A MAP Fitting Method to Approximate Real Traffic Behavior*

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Abstract

This paper provides a heuristic fitting method to capture some important features of real traffic sources by a MAP. The novelty of the proposed approach lies in the separate treating of short and long range behavior of the considered traffic sources. The proposed MAP is the superposition of two elementary processes. A Phase type renewal process, whose interarrival time exhibits heavy-tail behavior over some time scales, is used to capture the long range dependent behavior, i.e., the empirical Hurst parameter. While an IPP is applied to approximate the short range behavior. Different analysis techniques are used to evaluate the goodness of the proposed fitting method.

Keywords: Traffic source models, Self-similarity, Markovian Arrival processes (MAP), short and long range behavior.

1 Introduction

Traffic measurement on real high speed networks carrying the data packets of various applications shows high variability and burstiness of the traffic process over several time scales (references to many measurment studies may be found in [14]). It is commonly assumed that Markovian models are not appropriate to capture this "burst in burst" behavior and other models are proposed in the literature, e.g., fractals, multifractals [2]. These models are analytically hardly tractable and often computationally expensive. The analytical tractability of Markovian models initiated a research effort to approximate real traffic behavior with Markovian models.

A first step in this direction was the approximation of heavy-tail distributions by Phase type ones. It was shown that general PH fitting methods performs poorly in tail fitting [5], while specific heuristic methods can provide a tight tail fitting over several orders of magnitude even for heavy-tail distributions [3]. It was also recognized [9, 12] that Phase type renewal processes (i.e., renewal processes with PH distributed interarrival time) with heavy-tail behavior over some orders of magnitude exhibits some features shown by real traffic measurements. The commonly applied tests used for evaluating the Hurst parameter (variance-time plot, R/S plot) show long range dependence of these PH renewal processes. In this paper a fitting method is proposed to approximate any traffic process with a Markovian arrival process (MAP) that intends to consider not only the long range traffic behavior but also some of the short range ones.

A general approach of MAP fitting consists of defining an appropriate distance measure over the set of arrival processes, and minimizing the defined measure with respect to the finite number of MAP parameters. Unfortunately, it is still an unsolved problem what could be an appropriate distance measure that reflexes those features of a traffic process that play

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important role in telecommunication engineering. Due to the lack of commonly accepted appropriate distance measure and also to contribute to the research of characterizing the practically important features of traffic processes we followed a different approach. We choose a given set of traffic parameters and compose a MAP that exhibits the same (or close) traffic parameters. The set of traffic parameters is such that both long and short range behavior is considered. The long range behavior is fitted through the Hurst parameter, while the short range behavior is approximated by fitting the index of dispersion (IDC) parameter at some time points.

The rest of the paper is organized as follows. Section 2 provides a short summary on the background of traffic processes. Section 3 introduces the MAP fitting method, while Section 4 investigates the goodness of fitting. The paper is concluded in Section 5.

2 Definitions

The analysis of traffic processes are based on one of the following two approaches. When detailed information is available on the exact arrival instances (Y_i) with inter-arrival series $Z_i = Y_i - Y_{i-1}$, then $\mathcal{Z} = \{Z_i, i \geq 1\}$ is used to characterize the arrival process. It is assumed that $Y_0 = 0$ and $Y_n = \sum_{i=1}^{n} Z_i$. It is also possible to consider the number of arrivals in the $(0,t)$ interval, $N_t = \max_i(Y_i < t)$ and to use the continuous time counting process $\mathcal{N} = \{N_t, t \geq 0\}$.

There are cases when the exact arrival instances are not known, only the number of arrivals in consecutive time intervals. Considering intervals of length Δ, the number of arrivals in the ith interval, X_i, is $X_i = \#\{Y_k \in (i\Delta, (i+1)\Delta)\}$. $\mathcal{X} = \{X_i, i = 0, 1, \ldots\}$ is a discrete-time stochastic process. Its aggregated process is defined as follows:

$$\mathcal{X}^{(m)} = \{X_i^{(m)}\} = \{\frac{X_1 + \ldots + X_m}{m}, \ldots, \frac{X_{mk+1} + \ldots + X_{(m+1)k}}{m}, \ldots\}$$

