify the dropping probability calculations as suggested by Equation (24). In order to describe our approach, we use the simple multi-cell model and note that it is also applicable in the refined model when using the subset of the state variables that specify the state of the serving (own) cell.

A. Session Tagging and Modifying the State Space

In order to calculate the moments and the distribution of the holding time of successful (not dropped) sessions we modify the state space by introducing a trapping (absorbing) state and make the following considerations.

We will continue to think of an elastic session as one that brings with itself an exponentially distributed amount of work and, if admitted into the system, stays in the system until this amount of work is completed or the session gets dropped. The method we follow here is based on (1) *tagging* an elastic session arriving to the system, which, at the time of arrival is in one of the feasible states; and (2) carefully examining the possible transitions from the moment this tagged call enters the system until it acquires the required service or gets dropped and therefore leaves the system. Finally, un-conditioning on all possible entrance state probabilities, the distribution of the best effort service time can be determined.

For the purpose of illustration, we again concentrate on the part of the state space in which $U_1 = 8$ and tag a class-3 session. Figure 2 shows the state transition diagram from this tagged session's point of view an infinitesimal amount of time after this tagged session entered the system. Since we assume that at least the tagged session is now in the system, we exclude states where $U_3 = 0$. Figure 2 also shows the entrance probabilities for each state, with which the tagged session finds the system in *that* state. Thus, in Figure 2, the tagged arriving session will find the system in state (U_2, U_3) with probability $P(U_2, U_3)$, and will bring the system into state $(U_2, U_3{+}1)$ unless (U_2, U_3) is a Class-3 hard blocking state. For non hard blocking states the entrance probabilities have to be "thinned" with the passage factor (i.e. $\gamma(U_1, U_2, U_3)$). In order for the entrance probabilities to sum up to 1, they need to be re-normalized since we have excluded entrances in the hard blocking states.

In this modified state space, we also define a *trapping (absorbing) state*. Depending on how this trapping state is interpreted and how the transition rates into that state is defined, we can calculate the moments and the distribution of the holding time of successful sessions and the time until dropping of dropped sessions as well.

We first discuss the case of successful sessions. In this case, the trapping state corresponds to the state which the tagged session enters when the workload is completed ("the file has been transferred successfully"). The transition rates from each state are given by $\mu_3/a(\underline{U})$. The time until absorption corresponds to the time the tagged session spends in the system provided that it is not dropped. Indexing the modified state space in a similar manner as the original state space, the new generator matrix $\mathbf{\tilde{Q}_S}$ will have the following structure:

$$\mathbf{\tilde{Q}_S} = \begin{bmatrix} B_S & b_S \\ 0 & 0 \end{bmatrix} \quad (25)$$

where the B_S matrix represents the transitions between the non-trapping states, the b_S vector contains the transitions *to* the trapping state, the 0 vector indicates that no transitions are allowed *from* the trapping state. When the trapping state represents the state that the tagged session enters when it is dropped, the transition rates to the trapping state are given by $\frac{\gamma_3(\underline{U})-1}{a_3(\underline{U})}\mu_3$ and the generator matrix takes the following form:

$$\mathbf{\tilde{Q}_D} = \begin{bmatrix} B_D & b_D \\ 0 & 0 \end{bmatrix} \quad (26)$$

where the B_D matrix represents the transitions between the non-trapping states, and the b_D vector contains the transitions *to* the trapping state. Once the structure of the expanded state space and the associated transition rates together with the (thinned) initial probability vector, $P_R(0)$, are determined, we can determine the r^{th} moment of T_S:

$$E[T_S^r] = r! \cdot P_R^t(0) \cdot (-B_S)^{-r} \cdot e \quad (27)$$

We note that the procedure to calculate the moments of T_D is the same as that for T_S, except that we now have to make use of the B_D matrix instead of B_S. The distributions of T_S and T_D are given by:

$$Pr\{T_S < x\} = 1 - P_R^t(0) \cdot e^{xB_S} \cdot e;$$
$$Pr\{T_D < x\} = 1 - P_R^t(0) \cdot e^{xB_D} \cdot e. \quad (28)$$

B. Verifying Equation (24): An Alternative Way to Calculate the Dropping Probabilities

The trapping state approach can also be used to determine the dropping probabilities, which can be used to

