Compendium of Abstracts

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Message from Manager

The postgraduate department of mathematics in St Aloysius college is blessed with a vibrant team of teachers. This compendium of Abstracts is the result of their concerted effort in organizing an International Conference on Advances in Applied Probability and Stochastic Processes - 2024. This is an opportunity for scholars in this subject for sharing their studies with others in the same area and update and refine their knowledge and continue their studies at farther. I appreciate the delegates in sending the papers in advance so that the organizing committee could get it printed and share with the participants in time. Probability and Stochastic processes has wide practical applications and this international conference will discuss some of these aspects so that the emerging scholars among the post graduates may fix the topics for their research. The success of this conference will give additional impetus to the teachers to continue their research and research publication. Congratulations to all the members of the department and all the contributors for their sincere and hard work in bringing out this publication.

Manager
(Fr. Thomas Chakramakkil)

Message from Principal

In the ever-evolving landscape of academia, the pursuit of knowledge and the exchange of ideas are essential for progress.

St Aloysius College is extremely proud that the Department of Mathematics in association with 'Indian Society for Probability and Statistics' is hosting the 'International Conference on Advances in Applied Probability and Stochastic Processes - 2024'. The hosting of such an event aligns with our institution's commitment to promoting excellence in education and research and I wholeheartedly congratulate the Department of Mathematics. We believe that this conference will not only facilitate the exchange of ideas but also encourage networking and collaboration among participants from diverse backgrounds.

'ICAAP & SP 2024' serves as a platform for bringing together some of the brightest minds in the field of applied probability and stochastic processes. The distinguished keynote speakers and presenters will share their valuable insights and experiences, providing all participants with a unique opportunity to enhance their understanding and contribute to the collective growth of our academic community.

St Aloysius College proudly acknowledges the collaboration and financial assistance of 'The Science and Engineering Research Board' (SERB), 'The National Board for Higher Mathematics' (NBHM) and 'The Kerala State Council for Science, Technology and Environment' (KSCSTE) in the organisation of this seminar.

I encourage all attendees to actively participate in the various sessions, engage in fruitful discussions, and take advantage of the networking opportunities available throughout the conference. Together, let us explore the frontiers of Applied Probability and Stochastic Processes, contributing to the advancement of knowledge and the development of innovative solutions to real-world challenges.

Dr Chacko Jose P
(Principal)

Message from Administrator

Indeed, it's my proud privilege and pleasure to congratulate the Post Graduate Department of Mathematics of St. Aloysius College for organizing an International Conference on Advances in Applied Probability and Stochastic Processes (ICAAP & SP-2024) in Collaboration with Indian Society for Probability and Statistics (ISPS).

I am glad that the organisers have chosen a topic whose applications are so multifaceted and capable of contributing to and transforming domains ranging from engineering and computer science to finance and insurance. Its gratifying to know that renowned scholars on the topic from across the globe along with erudite participants make this a platform for stellar academic exchange. Let's hope that the discourses that are bound to happen here will encourage young researchers to pursue further in their intellectual quest. I extend my wholehearted wishes for this innovative and informative academical platform.

Fr. Arun Jose K CMI
(Administrator)

Message from IQAC Coordinator

I would like to extend my heartfelt congratulations on the successful organization of the International Conference on Advances in Applied Probability and Stochastic Processes (ICAAP&SP - 2024). Your hard work and dedication to make this event a reality is commendable and I am proud to be associated with such a talented team.

I would also like to acknowledge the support of Chair of the International Conference Prof. Achyutha Krishnamoorthy, Emeritus Professor and Honorary Director, CMS College, Kottayam, Retd. Professor, CUSAT and Hon'ble visiting Professor, CUK, Kasaragod for his invaluable support and guidance.

Finally, I would like to extend my sincere thanks to DST-SERB, NBHM and KSCSTE for their support in organizing this event.

Once again, congratulations to everyone involved and I look forward to future endeavors.

Dr. Libison K. B
(IQAC Coordinator)

Key Note Address

MARKOV SELECTION REVISITED

Vivek S. Borkar, IIT Bombay

Abstract. This talk will describe some recent work on an alternative approach to selecting a Markov family of solutions for a degenerate diffusion with continuous coefficients, based on small noise limits and viscosity solutions. (Joint work with Anugu Sumith Reddy)

ON WIENER-HOPF FACTORIZATION FOR LEVY PROCESSES

Søren Asmussen, Aarhus University, Denmark

Abstract. The Wiener-Hopf factorization for a Levy process states that the maximum over an exponential time horizon and the displacement from the maximum to the terminal value are independent, and that the displacement has the same distribution as the minimum. Finding the distribution of the maximum and/or the displacement opens up the door to solving many problems, both of theoretical and applied nature. The main case where this can be done is when jumps in one or both directions are compound Poisson with a jump size distribution that is phase-type or, more generally, has a rational Laplace transform. The traditional solutions are then lengthy expansions in terms of roots determined by a scheme involving analytic continuation and Rouche's theorem. As alternative, we give simple matrix formulas derived by probabilistic arguments. Further, an extension from exponential to phase-type horizon is given and applied to the prizing of certain equity dependent products in life insurance.

Joint work with Jevgenijs Ivanovs and Mogens Bladt.

Plenary Talks

ANALYSIS OF QUEUEING-INVENTORY SYSTEM WITH QUEUE-DEPENDENT REPLENISHMENT POLICY

Agassi Melikov[a] and Serife Ozkar[b]

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Abstract. In classical inventory management systems, the amount of time required to serve the customer is zero. However, in real-life situations, a positive time is needed for some necessary procedures to delivery of items in the inventory. Inventory systems in which the service time is positive called queueing-inventory systems (QISs). The pioneers in the study of QISs models are Sigman and Simchi-Levi [1] and Melikov and Molchanov [2]. A detailed survey of the literature for QISs can be found in Krishnamoorthy, Shajin and Narayanan [3].

Classical QIS models are based on some assumptions. The main assumption is the following: the replenishment policy (RP) do not take into account the number of customers in the system (or in the queue), i.e. RP is queue-independent.

Obviously, queue-independent RPs are inefficient, since over-ordered stocks may not be required, which means that the system will spend useless economic costs on holding stocks. In other words, in order to increase the efficiency of the QISs, when ordering stocks, it is also necessary to take into account the current number of customers in the queue. To our best knowledge, the QISs with queue-dependent RP have not been studied in the literature. Our goal is developing the simple, implementable, and yet effective RP that take into account simultaneously both the current number of customers in queue and the inventory level.

Melikov and Molchanov [2] was among the early pioneers to introduce the notion of state-dependent RP within QISs and examined the following model. Threshold s, $0 \leq s \leq S-1$, where S denotes maximum capacity of warehouse, is defined and if the inventory level is greater than s, then no restocking order is sent; otherwise, the queue-dependent randomized replenishment policy is determined as follows: the system orders an inventory of size m, $1 \leq m \leq S-s$, with probability $\alpha_m(n)$, where n is the number of customers in queue, $0 \leq n \leq N$, N denotes the maximum queue capacity. It is assumed that $\sum_{m=1}^{S-s} \alpha_m(n) = 1$ for each n. The authors use Markov Decision Process approach to minimizing the total cost associated with waiting time and loss of customers and the holding of stock in the warehouse. The problem is solved by selecting optimal values of the probabilities $\alpha_m(n)$, $1 \leq m \leq S-s$, $0 \leq n \leq N$. It is shown that the optimal RP is in the class of nonrandomized (deterministic) policies, i.e. for each n, there exist only one $m^* \in \{1, \ldots, S-s\}$ such that $\alpha_{m^*}(n)$.

The purpose of this work is to draw the attention of researchers to the study of an important

problem: the study of QIS models in which the replenishment policy takes into account not only the level of stocks, but also takes into account the current state of the queue.

The main contributions of this paper can be summarized as follows:

- A novel RP in infinite QIS that based on the switching the order replenishment from slow rate to fast one depending on the current level of queue is proposed.
- We consider a model with a Markov arrival process (MAP), phase distribution of customer service time and exponential order replenishment time with different rates for different types of orders.
- Mathematical model of the investigated QIS is formulated as a multi-dimensional Markov chain; an ergodicity condition is obtained and the method to derive the closed-form expressions for the system's steady-state probabilities and for its performance measures is developed.
- The developed approach can be easily applied to QIS models with any (known) RP.
- The behavior of performance measures versus on the load and structural parameters of the system is demonstrated, and the results of solving the problem of minimizing the total costs are presented.