The autocorrelation function of $\mathcal{X}^{(m)}$ is:

$$r^{(m)}(k) = \frac{E\{(X_n^{(m)} - E(X^{(m)})) \cdot (X_{n+k}^{(m)} - E(X^{(m)}))\}}{E\{(X_n^{(m)} - E(X^{(m)}))^2\}}$$

Then $\mathcal{X}$ is

a) exactly self-similar if $\mathcal{X} \stackrel{d}{=} m^{1-H}\mathcal{X}^{(m)}$, i.e., if $\mathcal{X}$ and $\mathcal{X}^{(m)}$ are identical within a scale factor in finite dimensional distribution sense.

b) exactly second-order self-similar if $r^{(m)}(k) = r(k), \quad \forall m\ , \ k \geq 0$

c) asymptotically second-order self-similar if $r^{(m)}(k) \to r(k), \quad (k, m \to \infty)$

where H is the Hurst parameter, also referred to as the self-similarity parameter.

The process $\mathcal{X}$ exhibits long-range dependence (LRD) of index β if its autocorrelation function can be realized as:

$$r(k) \sim A(k)k^{-\beta}, \qquad k \to \infty$$

where A(k) is a slowly varying function at infinity, i.e., $A(tk)/A(k) \to 1, \ \ t > 0, k \to \infty$.

Having only a finite number of samples from real traffic it is impossible to check the presence of self similarity or LRD. In practice *pseudo* self-similarity may be checked by statistical tests. An overview of tests may be found in [1].

2.1 Variance-time plot

One of the tests for *pseudo* self-similarity is the variance-time plot. It is based on the fact that for self-similar time series $\{X_1, X_2, \ldots\}$

$$\mathrm{Var}(X^{(m)}) \sim m^{-\beta} \quad , \quad \text{as} \;\; m \to \infty, \quad 0 < \beta < 1$$

The variance-time plot depicts $Log(\mathrm{Var}(X^{(m)}))$ versus $Log(m)$. For *pseudo* self-similar time series, the slope of the variance-time plot $-\beta$ is greater than -1. The Hurst parameter can be calculated as $H = 1 - (\beta/2)$. A traffic process is said to be *pseudo* self similar when the empirical Hurst parameter is between 0.5 and 1.

2.2 R/S plot

The R/S method is one of the oldest tests for self-similarity, it is discussed in detail in [6]. For interarrival time series, $\mathcal{Z} = \{Z_i, i \geq 1\}$, with partial sum $Y_n = \sum_{i=1}^{n} Z_i$, and sample variance

$$S^2(n) = \frac{1}{n} \sum_{i=1}^{n} Z_i^{\,2} - \frac{1}{n^2} \cdot Y_n^2,$$

the R/S statistic, or the rescaled adjusted range, is given by:

$$R/S(n) = \frac{1}{S(n)} \left[\max_{0 \le k \le n} \left(Y(k) - \frac{k}{n} Y(n) \right) - \min_{0 \le k \le n} \left(Y(k) - \frac{k}{n} Y(n) \right) \right].$$

$R/S(n)$ is the scaled (by $\frac{1}{S(n)}$) difference between the fastest and the slowest arrival period considering n arrivals. For stationary LRD processes $R/S(n) \approx (n/2)^H$. To determine the Hurst parameter based on the R/S statistic the data set is divided into blocks, $\log[R/S(n)]$ is plotted versus $\log n$ and a straight line is fitted on the points. The slope of the fitted line is the estimated Hurst parameter.

It is important to note that the introduced statistical tests of self- similarity, based on a finite number of samples, provides an approximate value of H only for the considered range of scales ($\log n$). Nothing can be said about the higher scale and the asymptotic behavior based on these tests.