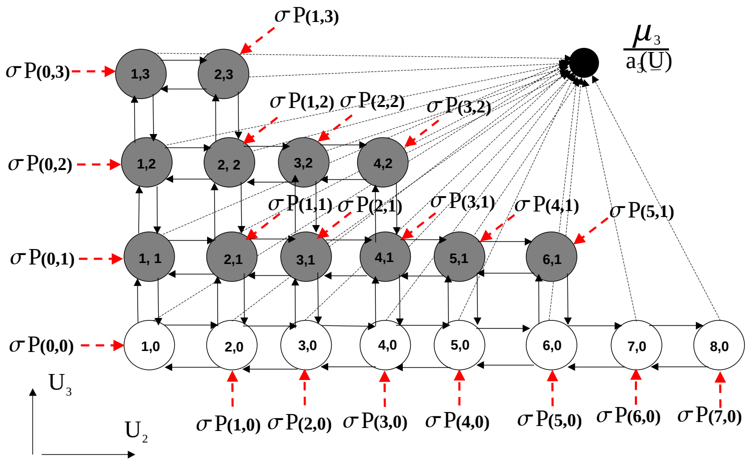


Figure 2. Modified state space with a trapping state that represents successful session termination. The transition rates to this trapping state correspond to the transition rates with which the tagged session enters the trapping state. The initial probability vector can be determined from the steady state by normalization and taking into account the 'thinning' affect of the passage factors.

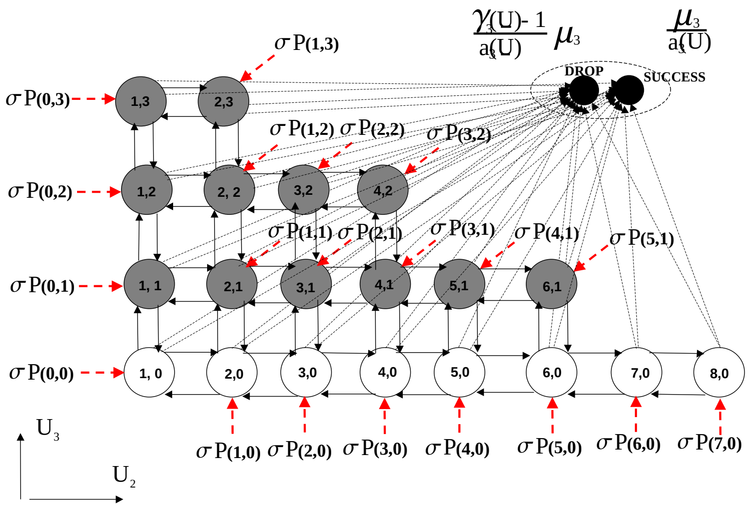


Figure 3. Modified state space with two trapping states representing successfully terminated and dropped sessions respectively. Seen from the transient states, the total transition rates with which the tagged session enters either of these states is the sum of the two transition rates. This modified state space can be used to determine the probabilities of success and drop.

verify results obtained from Equation (24). In order to do this, we consider the modified state space with two trapping states illustrated in Figure 3. From each state, the tagged class-i session can enter any of the two trapping states corresponding to the case when the tagged session successfully terminates or gets dropped. The generator matrix of this state space is given by:

$$\tilde{\mathbf{Q}}_{\mathbf{i}} = \begin{bmatrix} B_i & b_{S,i} & b_{D,i} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (29)$$

where $\underline{b}_{drop,i}$ is the column vector containing the transition rates to the trapping state representing the session drops. The B_i matrix has to be determined considering the total transition rates to the two trapping states.