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PARALLEL PROCESSING QUEUES: A SUMMARY AND BEYOND

Alexander Rumyantsev

Karelian Institute , Russia

Abstract. In this talk, we summarize the results of our group in studying the parallel processing systems, with the main emphasis put on the so-callled multiserver job model a.k.a. simultaneous service multiserver model or supercomputer model. In such multiserver systems, the key challenge in analysis is the non-work-conserving property which means that non-empty queue may be in the system with idle servers. We outline some recent results and summarize the key challenges in this research area.

CONTINUITY OF SOLUTIONS TO DIFFUSION-SPDES WITH RESPECT TO INITIAL CONDITIONS AND COEFFICIENTS

Rajeev Bhaskaran

Indian Institute of Science,Education and Research, Thiruvanantapuram, Kerala

Abstract. In this talk we will present some results on the continuity of solutions of SPDEs corresponding to d-dimensional diffusions with coefficients in the Hermite-Sobolev spaces $\mathcal{S}_p, p > \frac{d}{4}$. The proof is an extension of the proof of pathwise uniqueness property of such equations via the 'monotonicity inequality'.

ANALYSIS OF THE MULTI-SERVER RETRIAL QUEUEING SYSTEMS WITH MARKOV ARRIVAL PROCESS AND SERVERS RESERVATION FOR PRIMARY CUSTOMERS

S. Dudin, A. Krishnamoorthy, A. Dudin, O. Dudina

Abstract. Multi-server retrial queueing systems are suitable for modelling the variety of important real-world systems, including telecommunication networks and contact centers. Arrival flows in modern systems are well described by the versatile Markov arrival process (MAP). In the MAP, periods of active arrival of customers alternate with periods when the arrival rate is lower. This creates the possibility of different treating of the retrying customers, who did not succeed to enter the service immediately upon the arrival, depending on the current activity of primary customers arrival. For example, admission of retrials should be restricted when the primary customers arrive intensively. We assume that a certain threshold corresponds to each state of the underlying process of the MAP flow of primary customers. A retrying customer is admitted for the service only if, upon its arrival, the number of the idle servers is not less than the corresponding threshold. Otherwise, the customer has to continue retrials. Such type of control allows to increase the probability of immediate access to service for the primary customers. But it is a bit discriminative with respect to the retrying customers and the proper choice of the thresholds have to allow to reach a certain trade-off between the desire to provide easy access for the primary customers and the necessity to be enough fair with respect to those of them who will be compelled to become the retrying customers. Under an arbitrary set of the thresholds, the stationary behavior of such a system is analyzed via its description by a multidimensional asymptotically quasi-Toeplitz Markov chain. The problem of the optimal choice of the thresholds under a fixed form of cost criterion is considered and numerically solved.

STABILITY ANALYSIS OF REGENERATIVE QUEUES

Evsey Morozov

Karelian Research Center of the Russian Academy of Sciences, Russsia

Abstract. We discuss the regenerative approach to stability analysis of a wide class of queueing systems, including multiclass multiserver systems, state-dependent systems and multiclass retrial systems with constant and state-dependent rates.

LARGE DEVIATIONS FOR STOCHASTIC APPROXIMATIONS

Henrik Hult

Professor (Full) at KTH Royal Institute of Technology, Sweden

Abstract. Stochastic approximation is a general and useful random iterative algorithm originating from the work of Robbins and Monro in the 1950s. Many popular training algorithms in machine learning can be formulated as stochastic approximations, including stochastic gradient descent, reinforcement learning, contrastive divergence, adaptive MCMC, and various adapted extended ensemble methods such as Wang-Landau and accelerated weight histograms. In this talk we will present recent work on large deviations for stochastic approximations and provide a new representation of the rate function. An interpretation that learning algorithms can forget will be discussed and the rate function reveals how the forgetting occurs.

The talk is based on joint work with Adam Lindhe, Pierre Nyquist and Guo-Jhen Wu.

DIFFUSION APPROXIMATION OF THE STATIONARY DISTRIBUTION OF A TWO-LEVEL SINGLE SERVER QUEUE

Masakiyo Miyazawa

Tokyo University of Science

Abstract. We consider a single server queue which has a threshold to change its arrival process and service rate by its queue length, which is referred to as a two-level single server queue. This model is motivated by an energy saving problem for a single server queue whose arrival process and service rate may be changed. To get its performance in tractable form, we study in the limit of the stationary distribution of this two-level queue under a certain scaling in heavy traffic, which may be considered as a diffusion approximation. It is shown that this limiting distribution is truncated exponential below the threshold level and exponential above it under suitably chosen system parameters. This result is proved using the BAR approach studied in [1,2,3,4]. We also intuitively discuss about a diffusion process corresponding to the limit of the stationary distributions.

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SENSITIVITY ANALYSIS OF THE WARNING VACCINATION LEVEL FOR A STOCHASTIC SIVS MODEL WITH IMPERFECT VACCINE

M J Lopez Herrero

Abstract. This talk deals with a stochastic Susceptible-Infected-Vaccinated-Susceptible (SIVS) model with infection reintroduction. Health policies depend on vaccine coverage, v0, that guarantees herd immunity levels in the population. Vaccine failures occur when an organism develops a disease despite of being vaccinated against it. After vaccination, a proportion of healthy individuals unsuccessfully tries to increase antibody levels and, consequently these individuals are not immune to the vaccine preventable disease. When an infectious process is in progress, the initial vaccine coverage drops down and herd immunity will be lost. Our objective is to introduce a threshold for vaccination level and study the so-called sleeping period, that is the time until the number of vaccinated descends to this warning vaccination level. A sensitivity analysis is performed to assess the influence of the model parameters on the variation and robustness of the sleeping period.

LARGE DEVIATIONS FOR MARKOV CHAIN MONTE CARLO METHODS: THE SURPRISINGLY CURIOUS CASE OF THE METROPOLIS-HASTINGS ALGORITHM

Pierre Nyquist

PhD Professor (Associate) at KTH Royal Institute of Technology, Stockholm, Stockholm, Sweden

Abstract. Markov chain Monte Carlo (MCMC) methods have become the workhorse for numerical computations in a range of scientific disciplines, e.g., computational chemistry and physics, statistics, and machine learning. The performance of MCMC methods has therefore become an important topic within (applied probability) and statistics. In particular, when the underlying distribution one is trying to sample from becomes sufficiently complex, convergence speed and/or the cost per iteration becomes an issue for most MCMC methods.

The analysis, and subsequently design, of MCMC methods has to a large degree relied on classical tools used to determine the speed of convergence of Markov chains, e.g., mixing times, spectral gap and functional inequalities (Poincaré, log-Sobolev). An alternative avenue is to use the theory of large deviations for empirical measures. In this talk I will first give a general outline of this approach to analysing MCMC methods, along with some recent examples. I will then consider the specific case of the Metropolis-Hastings algorithm, the most classical amongst all MCMC methods and a foundational building block for many more advanced methods. Despite the simplicity of this method, it turns out that the theoretical analysis of it is still a rich area, and from the large deviation perspective it is surprisingly difficult to treat. As a first step we show a large deviation principle for the underlying Markov chain, extending the celebrated Donsker-Varadhan theory. Time permitted I will also discuss ongoing and future work on using this result for better understanding both the Metropolis-Hastings method and more advanced methods, such as approximate Bayesian computation (ABC-MCMC) and the Metropolis-adjusted Langevin algorithm (MALA).

The talk is primarily based on join work with Federica Milinanni (KTH).