3 A MAP fitting method

In this section a procedure is given to construct a MAP such a way that some parameters of the traffic generated by the model match predefined values. The following parameters are set:

- The fundamental arrival rate $E(N_1)$ describes the expected number of arrivals in a time unit.
- In order to describe the burstiness of the arrival stream the index of dispersion for counts $I(t) = \mathrm{Var}(N_t)/E(N_t)$ is set for two different values of time: $I(t_1)$ and $I(t_2)$. The choice of these two time points significantly affects the goodness of fitting. This issue will be discussed in Section 4.
- The degree of *pseudo* self-similarity is defined by the Hurst parameter H. The Hurst parameter is realized in terms of the variance-time behavior of the resulting traffic, i.e., the straight line fitted by regression to the variance-time curve in a predefined interval (L_1, L_2) has slope $2(H-1)$.

The MAP resulting from our procedure is the superposition of a PH arrival process and an Interrupted Poisson Process (IPP). In the following we shows how to construct a PH arrival process with *pseudo* self-similar behavior, describe the superposition of the PH arrival process with an IPP, and finally provide the proposed fitting algorithm itself.

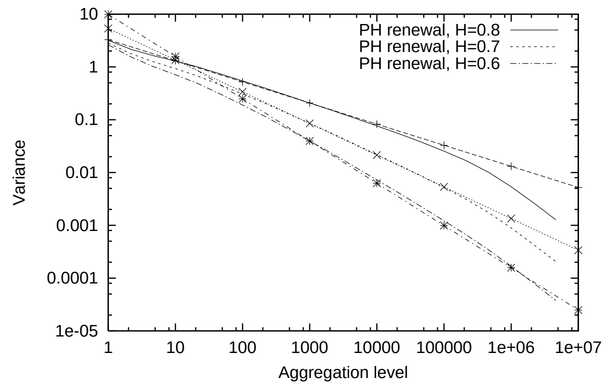

Figure 1: Variance-time plot of *pseudo* self similar arrival processes with i.i.d. PH interarrival

3.1 *Pseudo* self-similar PH arrival process

Let us consider an arrival process whose interarrival times are independent random variables with heavy-tail probability density function (pdf) of Pareto type

$$f(x) = \frac{c \cdot a^c}{(x+a)^{c+1}}, \; x \geq 0. \tag{1}$$

The process X_n $(n > 0)$ representing the number of arrivals in the nth timeslot is asymptotically second-order self-similar with Hurst parameter ([13]):

$$H = \frac{3-c}{2}. \tag{2}$$

Feldmann and Whitt propose a heuristic fitting algorithm in [3] to approximate heavy-tail distributions by Phase Type (PH) distributions. Using their method one may build an arrival process whose interarrival times are independent, identically distributed PH random variables with pdf approximating (1). In order to show that this arrival process exhibits *pseudo* second-order self-similarity, let us recall some properties of MAPs from [7]. Having a PH distribution with initial probability vector $\mathbf{b}$ and generator $\mathbf{T}$, the corresponding PH arrival process may be described with MAP notation as $\mathbf{C}_{PH} = \mathbf{T}$ and $\mathbf{D}_{PH} = \mathbf{T}^0\mathbf{b}$ with $\mathbf{T}^0 = -\mathbf{T}\mathbf{e}$, where $\mathbf{e}$ denotes the column vector of 1s. The variance of the number of arrivals in the interval $(0,t)$ is given by

$$Var(N_t) = \frac{\nu_2 - \nu_1^2}{\nu_1^3} t + 2\mathbf{b}\left[\mathbf{I} - e^{(\mathbf{C}_{PH}+\mathbf{D}_{PH})t}\right]\left[\nu_1^{-2}\mathbf{C}_{PH}^{-2}\mathbf{e} + \frac{1}{2}\nu_1^{-3}\nu_2\mathbf{C}_{PH}^{-1}\mathbf{e}\right], \tag{3}$$

where ν_i denotes the ith moment of the PH interarrival time distribution and is given by

$$\nu_i = (-1)^i \; i! \; \mathbf{b} \; \mathbf{T}^{-i}\mathbf{e}. \tag{4}$$

The variance of the aggregated arrival process $\mathcal{X}^{(m)}$ may be expressed as

$$Var(X^{(m)}) = \frac{Var(N_{m\Delta})}{(m\Delta)^2}, \tag{5}$$

where Δ is the length of a timeslot.