The class-wise dropping probabilities can be calculated using Equation (30):

$$P_{drop,i} = P_R^t(0) \cdot (-B_i)^{-1} \cdot \underline{b}_{D,i}. \qquad (30)$$

VI. Numerical Results

A. Input Parameters

The input parameters for the two cases that we study are summarized by Table I. In Case I, Class-1 is a rigid class, whereas in Case II, Class-1 is elastic with a maximum slow down factor $\hat{a}_1 = 3$. In both cases we change the maximum slow down factor of Class-2 $\hat{a}_2 = 1 \dots 4$. ($\hat{a}_2$ is changed along the x axis in each Figure.) The offered traffic is set to 2.72 Erlang per each class and the required Δ_i value for sessions of each class is ≈ 0.15. The function $\gamma_i(\underline{U}) = f(\underline{U})$ is set such that it does not depend on the slow down factors, according to the discussion at the end of Section IV-A. Specifically, in this paper we choose the following dropping factor: $f(\underline{U}) = 1 + \nu \ln(1 + U_1 \cdot \Delta_1 + U_2 \cdot \Delta_2)$, expressing that the dropping factor is a function of the total load in the system (see also Table I). For the refined multi-cell model, we study a 2-cell model and let the per-MS coupling factor vary between 0.09 and 0.39.

TABLE I.
MODEL (INPUT) PARAMETERS

I	2
$\hat{R}_i$	128 [kbps]
λ_i	87.2613 [1/s]
μ_i	32.03 [1/s]
$\hat{a}_1$	1 (Case I); 3 (Case II)
$\hat{a}_2$	$1 \ldots 4$ (along the x axis)
φ	0.25
E_i/N_0	7 [dB]
Dropping factor	$f(\underline{U}) = 1 + \nu \ln(1 + U_1 \cdot \Delta_1 + U_2 \cdot \Delta_2)$, $\nu = 1$; [17]

B. Numerical Results

1) Blocking Probabilities: Figures 4-5 and Figures 6-7 show the impact of state dependent blocking on the total blocking probabilities. State dependent blocking implies that the admission control takes into account the instantaneous value of the noise rise at the base station rather than just the state of the own cell. This increases the class-wise total blocking probabilities from around 7% and 2% to 10% and 6% in Case I when $\hat{a}_2 = 4$. We also note that when both classes are rigid (Case I, $\hat{a}_2 = 1$), the total blocking values are high, but these high values are brought down to reasonably low blocking probability values when either one and especially when both classes tolerate slowing down of the instantaneous transmission rates (Case II, $\hat{a}_2 = 4$).

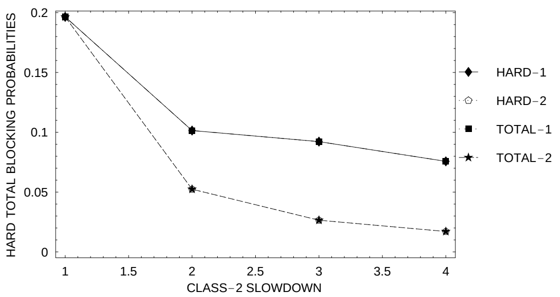


Figure 4. Case I, no soft blocking, blocking probabilities (total and hard blocking probabilities being equal)

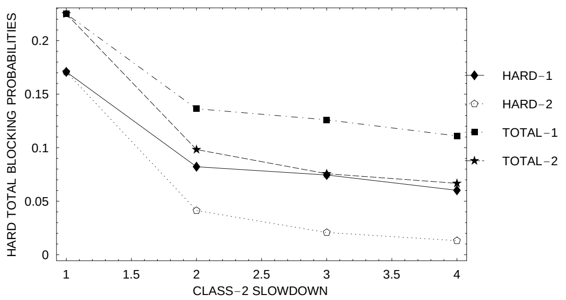


Figure 5. Case I, soft blocking, blocking probabilities

2) Dropping Probabilities: Figures 8-9 and Figures 10-11 show the impact of soft blocking on the session drop probabilities. First, we note that the session drop probabilities slightly (less than 2%) increase as traffic becomes more elastic. The reason is that the system utilization increases when traffic is elastic and the system operates in "higher states" with a higher probability than when traffic is rigid.