A UNIFORM APPROACH FOR ANALYZING QUEUES WITH CORRELATED INTERARRIVAL AND SERVICE TIMES

Qi-Ming He

University of Waterloo, Canada

Abstract. Using the Markov modulated fluid flow (MMFF) process, we introduce a framework to analyze queues with correlated interarrival and service times. An algorithm is developed for computing quantities such as the distributions, means, and variances of age, waiting time, sojourn time, and queue length. System stability conditions are found. A number of special cases are presented to demonstrate the versatility of the queueing model and the power of the solution approach. (Joint work with Haoran Wu and Li Xia)

A SINGLE-SERVER DELAY/RETRIAL MODEL WITH TWO PHASES OF SERVICE AND NON-PREEMPTIVE PRIORITY FOR THE QUEUED CUSTOMERS

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Abstract. In mobile cellular communication systems handover calls must be given priority over orig- inating calls to avoid undesired interruptions of handover calls when entering the region of a new base station. One protocol is to queue the handover calls and block the originating calls in case upon arrival at the base station no channels are free. In this paper a one-server (channel) mixed delay/retrial queueing model is considered to study the performance of this protocol. Calls arrive at the base station according to a Poisson process, and if the single server (channel) is busy the handover calls are put in a queue and the originating calls are sent to an unlimited virtual waiting place, called the orbit. Any call which finds the server idle upon arrival starts its service immediately. From the orbit calls try to approach the server anew after a random time and because non-preemptive priority is given to the queued (handover) calls, calls in the orbit can approach the server successfully only when the queue is empty and the server is idle. Every call requires two service phases. The First Essential Service phase (FES) can fail, and upon failure the call is (re)sent into the orbit (in that case a handover call looses his handover status). The Second Essential Service phase (SES) is always successful. Both phases are exponentially distributed with different parameters. The prob- ability generating function (PGF) of the joint steady-state distribution of the queue length and the orbit size is derived, and from this PGF several performance measures are calculated, such as the queue-length distribution, the mean queue length, the waiting-time distribution of a handover call in the queue, and the mean orbit size, amongst others. Extensive numerical results illustrate the sensitivity of these performance measures for the parameters, such as the fraction of handover calls and the success probability of the FES.

BEST ARM IDENTIFICATION IN MULTI-ARMED BANDITS – OPTIMAL ALGORITHM BASED ON FLUID ANALYSIS

Sandeep Juneja

Abstract. We are given finitely many unknown probability distributions that can be sampled from and our aim is, through sequential sampling, to identify the one with the largest mean. This is a classical problem in statistics, simulation and learning theory. Lately, methods have been proposed that identify a sample complexity lower bound that any algorithm providing probabilistic correctness guarantees must satisfy, and algorithms have been developed that asymptotically match these lower bounds even for general sampling distributions, as the probabilistic error guarantees converge to zero. We review these ideas and propose a novel algorithm that relies on exploiting the underlying fluid structure in the evolution of the optimal sampling process and improves upon existing asymptotically optimal algorithms

MATRIX-ANALYTIC METHODS IN STOCHASTIC MODELING

Srinivas R. Chakravarthy
Kettering University, Flint, MI-48504, USA

Abstract. Matrix-analytic methods (MAMs) have been introduced by Neuts almost five decades ago. Since then the subject has grown significantly with applications from a variety of areas. Neuts in his famous article mentions, "..The history of the matrix methods (so called for brevity) is short, but worth telling. ..I tackled a number of models involving embedded Markov renewal processes, evidently with some measure of success, since the papers were published in noted journals and some academic recognition came my way. It privately bothered me that, as the papers grew longer and the analysis more complex, the explicit or qualitative results in them became fewer and fewer." He continues further, "..In the history of mathematics, a similarity of formalism has always indicated similarity of structure and an ultimate level of understanding is that of unifying structure."

The following quote, taken from the recently published 2-volume book of this presenter, aptly summarizes the development of MAMs. "Out of a genuine concern for the type of analysis and solutions emanating from the published works on queueing models which were driving the practitioners away from using queueing models to understand and apply them for the improvement of the processes, (late) Prof. Neuts developed phase type (PH-) distributions first in early 1970s. This helped him to lay the path for future researchers in stochastic models to study models useful in practice. These stochastic models include queues, reliability, and inventory. The impetus in developing phase type distributions resulted in Neuts to identify queues possessing matrix-geometric solutions and then to introduce a versatile class of Markovian point process (VMPP) a few years after developing the theory of PH-distributions. In 1990, this VMPP was reintroduced as a batch Markovian arrival process (BMAP) by Neuts along with his two doctoral students, David Lucantoni and Kathy Meier-Hellstern. Since then VMPP has been used as BMAP and Markovian arrival process (MAP) in the case when the arrivals occur singly. The introduction of VMPP in late 1970s paved the way for Neuts to introduce matrix-analytic methods (MAM). Ever since these methods have been extensively studied both theoretically and computationally."

In this talk, which is based on the recently published 2-volume book on this topic to enable students and practitioners to implement the methods to study stochastic models of interest

in practice, we will present an overview of matrix-analytic methods in stochastic modeling and also discuss a few examples using such methods.

ON SECOND ORDER FLUID QUEUES

V. Thangaraj

Formerly at Ramanujan Institute for Advanced Study in Mathematics, University of Madras

Abstract. Conventional stochastic modelling techniques have some limitations due to State space explosion, Granularity and size, Modelling power limitations, Inaccurate distribution approximation, etc. Continuous modelling technique can help overcome these limitations. Fluid techniques use additional continuous variables which are not part of the conventional state space, leading to smaller sets. Many systems are characterized by huge amount of very small elements (i.e. the packets in a broadband router, raw parts in a flexible manufactory system). Continuous variables may very naturally approximate these large discrete numbers. In some cases, physical quantities like time, temperature, or speed must be modelled explicitly. Conventional modelling technique discretise those quantities by choosing a finite set of possible values. Continuous variables can instead exactly model these quantities. Fluid model can directly embed more complex distributions and non-homogenous Poisson process, without the need of using approximate techniques. Here, in this survey talk, we discuss some Reward Models and Fluid Models.

Invited Talks

CONTINUOUS-REVIEW FLUID INVENTORY SYSTEMS WITH LOST SALES AND VARIABLE LEAD TIME

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Abstract. Inventory systems of fluid matter have received only little attention in the literature. In this paper, we present continuous review inventory systems in which both the items stocked and the size of demands do not take integer values, indicating that the stock type is fluid, such as oil, flour etc. We assume that the arrival times of demands form a Poisson process and that the demand sizes have i.i.d. exponential distribution. We assume (s, S) ordering policy to replenish the inventory. We consider two models: in the first one we assume exponential lead times for the order and in the second one we assume Erlang lead times. In both models, we assume that the demands that occurred during stock out periods and the excess demand are lost. Using the level crossing method, we derive the steady state probability distribution of inventory level for both models. After deriving some system performance measures, we computed the total expected cost rates. For inventory model with exponentially distributed lead times, we derive a closed form expressions for stationary distributions and some remarks are made as to the optimality of cost function with respect to each of decision variables. For both models, we numerically analyze the sensitivity of the parameters on the system performance measures and influence of system costs and parameters on the total costs

Keywords: Continuous review fluid inventory; random lead times; lost sales; level crossing theory; stationary distribution.

ON THE HEURISTIC COMPUTATIONAL PROCEDURES FOR THE VIRTUAL WAITING-TIME DISTRIBUTION IN NON-RENEWAL INPUT FINITE-BUFFER BULK-SERVICE QUEUES: $MAP/G^{(a,b)}/1/N$

Abhijit Datta Banik, M. L. Chaudhry, Sitaram Barik, Gagandeep Singh Rahi

Abstract. In this paper, we discuss a finite-buffer single-server queue wherein arrivals occur according to a Markovain arrival process. The server serves customers in batches according

to (a, b)-bulk service rule, that is, the server serves in batches of maximum size 'b' with a minimum threshold size a. The service time of each batch follows general distribution and the queue capacity is N excluding the batch of customers with the server. An alternative approach for finding the queue-length probability vector at a post-departure epoch has been calculated using the roots of a characteristic equation. Queue-length probability vectors at random and pre-arrival epochs have also been obtained using the embedded Markov chain, Markov renewal theory, and the semi-Markov processes. Approximation to the virtual waiting-time distribution (in the queue as well as in the system) for each arrival phase of a random customer is derived using the functional relation between the vector generating function (vgf) for the queue-length probability vectors at a random epoch and the vector containing the phase-wise Laplace-Stieltjes transform (LST) for the queueing-time distribution of a random customer. Using LSTs, we discuss the derivation of the approximate phase-wise probability density function for the virtual waiting-time of a random customer as well as its numerical implementations.