To check pseudo self-similarity of PH renewal processes Figure 1 plots $Var(X^{(m)})$ of PH arrival processes whose interarrival time is a 6 phase PH approximation of the pdf given in

(1) for different values of c. As it can be observed $Var(X^{(m)})$ is close through several orders of magnitude to the straight line corresponding to the self-similar case with slope $2(H-1)$. The aggregation level where $Var(X^{(m)})$ drops compared to the straight line may be increased by changing the parameters of the PH fitting algorithm.

3.2 Superposition of the PH arrival process with an IPP

The superposition of a PH arrival process with an Interrupted Poisson Process (IPP) results in a MAP. In order to have the desired parameters for the MAP resulting from the superposition the following equations have to hold for the IPP:

$$E_{IPP}(N_1) = E(N_1) - E_{PH}(N_1), \tag{6}$$

$$Var_{IPP}(N_{t_1}) = I(t_1)E(N_{t_1}) - Var_{PH}(N_{t_1}), \tag{7}$$

$$Var_{IPP}(N_{t_2}) = I(t_2)E(N_{t_2}) - Var_{PH}(N_{t_2}). \tag{8}$$

The latter two equations are the consequence of the fact that the index of dispersion $I(t)$ for the superposed model may be written as

$$I(t) = \frac{Var_{PH}(N_t) + Var_{IPP}(N_t)}{(E_{PH}(N_1) + E_{IPP}(N_1))t}. \tag{9}$$

The IPP may be described by MAP notation as

$$\mathbf{C}_{IPP} = \begin{bmatrix} -r_1 - \lambda & r_1 \\ r_2 & -r_2 \end{bmatrix}, \; \mathbf{D}_{IPP} = \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix}. \tag{10}$$

The mean number of arrivals of the IPP is given as

$$E(N_t) = \frac{\lambda r_2}{r_1 + r_2} t, \tag{11}$$

while its variance may be expressed as

$$Var(N_t) = \left(\frac{\lambda r_2}{r_1 + r_2} + \frac{2\lambda^2 r_1 r_2}{(r_1 + r_2)^3} \right) t - \frac{2\lambda^2 r_1 r_2}{(r_1 + r_2)^4} \left(1 - e^{-(r_1 + r_2)t} \right). \tag{12}$$

Let us denote $r_1 + r_2$ by S and r_1/r_2 by Q. Substituting (11) and (12) into the equations describing the requirements on the IPP ((6), (7) and (8)) and manipulating the resulting set of equations one may arrive to the following implicit expression for S and Q

$$S = \frac{K(1 - e^{-St_2}) - (1 - e^{-St_1})}{Kt_2 - t_1}, \tag{13}$$

$$Q = \frac{S^2(Var(N_{t_1}) - t_1 E(N_1))}{2E(N_1)^2(e^{-St_1} + St_2 - 1)}, \tag{14}$$

where

$$K = \frac{Var(N_{t_1}) - Var_{PH}(N_{t_1}) - t_1(E(N_1) - E_{PH}(N_1))}{Var(N_{t_2}) - Var_{PH}(N_{t_2}) - t_2(E(N_1) - E_{PH}(N_1))}. \tag{15}$$

Applying fix point iteration S may be found by (13) and than Q is given by (14). Having S and Q the transition intensities are simple given by

$$r_1 = S - \frac{S}{Q+1}, \; r_2 = \frac{S}{Q+1}, \tag{16}$$

while the arrival intensity can be found as

$$\lambda = (1 + Q)E(N_1). \tag{17}$$

The MAP resulted as the superposition of a PH arrival process (with descriptors $\mathbf{C}_{PH}$, $\mathbf{D}_{PH}$) and an IPP ($\mathbf{C}_{IPP}$, $\mathbf{D}_{IPP}$) has descriptors

$$\mathbf{C} = \mathbf{C}_{PH} \oplus \mathbf{C}_{IPP}, \ \mathbf{D} = \mathbf{D}_{PH} \oplus \mathbf{D}_{IPP}, \tag{18}$$

where $\oplus$ stands for the Kronecker sum.