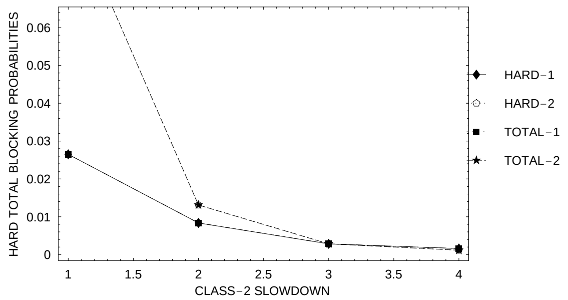


Figure 6. Case II, no soft blocking, blocking probabilities (total and hard blocking probabilities being equal)

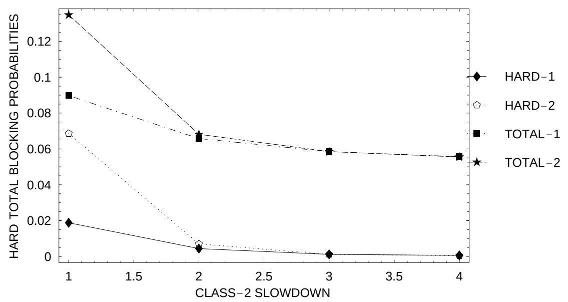


Figure 7. Case II, soft blocking, blocking probabilities

We also see that state dependent blocking decreases the session drop probabilities in both cases (for example from around 7% to 5% in Case I when $\hat{a}_2 = 4$). This is because soft blocking entails that in average there are fewer sessions in the system that decreases session drops.

3) Mean Holding Time of the Successful (Not Dropped) Sessions: Figures 12-13 show the mean holding times of successful sessions (normalized to the nominal expected holding time, that is when the slow down factors are 1). In Case I, Class-1 sessions are rigid and there is no increase in their mean holding times. In this case, Class-2 sessions benefit from soft blocking (keeping in mind that we are now only taking into account the sessions that are successful). Their holding time is somewhat lower in the case of soft blocking.