SYSTEM POINT LEVEL CROSSING APPROACH TO A QUEUEING-INVENTORY SYSTEM

B. Sivakumar and A. Shophia Lawrence

School of Mathematics Madurai Kamaraj University Madurai

Abstract. The system point level crossing method is used in this article to analyze a queueing-inventory system. Customer arrive according to a Poisson process. A random amount of items are delivered to customers based on their demands after a random service time. It is assumed that service times and size of the demand have independent exponential distributions. The inventory has a fixed maximum capacity of S. An order is placed if the inventory level falls to s or less. It is assumed that the lead times for the orders are exponential. We obtain the system of integral equations that govern the stationary probability distributions by employing system point level crossing techniques. A number of system performance metrics are derived, and the total expected cost rate is calculated using these metrics.

M/M/1/N QUEUES WITH ENERGY REQUIRED SERVICE AND PHASE-TYPE VACATION TIMES

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Abstract. Here we consider an M/M/1/N queueing model where each customer needs a random amount of fluid for its service say, in the form of energy or power. The rates at which the fluid flow to the system is governed by a continuous time Markov process having a finite number of states. During service time, fluid is consumed at a constant rate. Service to customers will go on continuously as long as customers are there in the finite capacity waiting space and the fluid buffer is nonempty. At each time the buffer becomes empty, the server waits for a random amount of time in order to accumulate the fluid and then restart the

service of the customer for which the service got interrupted due to lack of fluid. We call this time as the vacation time and it is assumed to be phase-type distributed. Matrix analytic method is used for the stationary analysis of the model. Laplace transform of the steady state vector and the mean fluid level at an arbitrary epoch are derived. Some performance measures related to queue size are computed. Numerical illustrations are provided to support the theoretical findings.

STUDY OF LIMIT THEOREMS ON EXTENDED INVERSE HAWKES PROCESSES

S Dharmaraja

Professor and Former Head
Department of Mathematics
Indian Institute of Technology Delhi

Abstract. An inverse Hawkes process is a process having constant intensity and stochastic jump size, depending on the past number of jumps, while a Hawkes process has the intensity which is stochastic. An extended inverse Hawkes process is a process obtained by combining a Hawkes process and an inverse Hawkes process. The focus of this talk is to investigate the asymptotic behaviour of an extended inverse Hawkes process with general structure of the exciting functions. In particular, the results obtained are the generalized versions of the Law of Large Numbers and of the Central Limit Theorem.

STATIONARY SYSTEM-LENGTH DISTRIBUTION OF A MARKOVIAN BULK SERVICE QUEUE WITH BULK ARRIVALS, MODIFIED BULK SERVICE RULE AND DYNAMIC SERVICE RATES

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Abstract. Queueing theory, derived from the study of phone network congestion, facilitates a systematic understanding and prediction of queues. It plays a crucial role in modern systems, such as the internet, explaining the efficient travel of data. Bulk queues, where people or services come in groups, find applications in various fields like telecom and traffic systems. Excessive bulk activity, such as emails or calls, in networks can lead to service issues. Commonly used rules for managing tasks in batches, such as general bulk service rule (G-BSR) or (L, K) rule, involve a server providing services for a minimum batch size of L and up to a maximum batch size of K, as illustrated by Neuts [1]. Once a batch of tasks begins, new tasks cannot join, even if there is available capacity. Late arrivals are typically considered for service within the ongoing batch up to a specified limit. For instance, one might be able to join an ongoing conference call up to a predetermined participant limit. Recently, this approach has been modified and termed as the modified bulk-service rule (M-BSR), as discussed by Singh et al. [2]. In M-BSR, customers are served in batches with a

minimum threshold of L, and individuals arriving during service can join until the server reaches its maximum serving capacity of K. To distinguish between G-BSR and M-BSR, we employ the notation $(L \to K)$ for the M-BSR rule instead of (L, K).

In this paper, we analyze a single-server, infinite-buffer queueing system where customers arrive in batches of size X, following the compound Poisson process with a batch arrival rate of λ. The arriving batch size X is a random variable with a probability mass function (PMF), denoted as $P(X = i) = g_i$ for $i = 1, 2, 3, \ldots$. The service times are independent and exponentially distributed random variates with a dynamic service rate, μ_n, which depends on the size of the batch with the server, where $L \leq n \leq K$. Arriving batches are served under a modified bulk-service rule (M-BSR) with batch-dependent service, and we represent such queues as $MX/M(L \to K)/1$ queueing systems.

We obtained the bivariate probability generating function for the queue-length and server-content by employing the steady-state equations. Using the roots' method, we derived the closed-form joint probabilities of the queue-length and server-content of a batch at an arbitrary epoch in terms of the roots of the characteristic equation. Additionally, we obtained the marginal probability distribution of the queue-length, system-content, and the number of customers with the server. Furthermore, we computed various performance measures such as the average number of customers in the queue, system, with the server, etc. To illustrate the computational process, we presented numerical results, including various discrete arriving batch-size distributions and different parameters, using tables and graphs.

Once the expected numbers of customers in the queue, system, and with the server are known, one can make better decisions (policies), such as increasing the maximum batch size with the server and fixing the arrival batch size. These decisions will help reduce the average waiting time and average queue length, thereby controlling system congestion.

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ALGORITHMIC ANALYSIS OF COGNITIVE WIRELESS MULTI-CHANNEL RETRIAL QUEUEING NETWORK SYSTEM FOR FINITE-SOURCE PRIMARY USERS WITH ADMISSION CONTROL OF SECONDARY USERS

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Abstract. This article deals with a new class of multi-channel cognitive radio retrial queue-

ing network in which a waiting line buffer is deployed for secondary users (Sus). The primary users (PUs) are generated by a finite-source system. Apart from PUs have pre-emptive priority overs Sus, the effect of retrial attempts of PUs have been considered. In such a versatile complex system, the stability condition and the joint-steady state probability distribution of PUs and Sus are determined by using the matrix-analytic methods. Several vital performance measures of interest such as blocking probability, throughput, mean waiting time, etc., have been studied. By employing the first-step analysis, the first- passage time to reach a congestion level in the Sus waiting line buffer and its related characteristics are analysed. Finally, extensive numerical results are presented graphically for the system insight.

Keywords: Cognitive radio network; Multi-Channel; Retrial queue; Stability; Stationary distribution; First-passage time; Performance indices.

STOCHASTIC MODELING OF MULTISTATE DISEASE DYNAMICS UNDER RANDOM ENVIRONMENT

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Abstract. In some medical treatments, decision must be made sequential and in an uncertain environment. A physician determining a course of treatment must consider patient's health as well as the best treatment decision in the future. Often decisions are to be taken in a dynamic environment. We provide insight into the nature of optimal decision on the treatment strategy through the application of Semi-Markov decision Process (SMDP) for such a problem. The subject/patient lives in varying random environments, imparting significant effects on performance/health status. While the environment evolves according to a Semi-Markov Process, in each environment state, the subject goes through several states of disease according to a Semi-Markov Process. In an environment 'k' when the patient state is 'i', one of the following two actions are available: continue the present treatment strategy (C) with a given cost rate $h^k(i)$ or initiate a rejuvenating treatment strategy (R) with a cost rate $c^k(i)$. In this complex model the optimal strategy is found out minimizing the expected discounted total cost. A special case of Markov environment is discussed indicating the feasibility of the computation of optimal policy. A numerical illustration is also provided to support the viability of the analysis and results. The model provides a useful and flexible representation of acute and chronic events and can be used to explore the economic impact of changes in therapy

Keywords: Stochastic model, Semi-Markov Process, Semi-Markov Decision Process (SMDP), Random environment, Optimal Treatment strategy.