3.3 The applied fitting algorithm

Before putting down the fitting algorithm we introduce how the alteration in the parameters of the PH arrival process affects the resulting MAP:

- The Pareto type pdf (1) that is approximated by a PH distribution has mean $a/(c-1)$. Even if the fitting method of Feldmann and Whitt [3] does not preserve the mean, increasing a increases the mean of the approximating PH distribution, and so that decreases $E_{PH}(N_1)$. According to our experiences increasing a decreases $Var_{PH}(N_t)$ as well. The PH fitting may result in such $E_{PH}(N_1)$, $Var_{PH}(N_{t_1})$ and $Var_{PH}(N_{t_2})$ that the requirements in (6), (7) and (8) are not feasible, this situation may be resolved by changing a. Experiments suggest $E(N_1)(c-1)*2$ as an initial value for a, which means that approximately every 2nd arrival arises from the PH arrival process.

- Parameter c of the Pareto type pdf (1) is used to set the Hurst parameter of the arrival process. An appropriate initial value for c is given by (2) as $c = 3 - 2H$. Superposing the PH arrival process with the IPP may change the degree of self-similarity of the superposed model. The predefined Hurst parameter may be reached by adjusting appropriately c.

- Using the fitting method of Feldmann and Whitt one has to define the limit of tail fitting L_{fit}, i.e., the time point until which the pdf of the PH distribution follows the Pareto pdf. The higher the limit for the tail fitting the longer the PH arrival process exhibits self-similar nature in terms of the variance-time plot. This fact is depicted in Figure 3 with Hurst parameter 0.9; all the three PH arrival processes have 6 phases. As a rule $L_{fit} = L_2/10$ seems to be a good choice.

- The effect of the choice for the number of phases of the approximating PH distribution (D) is illustrated in Figure 2. Having a large value for the limit of tail fitting, low number of phases may lead to an irregular behavior in terms of the variance-time plot.

Based on the above considerations the applied MAP fitting algorithm is the following:

1. Set initial values as
   $a = E(N_1)(c-1)*2$, $c = 3 - 2H$, $L_{fit} = L_2/10$,
   if ($L_2 \leq 10^4$) $D = 4$,
   if ($10^4 < L_2 \leq 10^6$) $D = 6$,
   if ($10^6 < L_2$) $D = 8$.

2. Perform PH fitting.

3. If (6), (7) and (8) are not feasible change a accordingly and go back to 2.

4. Compute the IPP parameters based on (13), (14), (16) and (17).

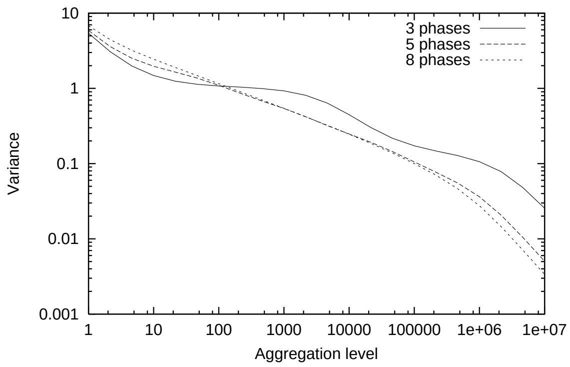

Figure 2: PH arrival processes of different order

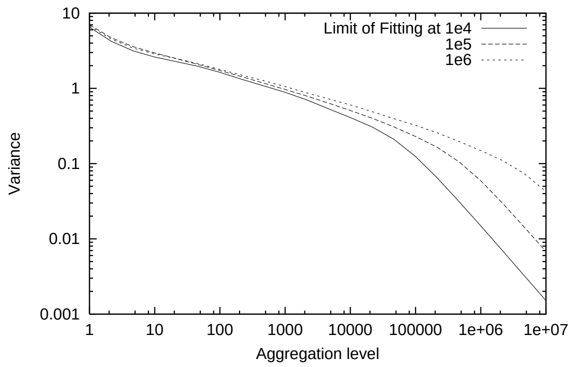

Figure 3: PH arrival processes with different limits of tail fitting

5. Check the Hurst parameter by applying regression in the interval (L_1, L_2) to the variance-time plot of the superposed MAP. Decrease (increase) c if the actual Hurst parameter is lower (greater) than the desired value.