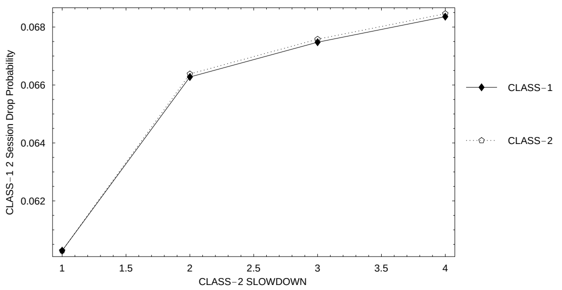


Figure 8. Case I, no soft blocking, session drop probability

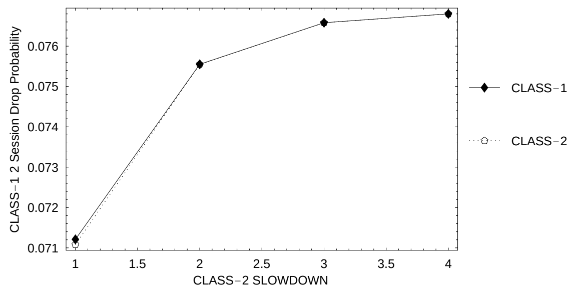


Figure 10. Case II, no soft blocking, session drop probabilities

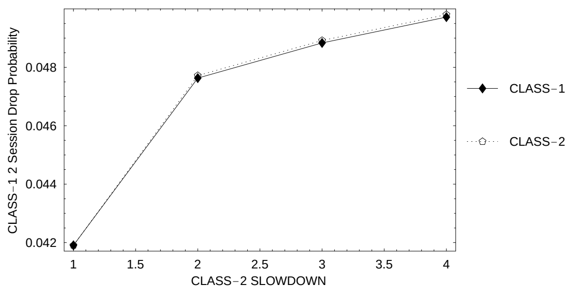


Figure 9. Case I, soft blocking, session drop probability

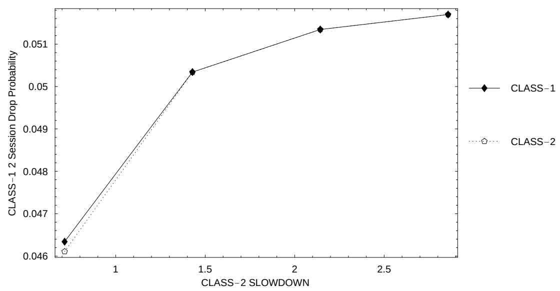


Figure 11. Case II, soft blocking, session drop probabilities

4) The Impact of the per-MS Coupling Factor: Figures 14-21 illustrate the impact of an increasing per-MS coupling factor on the blocking probabilities, the session drop probabilities and the successful sessions mean holding time. In these figures, the per-MS coupling factor increases from 0.095 to 0.38 along the x axis. The load in the system is quite high, in fact as the per-MS coupling factor increases and when $\hat{a}_1 = \hat{a}_2 = 1$ (both service classes are peak allocated), the blocking probabilities increase from around 7% to 15% (not shown in the figures). When $\hat{a}_2 = 2$, the blocking probabilities become significantly lower (see Figure 14 and less sensitive to the increase in of the coupling factor. (This effect is even more visible when $\hat{a}_2 = 4$ in Figure 19.) We also note that the blocking probability of the peak allocated class is significantly higher than that of the elastic class, especially at high coupling factor values. The admission control algorithm in this refined model is such that the session drop probabilities basically remain at the same level (or even decrease a little bit) as the coupling factor increases (the session drop probabilities remain around 4%). The mean holding time of the elastic class (in this example Class-2) increases somewhat, especially (as expected) in the case when $\hat{a}_2 = 4$ (see Figure 21). This is because the throughput of the system degrades at increasing coupling factor and a highly elastic traffic class becomes sensitive for such throughput degradation. (The peak allocated class mean time remains of course unit, irrespective of the coupling factor.)

VII. Conclusions

In this paper we have proposed a model to study and analyze the trade-off between the blocking and dropping probabilities in CDMA systems that support elastic services. The model of this present paper captures the impact of state dependent blocking, which is a consequence of the CDMA admission control procedure that takes into account the actual noise rise value at the base station (including the interference coming from surrounding cells) rather than just the state of the serving cell. Session drops happen with a probability that increases with the overall system load.

As traffic becomes more elastic, the session drop probability increases, but this increase can be compensated for by a suitable admission control algorithm. Such state dependent admission control algorithms increase the blocking probabilities somewhat, but this increase can be mitigated if sessions tolerate some slow down of their sending rates. Thus, the design of the CDMA admission control algorithm should take into account the actual traffic mix in the system and the per-class blocking and session drop probability targets.

An important consequence of the presence of elastic traffic is that the blocking probabilities decrease as the maximum slow down factors increase. This is a nice practical consequence of one of the key findings in [3], namely that the Erlang capacity increases. Another consequence of elasticity is that the dropping probabilities increase somewhat, but this increase is not significant (the exact value would depend on the model assumptions, for instance the value of ν).

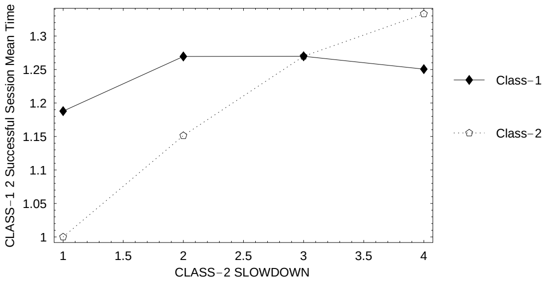


Figure 12. Case II, no soft blocking, successful sessions' mean holding time

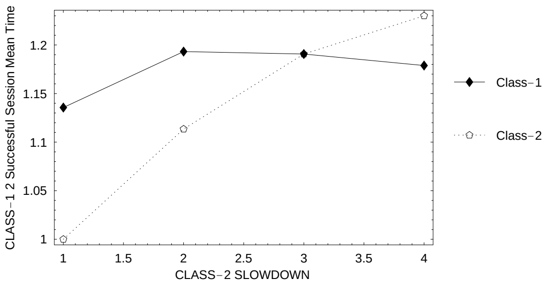


Figure 13. Case II, soft blocking, successful sessions' mean holding time

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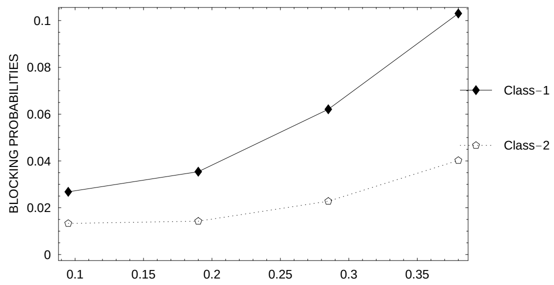