ANALYSIS OF POWER MANAGEMENT IN WIRELESS SENSOR NETWORK WITH START-UP TIMES AND THRESHOLD POLICY

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Abstract. In recent years, in mobile communications, power savings in the User Equipment (UE) and Quality of Service(QoS) provided are important issues. It is critical for the UE to maximize its energy efficiency. For instance, there is a large literature on power-awareness in mobile and wireless networks. The stochastic processes are used for the modeling of wireless networks. Queueing models play a significant role in analyzing the performance of power management systems in various electronic devices and communication systems. We adopt a multiple vacation queueing model with a threshold policy to analyze the power-saving mechanisms of the wireless sensor network (WSN) using the Dynamic Power Management technique. The proposed system consists of a busy state (transmit state), wake-up state, shutdown state and inactive state. In this model, the server switches to a shutdown state for a random duration of time after serving all the events (data packets) in the busy state. Events that arrive during the shutdown period cannot be served until the system size reaches the predetermined threshold value of k and further it requires start-up time and a change of state to resume service. At the end of shutdown period, if the system size is less than k, then the server begins inactive period; otherwise, the server switches to the wake-up state. For this system, an explicit expression for the transient and steady-state probabilities is computed in closed form. Furthermore, performance indices such as mean, variance, probability that the server is in various stages of power management modes and mean power consumption are computed.

Keywords:Single server, transient probabilities, steady-state probabilities, threshold policy, start-up times, mean power consumption

ON SOME BIVARIATE RANDOM SUM MODELS

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Abstract. Univariate random sum models have received much attention in statistical literature due to their practical relevance in several areas of scientific research, especially in the areas like actuarial science, physical sciences, biological sciences etc. But bivariate random sum distributions have not much studied in the literature. The present talk concentrates on this direction and discusses how to develop such bivariate random sum models and their related mixture models. Further, we investigate several interesting properties of these classes of distributions along with certain recently developed bivariate random sum models.

ON SOME INVENTORY MODELS WITH/WITHOUT LOST SALES

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Abstract. Several researchers have analyzed inventory models which assume a non-instantaneous service time. However, such models pose several challenges in their study, when compared to models with instantaneous service. Hence, researchers have searched for assumptions under which the steady state distribution of the former models is obtained from that of the latter, preferably in a product form. One such assumption is of lost sales, which refers to not allowing customers to join the queue when the inventory level is zero. We discuss some inventory models, which do not possess a product form solution even under the lost sales assumption.

STEADY-STATE ANALYSIS OF A SINGLE SERVER RETRIAL QUEUE SUBJECT TO RANDOM SHOCKS AND ASSIGNMENTS

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Abstract. In this talk, a single server retrial queueing system operating in a multi-level random environment subject to randomly occurring disasters is analysed. Customers arrive according to a Poisson process. These customers are called primary customers. An arriving primary customer gets into service immediately if the server is idle, otherwise joins an orbit of infinite capacity and retry later for service under constant retrial policy. The customers in the orbit are called secondary customers. Shocks arrive to the system according to an independent Poisson process and damages the server and also wipes out the customer (if any) getting service at that instant. The damaged server is immediately taken to a repair facility and the repair commences instantaneously. All primary customers and secondary customers arriving during the repair are lost. The repair time is exponentially distributed and immediately after repair the server is assigned to operate in one of the levels of a multi-level environment. Both the arrival and service rates depend on the level of the environment. For this model, steady- state probabilities are found using generating function approach. A performance analysis is also made with numerical example.

AMS Subject Classification: 60K25,90B22

Key Words and Phrases: Single server queue, orbit, retrial, disaster, repair, multi-level random environment

STOCHASTIC DYNAMICS AND PASSAGE TIMES FOR DIFFUSION APPROXIMATIONS

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Abstract. We discuss the first passage time problems for a class of one dimensional master equations with separable kernels. For this class of master equations, the integral equations for first passage time moments (FPTM) have been transformed exactly into ordinary differential equations by Weiss and Szabo. There the boundary conditions for the first passage time moments have been introduced separately. In our study we use the imbedding method to derive the first passage time densities (FPTD) by imbedding the boundary conditions for crossing the barriers into the master equations. We also derived the results of Weiss and Szabo model from our results for first passage time densities and FPTM for crossing the barriers. In our study we also derived the results the FPTD and FPTM by taking one of the barriers as absorbing and the other barrier as reflecting. When the separable kernel has only single term the equation for FPTD and FPTM obtained are exactly the same as that of simple diffusion. Generalization of the model is studied to obtain the differential equations for FPTD and FPTM. The generalization in the separable kernel leads to higher order differential equation for FPTD and FPTM. The differential equation for FPTM is identified with that of time homogeneous Fokker-Plank equation.

Keywords: Imbedding method ; absorbing barrier ; Reflecting barrier ; Diffusion Approximation ; Generalised Separable Kernel

Contributory Talks

HORIZONTAL DIVIDEND RESTRICTION ON A COMPOUND POISSON RISK PROCESS WITH COMPOUND POISSON PREMIUMS AND DELAYED BY-CLAIMS

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Abstract. We consider a queueing inventory system with a positive lead time. The system operates with two types of replenishment policies: normal replenishment and emergency replenishment. Customers arrive according to a Poisson process with a rate λ and the inventory is served based on an exponential distribution with a parameter μ . When the inventory level drops to a certain threshold, denoted as s, a replenishment order (normal replenishment) is placed. The time it takes for normal replenishment to be realized follows an exponential distribution with a parameter β_n. However, if the normal replenishment is not realized when the inventory level reaches a lower threshold r (where $r < s$ and r is close to zero), an emergency replenishment order is placed. The lead time for an emergency replenishment order follows an exponential distribution with a rate parameter β_e. Whenever a replenishment is realized, a specific quantity, denoted as Q_n or Q_e, is added to the inventory level, corresponding to normal replenishment and emergency replenishment, respectively. The system has been studied in detail to determine its stability condition. The steady-state probabilities in the system can be calculated using a product-form solution, with the assumption that customers are blocked when there is no inventory available. Various performance measures of the system are discussed. Additionally, a cost function is formulated, and optimal reorder levels and reorder quantities for both normal and emergency replenishment policies are estimated using numerical illustrations.

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ANALYSIS OF RELIABILITY IN A REPAIRABLE k-OUT-OF-n: G SYSTEM WITH DUAL REPLENISHMENT STRATEGIES

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Abstract. This paper explores a k-out-of-n : G COLD system incorporating repair and component replenishment. Repair of failed components commences when the count of working components reaches L. The system employs two replenishment types: normal, triggered once the number of working components turn N and emergency, initiated at S operational components. If normal replenishment is delayed and failures reach a threshold, an order for emergency replenishment with a shorter lead time is placed. If the order for normal replenishment materializes after placing an emergency order, the latter is cancelled. System failure is defined as the count of operational components falling to $k-1$. Using Chapman-Kolmogorov difference-differential equations, we derive the steady state probabilities explicitly. Key performance measures are computed, and numerical examples are provided. The analysis of the model yields related distributions, and an optimization problem for minimal cost is obtained.

Keywords: Reliability-queueing-inventory system, k-out-of-n: G system, COLD system, System repair, Normal replenishment, Emergency replenishment, Lead time

PERFORMANCE ANALYSIS OF THE BASE STATION CONSIDERING DIFFERENT FREQUENCY BANDS USING STOCHASTIC MODELLING IN 5G NETWORKS

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Abstract. Cellular networks, evolving since the late 1970s, have seen remarkable progress across generations, spanning from 1G to 4G. Initially, 1G networks catered primarily to voice calls, featuring a 30KHz channel and 2.4 kbps data speed. The advent of 2G in 1991 expanded bandwidth to 200KHz, enabling messaging at speeds up to 64 kbps. Subsequent leaps in bandwidth and speed occurred with 3G in 2000 and 4G in 2008. While 3G provided bandwidths of 5 MHz to 20 MHz and speeds of 14 Mbps, 4G soared to 100 Mbps, albeit with slightly higher latency. A study emphasized the distinctions between 5G and its predecessors, highlighting advancements like reduced latency, multi-Gbps peak speeds, improved reliability, and an expanded network infrastructure.

To meet the demand for connectivity, 5G networks emerged in early 2019, promising reduced latency, higher peak speeds, improved reliability, and expanded bandwidth capacities. However, optimizing energy use in base stations has become a critical challenge. Maximizing

base station efficiency is crucial, as these stations utilize various frequency bands to adapt to changing communication needs and provide coverage.