Since not all combinations of the input parameters may be realized the implementation of the algorithm has to be complemented by some checks to recognize these situations.

4 Application of the fitting procedure

The fitting method described in the previous section was applied to approximate two real measured traffic traces. The traces are taken from the WEB site collecting traffic traces [15] and they are called *BC* and *dec-pkt*. We studied the first data set of these collections. These traces are analyzed in [4] and [11].

Variance-time plots of the traffic generated by the MAPs resulted from fitting for the first trace are depicted in Figure 4. The length of the interval Δ that is used to generate the series $\mathcal{X} = \{X_i, i = 0, 1, ...\}$ equals the expected interarrival time. The curve signed by (x_1, x_2) belongs to the fitting when the first (second) time point of fitting the IDC value, t_1 (t_2), is x_1 (x_2) times the expected interarrival time. The Hurst parameter of this trace (approximated by the variance-time plot) consisting of one million arrivals is 0.8367. The interval (L_1, L_2) is $(10,\ 5 \cdot 10^5)$ for the first three fitting, while it is $(500,\ 5 \cdot 10^5)$ for the last one. For the last fitting the interval had to be changed because the time point at which the IDC is set is so high that the IPP destroys the *pseudo* self-similar nature of the PH arrival process and the algorithm can not provide the desired Hurst parameter.

Since setting the IDC at a time point implies that the variance of the aggregated process is set at that time point as well. It can be observed in Figure 4 that the method is not capable of setting the IDC at t_2. The variation of this traffic trace for low values of t_2 is lower than the limit of this structure. The IDC at t_2 was set as close as possible to the IDC of the real source.

R/S plots of the traffic traces generated by the MAPs are plotted in Figure 6. Visual inspection suggests that for both the variance-time and R/S plot running the fitting algorithm with low values of t_1 and t_2 results in a close fitting of the real traffic trace behavior.

The second trace consisting of about 2 million arrivals has Hurst parameter 0.8012 given by the variance-time test. The interval (L_1, L_2) is set as for the first trace. For this data set the algorithm is able to set the IDC at both time points t_1 and t_2 exactly. Figure 5 shows the variance-time plots for the MAPs resulted from the fitting. R/S plots of the traces generated by the MAPs are depicted in Figure 7.

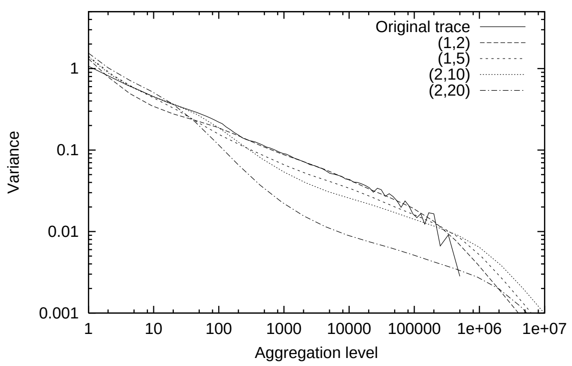

Figure 4: Variance-time plots of MAPs with different time points of IDC matching for the first trace

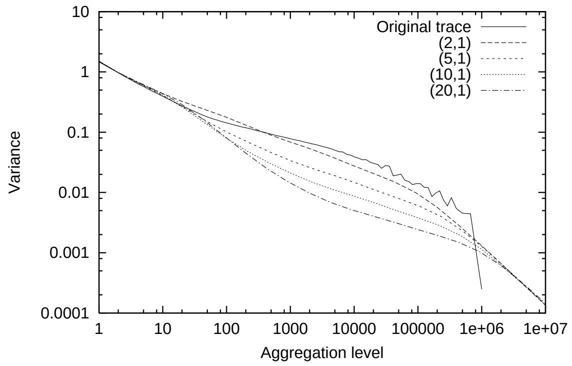

Figure 5: Variance-time plots of MAPs with different time points of IDC matching for the second trace