Figure 14. Blocking probabilities vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 2$

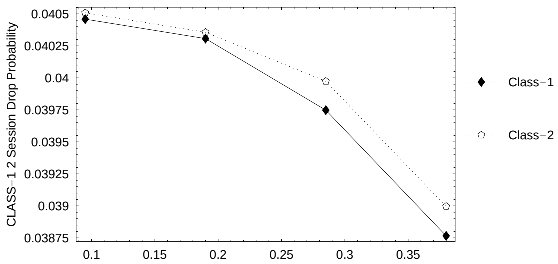

Figure 15. Session Drop probabilities vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 2$

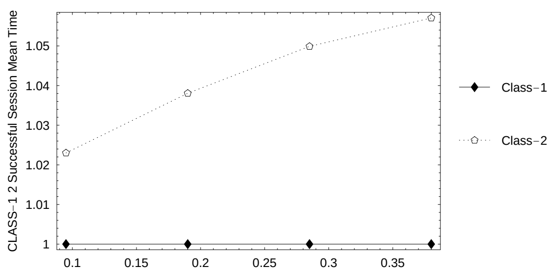

Figure 16. Successful Session Mean Time vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 2$

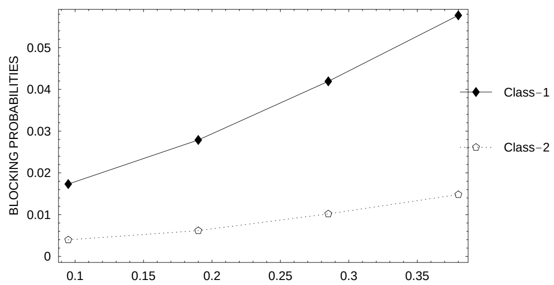

Figure 17. Blocking probabilities vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 3$

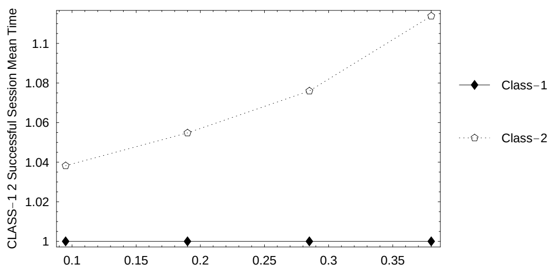

Figure 18. Successful Session Mean Time vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 3$

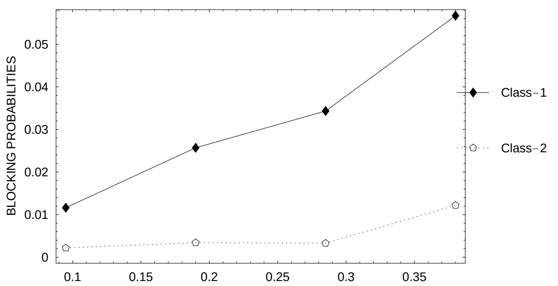

Figure 19. Blocking probabilities vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 4$

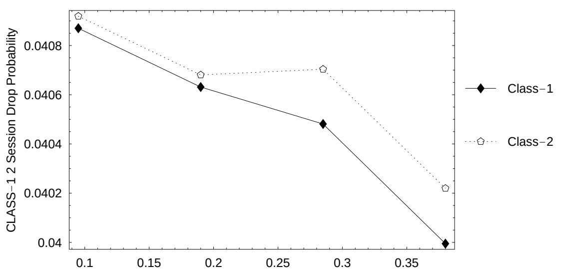

Figure 20. Session Drop probabilities vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 4$

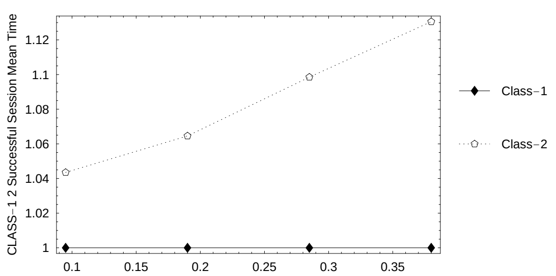

Figure 21. Successful Session Mean Time vs Coupling factor, $\hat{a}_1 = 1, \hat{a}_2 = 4$

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