The base station comprises three sectors, each positioned at 120-degree angles from the antenna to ensure service provision across the entire 360-degree coverage area. Additionally, every sector encompasses five layers, representing five distinct frequency bands. These frequency bands include 2300 carrier 1 Hz, 2300 carrier 2 Hz, 2100 Hz, 1800 Hz, and 900 Hz.

Frequency bands play a crucial role in base station communication, regulating signal ranges for efficient wireless communication. These bands, segmented by regulatory bodies, significantly affect network performance, coverage, and energy consumption. The selection of frequency bands directly impacts network performance, data rates, coverage, and signal propagation, influencing base station energy usage. In the realm of 5G, bands like 2300 carrier 1 Hz, 2300 carrier 2 Hz, 2100 Hz, 1800 Hz, and 900 Hz are available, chosen based on network traffic considerations.

This paper introduces stochastic models—the Markov and semi-Markov models—to manage frequency band selection based on system traffic. However, the dynamic reallocation of frequency bands among users occurs based on the prevailing load and service demands. The frequency bands 1800 Hz and 900 Hz will always be active to handle the load. Both transient and steady-state analyses have been conducted, deriving closed-form solutions for steady-state system size probabilities. Additionally, sensitivity analysis evaluated power consumption and other performance measures like delay and throughput across various scenarios. Comparison between analytical and simulation results reveals a strong convergence, signifying substantial agreement in estimating performance measures within the system.

ANALYSIS OF A RELIABILITY QUEUEING INVENTORY MODEL ASSOCIATED WITH ENERGY HARVESTING

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Abstract. The proposed work aims to model a communication unit in a solar powered $HAPS - BS$, High Altitude Platform full base Station, that can provide communication services even in remote or disaster stricken areas. Solar power is preferred currently over conventional energy sources as it is abundant in the stratosphere were the $HAPS$ are positioned.

Consider a single server queueing system with infinite waiting capacity, operating in a random environment. The system behavior depends on the state of the random environment, i.e., the continuous time Markov chain $E(t)$ $(CTMC)$ at time t. The arrival of customers to the queueing system is according to a Poisson process with parameter λ^i where $i = 1, 2, 3, ..., e$ are states assumed by $CTMC$ $E(t)$.The service process is exponentially distributed with parameter μ^i, where i is the state of the random environment. For providing service, inventory(energy) is required. We assume that, energy is stored in a buffer of finite capacity, M This assumption is made considering that during night time or during winters only secondary power sources like large batteries could be used for powering $HAPS$. Only one unit of inventory(energy) is required for service of a customer. The inventory(energy) is harvested from sun using solar panels, which can be thought of as a k-out-of-n : G system and stored in batteries. The k-out-of-n system is COLD; i.e., when the system fails in the absence of at least k working components, the working components do not deteriorate until system restarts with failed components replaced by new ones. The replenishment quantity

is $n-(k-1)$. The lifetime of a component is exponentially distributed with parameter γ. When the number of operational components reduces to N, $k \leq N \leq n$, an order is placed for $n-(k-1)$ components. It takes an exponentially distributed time(with parameter δ) for materialization of this order. During this time $1, 2, ..., N-(k-1)$ components may fail. The time for production of an energy unit in the random environment i is β^i. An energy unit that arrives when buffer is full is considered lost. If the buffer is empty or if the server is busy, the arriving customer joins the queue.

We derive the condition under which this system is stable and analyze the system in steady state. Various performance measures are computed. Numerical and graphical illustrations, providing insights into system characteristics are presented.

Keywords: Reliability; k-out of n system; N-Policy; HAP-BS; Breakdown

ANALYSIS OF MAP/PH/3 QUEUEING SYSTEM WITH BALKING, SERVER BREAKDOWN AND MULTIPLE VACATION

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Abstract.A Markovian queueing system with three heterogeneous servers, multiple vacation, server failure and repair along with impatient customers has been considered in this paper. The Markovian Arrival Process (MAP) is used for the customers arrival and phase type distribution are used for the service and repair time of the servers. During service time, the server may get breakdown with rate τ . All the server will take vacation with rate η, if system having empty cusomters. At the completion of vacation, each server takes another vacation until any customers reaches in the system. The system stableness and invariant probability vector of number of customers in the system was found by Quasi Birth and Death process and matrix analytic method is used to find stationary condition. Cost analysis and busy period analysis are also derived. Few system performance measures are found. By using system matrices, we represent the numerical results with graphically and numerically.

Keywords: Markovian arrival process, Phase type service, Multile vacation, Unreliable server, Balking.

ANALYSIS OF $MAP/PH/1$ QUEUEING INVENTORY SYSTEM WITH TWO TYPES OF COMMODITY, WORKING VACATION, (s, S) REPLENISHMENT POLICY AND ESSENTIAL AND OPTIONAL REPAIR

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Abstract. In this article, we examine a queueing inventory model with single server which can offer two types of inventory items namely, main item and gift item and assume that both commodities have a finite capacity. Customers reach the system by following the Markovian arrival process (MAP). The service times are considered to be phase-type (PH) distributed. We have considered no customer in the system even inventory level is positive, the server

will start the working vacation and any customer arrive during working vacation, the server provide to slow service. If item is not available, then the customer leave the system. When the server is affected by a breakdown during normal busy period, the server will move on to the repair process for rejuvenation, which is considered an essential repair. The server whereupon has the option to move on to another repair process if the server wants to go for another optional repair with some probability, or return to the service system with some probability. By utilising the matrix-analytic method, we have examined our model and investigated the stability condition of our model. We have also carried out the cost analysis for our model. At the end, some numerical is presented for clear insight into our model.

Keywords: Markovian arrival process, PH-distribution, working vacation, two types of commodity, breakdown, optional repair.

ENHANCING $HAPS - SMBS$ RELIABILITY: A COST-EFFICIENT APPROACH THROUGH RQI SYSTEM

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Abstract. This study aims to enhance the reliability of the High Altitude Platform System-Super Macro Base Station (HAPS-SMBS) with a focus on minimizing costs. We model the HAPS-SMBS as a k-out-of-n : F system (equivalently $(n-k+1)$-out-of-n : G system), with the repair of failed HAPS-SMBS commence when the number of components reaches L. The system comprises two types of units: $(n-k+1)$ active units and $(k-1)$ COLD standby. The investigation includes the perfect switching of one standby unit to an active unit when the total units in the system reach N due to failures. All distributions involved are assumed exponential, and system failure occurs when the number of operational components drops to $n-k$. Steady-state probabilities are explicitly derived using Chapman-Kolmogorov difference-differential equations. Key performance measures are computed and subjected to numerical analysis. The study further explores related distributions of interest that emerge from the system analysis. The article concludes with a discussion of an optimization problem focused on achieving minimal cost for the system.

Keywords: Reliability-queueing-inventory system, HAPS-SMBS, k-out-of-n: F system, COLD standby, Switching, System repair.

MULTI-SERVER MULTI-COMMODITY QUEUEING INVENTORY SYSTEM WITH ONE ESSENTIAL AND N OPTIONAL INVENTORIES

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Abstract. Queueing inventory theory one of the leading research areas in operational research became popular since 1992. Melikov and Molchanov [1] and Sigman and Simchi–Levi [2] were the first to research inventory with Positive service time. Berman et al. [3] investigated a non-deterministic inventory control model for service facilities. Berman and Sapna, K. P.[4] obtained an expression for optimal order quantity using an assumed cost structure. Berman and Kim[5] have discussed a paper on imposed revenue upon each service completion and pay for the service delay, replenishment setup, and inventory holding. The survey paper by Krishnamoorthy et al.[6] gives an overview of the research works related to the queueing inventory model. Queueing inventory theory has expanded in different directions. Works considering single-commodity, multiple commodities, single-server, multi-server, single-class customers, and multi-class customers in different combinations have been considered by researchers. The works listed here are relevant to the present work in the sense that these works consider multi-server and multi-commodity queueing inventory systems.