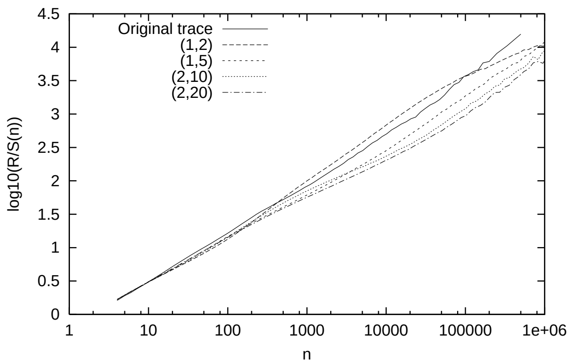

Figure 6: R/S plots of MAPs with different time points of IDC matching for the first trace

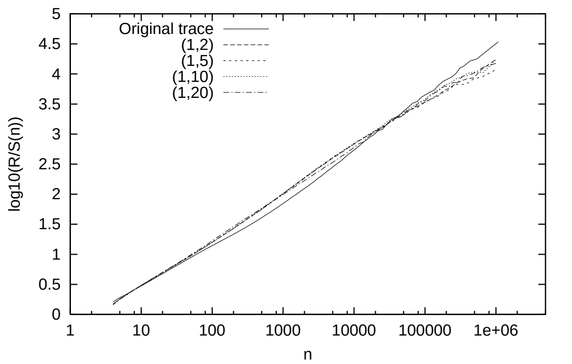

Figure 7: R/S plots of MAPs with different time points of IDC matching for the second trace

The fitting of the traces were tested by a •/D/1 queue, as well. The results are depicted in Figure 8 and 9. The •/D/1 queue was analyzed by simulation with 80 % utilization of the server. As one may observe the lower t_1 and t_2 the longer the queue length distribution follows the original one. The experiments suggests that the pair $E(Y_i)$, $2E(Y_i)$ is a good choice for t_1 and t_2. The cumulative distribution function (cdf) which is important when calculating loss probabilities is depicted in Figure 10 for the first trace. The cdf resulted by the original traffic trace crosses the cdf resulted by the MAP at about 140. This means that below 140 the MAP gives pessimistic results while over 140 it gives optimistic ones.

In the following we compare fitted MAP models to fractional Brownian motion (FBM) sources. The FBM is a self-similar continuous time continuous valued stochastic process whose increment is fractional Gaussian noise [8]. As a traffic source model it is defined by three parameters: the mean input rate ($E(N_1)$), the variance parameter ($Var(N_1)$), and the self-similarity parameter (H). One way to use FBM as a traffic model is to consider its increments in subsequent intervals as the amount of data arrived to the network. To compare the FBM source with a MAP the increments has to be rounded to an integer value and negative values has to be substituted by 0. This way we are given the number of arrivals in each timeslot. For queuing analysis the arrival instances have to be given. Given the number of arrivals in an interval, we assume that the arrival instance of each arrival is distributed uniformly in the interval. We used the method described by Paxson [10] to generate FBM traffic. The parameters of the traffic were $E(N_1) = 5$, $Var(N_1) = 50$ and $H = 0.8$. Figure 11 gives variance-time plots for the fitted MAP models for different timepoints of setting IDC,