Anoop et al. [7] have analyzed a multi-server Markovian queueing inventory model with zero lead time. In this model, each inventory is a server. Rasmi et al.[8] examines a queueing inventory system with k-service stations providing service to k-class customers. Dhanya et al. [9] discuss a multi-server queueing inventory system with emergency replenishment. The emergency replenishment will be there when the on-hand inventory level decreases to zero. A paper by Jeganathan et al. [10] investigates a multi-server queueing inventory system with asynchronous server vacation. Khamis et al. [11] developed a queueing inventory model with batch-arrival, batch-service process with multiple servers. Khami's work is the first work in the literature to discuss a multi-class, multi-server queueing inventory system. Work by Srinivas and Alexander [12] introduces the concept of retrial in a multi-server queueing system.

Araya-Sassi et al. [13] consider a model in which a single plant supplies commodities to ware-houses, where they serve a set of customers. The problem we have to solve here is to find which warehouse has to open, which customers should be served by each warehouse, and which commodity should assigned to warehouses. Jaison et al.[14] discuss a single server multi-commodity queueing inventory system with one essential and m optional inventories and a randomly changing environment. They have obtained a condition of system stability and various performance measures. shajin et al. [15] analysed a multi-commodity queueing inventory system with one essential and m optional inventory. The present paper is an extension to this paper in which we consider one essential commodity and n optional commodity and one server each for each commodity. This model has many real-life applications. A data transmission network is one such example. The customers are the data that is to be transmitted. The main station where all the data reaches is the essential server, where the data are analyzed and allowed to move through the channel. Power-up points are available in between where the transmitted data is boosted up so that the data can reach the target with the required power and within time. Amusement parks are another example of this model. There is an entry fee that is essential to enter the park. The rides and other facilities available are optional. Separate servers are there for each facility provided in the park. A person can choose the facilities as the customer wishes. It will be according to the demand that the maintenance and life span of the facilities are determined.

In this model, we analyze a multi-server multi-commodity queueing inventory system with one essential and n optional inventories. The replenishment policy is (s, S) and $(s_i, S_i), i = 1, 2, ..., n$ for essential and optional inventories respectively. The customer arrives in the system according to the Markovian arrival process, represented by $(D_0 D_1)$ of order m_1. The service time of the essential commodity follows phase-type distribution represented by (γT) of order m2 and the optional commodity follows exponentially distributed with parameter

$\beta_i, i = 1, 2, ..., n$. After the service of the essential commodity, the customer leaves the system with probability p or goes for the optional items with probability $1 - p$. Each customer demands exactly one essential item and more than one type of optional item with a maximum of one unit from each optional inventory. Only forward movement is allowed in the system. If a customer leaves any one of the optional inventory, the customer can't choose it again. The ith optional item demanded with probability p_i, ith and jth with probability p_{ij}, ith, jth and kth with probability p_{ijk}, going like this all the optional inventories together will be demanded with probability $p_{1,2,...,n}$. If the demanded optional items are unavailable, the customer leaves the system with the essential item and whatever optional items are available to the customer. The buffer size of ith optional item is n_i. The customer entering into an optional inventory area can choose an optional inventory or leave the system with the essential inventory. A customer who chooses an optional inventory can get the service immediately when the server is available to serve. If the server is busy the customer can enter into buffer if there is space in the buffer. The objective is to formulate different system performance measures, stability conditions, steady-state probability vectors, construction of a cost function and find the minimum expected cost. We use the Metric analytic method as the main tool in this work.

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Keywords: Multi-server, Multi-commodity, Optional inventory, Phase type distribution

AN M/M/2 HETEROGENEOUS QUEUE WITH MODIFIED MULTIPLE VACATION UNDER N-POLICY

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Abstract. In this paper we investigate a queueing system with two heterogeneous servers, one of which is always available and the other takes vacation under N-policy. The system is analyzed in the steady state using matrix geometric method and various performance measures are derived. A numerical illustrations are also provided.

ANALYSIS OF REPAIRABLE TWO PHASE TANDEM QUEUING MODEL WITH BACKUP SERVER AND CUSTOMER BALKING

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Abstract. In this paper, we address a two phase tandem queueing model, incorporating a backup server and customer balking. The first phase accommodates an infinite number of customers, while the second phase has a finite capacity. The system is subject to failure,

and incoming customers may opt to balk with a certain probability. Employing the matrix-geometric method, we conduct a steady-state analysis of the model. The study explores various performance measures such as steady-state probabilities, expected system size, cost analysis, presenting numerical and graphical examples for illustration. Additionally, we validate the efficacy of the proposed model by applying it to a real-world scenario involving packet transmission processes.

Keywords: Two service phases, Tandem queueing model, Server failure, backup server, Customer balking.

HORIZONTAL DIVIDEND RESTRICTION ON A COMPOUND POISSON RISK PROCESS WITH COMPOUND POISSON PREMIUMS AND DELAYED BY-CLAIMS

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Abstract. The objective of this paper is to introduce the delayed by-claims in a random income risk model with a horizontal barrier. Both the inter-premium and inter-claim arrival times of the risk model follow iid. exponential distribution, independent of the premium and claim size. The delayed by-claim strategy is setup such that the main claims induce the by-claims and they are settled together with probability θ. On the other side, the by-claim settlement is plausibly delayed with a probability $(1-\theta)$ The amounts of premiums, main claims and by-claims follow iid exponential and all these amounts are mutually independent. We develop a system of Volterra integro-differential equations (I-DEs) for the Gerber-Shiu discounted penalty function of risk model. We use the methodology of Laplace transforms to solve the I-DEs. Explicit expressions are obtained in terms of GSF in delayed time line. Finally, a numerical analysis illustrates the tractability and sensitivity of the expressions towards the suitable parametric values.

Keywords: Random income risk model, Delayed by-claims, Horizontal dividend barrier, Gerber-Shiu function, Volterra integro-differential equations.

ON A QUEUEING-INVENTORY PROBLEM WITH POSITIVE LEAD TIME IN TRANSPORT OF PASSENGERS

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Abstract. Consider a single server queueing-inventory system with a storage system having S items at the beginning of a cycle. Customers arrive according to the Poisson arrival process with rate λ. The system provides overbooking. It is a common feature in reservation of flight tickets/train tickets. We set an upper bound of V for overbooking. The inventoried items have a common life time which means that they all perish together on realization of common life time. The distribution of the duration of this time is $Erlang(K,\eta)$,that is, the common

life time progresses as $K \to K-1 \to ... \to 2 \to 1$.Each customer demands exactly one item from the inventory. Service time of customers is exponentially distributed with parameter depending on the stage of common life time; that is, the service time parameter while common life time is in stage j is μ_j for $1 \leq j \leq K$ with $\mu_1 > \mu_2 > ... > \mu_K$. Reservation of items and cancellation of sold items before the common life time realization is permitted. Cancellation takes place according to an exponentially distributed inter-occurrence time with parameter $i\theta$, when $(S-i)$ items are present inthe inventory $-V \leq i \leq S$. On realization of common life time the inventory level hits zero since the life time of all stored items expire and the customers waiting in the system stay back in it. Due to common life time realization, the inventory level drops to 0 (denoted by 0^* for identification purpose). The next schedule is announced on the realization of lead time which is exponentially distributed with parameter depending on the number of overbooking and the number of sold items on the previous schedule at the time of common life time realization epoch. In the present we consider 4 cases:

1. On realization of common life time, if the number of served items is less than $S/4$, then the schedule is cancelled; consequently these customers are provided that many items of the next schedule. The lead time follows exponential distribution with parameter β_1.
2. On realization of common life time, overbooked customers (if $>S/2$). The next schedule is announced on the realization of lead time and inventory level reaches its maximum S; consequently overbooked customers are immediately served with these new items. In this case lead time follows exponential distribution with parameter β_2.
3. On realization of common life time, overbooked customers (if $\geq S/2$), all the customers are served at the time of realization of common life time. Here the lead time follows exponential distribution with parameter β_3.
4. On realization of common life time, if the number of served items is $\geq S/4$, then the lead time follows exponential distribution with parameter β_4.

with $\beta_1 > \beta_2 > \beta_3 > \beta_4$.
We derive the stability condition for the system and analysis the system steady state distribution. From these we derive expressions for computing performance of the system. For this case an appropriate cost function is constructed and its properties investigated numerically.