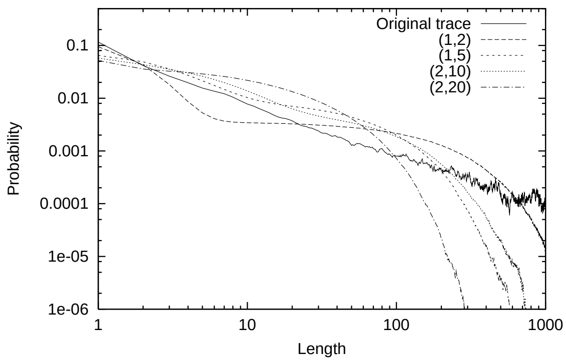

Figure 8: Queue length distribution for the first trace

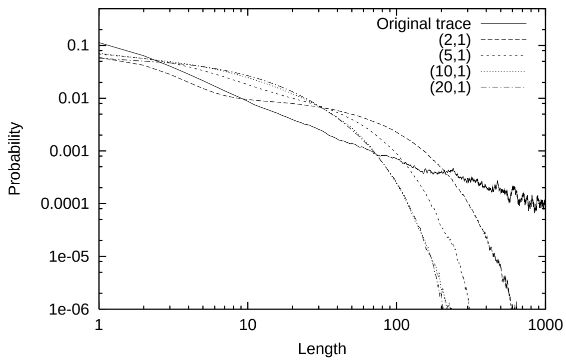

Figure 9: Queue length distribution for the second trace

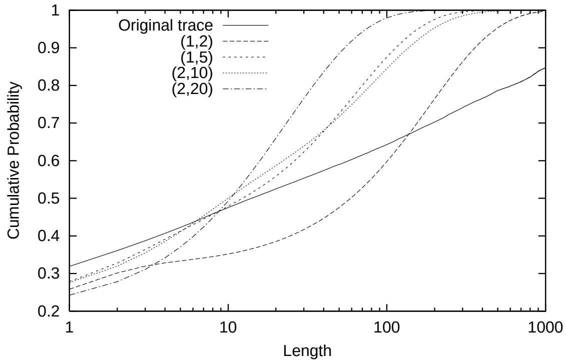

Figure 10: cdf of the queue length for the first trace

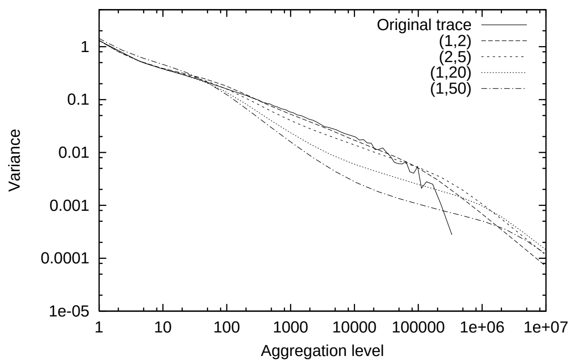

Figure 11: MAPs with different timepoints of IDC matching for the traffic generated by FBM

while Figure 12 depicts the queuing experiments with 80 % utilization. In Figure 13 one may observe how the variance-time plot of the arrival traces generated using a MAP approaches the analytically computed variance-time plot as the number of the generated arrivals increases.

5 Conclusion

The paper presents a heuristic MAP fitting method that fits some short and long range dependent parameters of the considered traffic process. The goodness of the fitting procedure is evaluated by commonly applied statistical tests and by the queue length distribution generated by the traffic processes.

The proposed fitting method provides a MAP whose fitted parameters are the same as the one of the original traffic process (or very close), but the applied statistical tests and the queue length distribution does not show a perfect match which means that other traffic parameters play also role in the traffic behavior. Further research is planned to investigate the effect of different parameters of traffic processes, and to find a dense but representative description of important traffic features.

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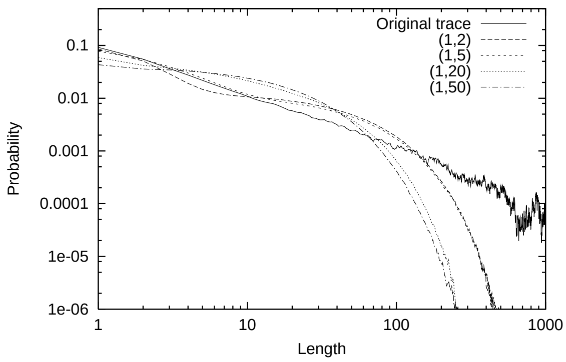

Figure 12: Queue length distribution for the traffic generated by FBM

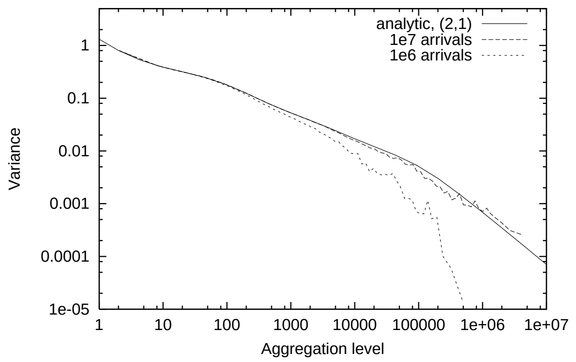

Figure 13: Variance-time plots of the generated traffic sources

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