Keyword: Overbooking, Lead time, Cancellation, Common life time, Queueing-inventory system.

A TWO-COMMODITY PERISHABLE INVENTORY SYSTEM WITH RANDOM COM MON LIFETIME, MARKOVIAN DEMAND ARRIVALS, AND N-POLICY

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Abstract. This article examines a perishable inventory system that deals with two commodities and has random common lifetimes, commodity-wise. We take into account two types of demands. Demands for commodity-1 follow a Markovian arrival process with the representation (D_0, D_1) of order n. Demands for commodity-2 follow a Poisson process. Commodity-1 is managed using an (s, S) type inventory control policy, and it experiences

a positive random lead time characterized by a phase type distribution. Additionally, an N policy is applied to manage inventory during the lead time. We consider the N-policy for commodity-1 in the following way. Whenever the backlogg of commodity-1 reaches a level N, a local purchase of $S + N$ items will be purchased by cancelling the pending order. Cost of local purchase will be large compared with the normal purchase. There is no lead time for commodity-2 and follow $(0, S)$ policy. The random common lifetimes of the two commodities are independent and follow exponential distributions. A four dimensional Markov process is used to model the system. Steady state system size distributions and performance measures, such as average inventory levels, the perishable rate of commodities, commodity reorder rates, etc., are calculated. A cost function is constructed based on the performance measures, and its convexity is examined numerically. Also, we consider that the lead time for commodity-1 follows an exponential distribution, characterized as a special case.

Keywords: local purchase, N-policy, inventory, two commodity, common lifetime, Markovian arrival process.

ANALYSIS OF A QUEUE SUBJECT TO DISCOURAGED ARRIVALS, CUSTOMER IMPATIENCE, AND SELF-SWITCHING SERVER DYNAMICS

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Abstract. This paper analyses a queue with a single self-switching server subject to discouraged arrivals and impatience of customers. Customers arrive with rate $\lambda/(n+1)$, where λ a positive constant is and n is the number of customers in the system at the epoch of arrivals. After joining the queue, customers will become impatient, and they will abandon the system with rate ζ. The server renders service with rate μ_1 until the size of the system is lesser than K and immediately the system size reaches K, the server switches to a fast rate of service μ_2. For this queueing system, the steady-state probability distribution is derived, and some performance measures such as the expected number of customers in the system or queue and the expected waiting time in the system or queue are derived. Numerical illustrations are provided.

Keywords: Single Server queue, Discouraged arrivals, Server switch, Customers impatience.

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APPLICATIONS OF WEIBULL LOMAX DISTRIBUTION: REVIEW

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Abstract. In this paper, we reviewed the various works by different authors on Weibull

Lomax distribution. The historical research based on some classical papers as well as ongoing research and advancements on Weibull Lomax distribution and its applications based on some latest papers were thoroughly reviewed and discussed in this paper. We mainly focused on various extensions of Weibull and Lomax models that have been used to model lifetime data with a bathtub shaped hazard function. We also gave importance to study about the wide applications of Weibull Lomax distribution in various fields including survival and lifetime data.

Keywords: Weibull distribution, Lomax distribution, Weibull Lomax distribution, Monotone hazard rate, Bathtub hazard rate, Lifetime data.

ON A MULTI-SERVER K-OUT-OF-N : F SYSTEM WITH N POLICY OF REPAIR

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Abstract. In this paper we analyse a multi-server k-out-of-n: F system with N policy of repair. We consider a multi-server queuing system with n identical servers operating in parallel. Customers arrive to the system according to a Poisson process with parameter λ. Service time of each customer is exponentially distributed with parameter μ. The servers are subjected to breakdown, which occurs at exponentially distributed time intervals with rate γ. As we are considering the k-out-n: F system, failure of any k servers results in the failure of the whole system. Here we also assume that the system is COLD; when the system fails due to the failure of any k servers, the servers that are still operational do not deteriorate until the system become operational. The repair process is according to the N policy. A repair facility starts repairing failed servers one by one when the number of failed servers reaches the threshold $N(1 \leq N < k)$. Repair process continues until all the failed servers are repaired. We assume that the time taken to repair each server is exponentially distributed with parameter δ. The mathematical formulation of the system is as follows. Let $X(t), t \geq 0$ denotes the number of customers in the system at time t and $J(t)$ denotes the number of failed severs in the system at time t. Let $S(t)$ denote the status of repair process at time t.

$$S(t) = \begin{cases} 0 \text{ when repair is off} \\ 1 \text{ when repair is on} \end{cases}$$

Then $(X(t), J(t), S(t)) : t \geq 0$ defines a three-dimensional Continuous Time Markov Chain (CTMC) with state space:

$$\Omega = \{(i, j, 0) : i \geq 0, 0 \leq j \leq N-1\} \cup \{(i, j, 1) : i \geq 0, 1 \leq j \leq k\}$$

Using the lexicographical sequence for the states, the infinitesimal generator matrix Q of

this process is of the form,

$$Q = \begin{pmatrix} Q_1^0 & Q_0 & & & & & & \\ Q_2^1 & Q_1^1 & Q_0 & & & & & \\ & Q_2^2 & Q_1^2 & Q_0 & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & Q_2^{n-1} & Q_1^{n-1} & Q_0 & & \\ & & & & Q_2 & Q_1 & Q_0 & \\ & & & & & \ddots & \ddots & \ddots \end{pmatrix}$$

The structure of Q indicates that it can be studied as a Level Independent Quasi- Birth-Death (LIQBD) process. Matrix Geometric Method is used to obtain the stationary distribution of the queuing process. With these distributions we compute performance mea- sures of the system and analyse it numerically. A suitable cost function is developed and optimal value of N is found numerically.

Keywords: Reliability, k-out-of-n: F system, N policy, breakdown, repair, Matrix Geometric Method, cost analysis.

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Keywords: Multi-server, Multi-commodity, Optional inventory, Phase type distribution

AN $M/M/1$ QUEUEING MODEL WITH DISASTERS AND AN UNRELIABLE REPAIRER

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Abstract.A single server Markovian queueing paradigm is studied. The server is prone to disaster when delivering service or during idle periods. When the server encountered problems, all waiting customers will be flushed away and the repair procedure began immediately. Arrivals are allowed to join the queue during the repair time. The likelihood of restoring the server is $p(p < 1)$ after an exponential repair time. All waiting users leave the system if the server cannot be fixed. The time-dependent likelihood for a single server queue with disaster and an unreliable repairer is explicitly estimated for the first time. The resulting results were also cross-checked with the available literature. To demonstrate the significance of parameter values, performance measures and numerical figures were also supplied.

Keywords: Random Breakdowns; Unreliable Repairer; Time dependent Probabilities; Idle Period; Abandon.

KUMARASWAMY HARRIS GENERALIZED FAMILY OF DISTRIBUTIONS AND ITS APPLICATIONS

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Abstract. In this paper, we introduced a new family of distributions based on the Harris Extended Family of distributions introduced by Aly and Benkherouf (2011) and Kumaraswamy G Family of distributions introduced by Cordeiro and de Castro (2011). The new fam- ily is called Kumaraswamy Harris Generalized family of distributions and is denoted by 'KwHG'. We explore the statistical properties such as probability density function (pdf), hazard rate function (hrf), expressions for cumulative distribution function (cdf), quantiles, survival function and Mill's Ratio, mixture representation, Rényi entropy, Order statistics, Record values and its shape properties. Method of maximum likelihood estimation is used for estimation of unknown parameters of the new distribution. Different special models which include Uniform, Exponential, Weibull, Kumaraswamy models are developed for this new family. The probability density function (pdf), cumulative dis- tribution function (cdf) and hazard rate function (hrf) are obtained and shape property is considered for the new models. Monte Carlo Simulation study is conducted based on Kumaraswamy Harris Generalized Kumaraswamy distribution (KwHGKw). KwHGKw distribution is applied to the real data set on financial modeling and survival analysis to show the effectiveness of the new model. Based on the analysis, it is concluded that the Kumaraswamy Harris Generalized Kumaraswamy distribution (KwHGKw) is a better model for the given data sets than the Beta Kumaraswamy Weibull distribution (BKwW). This distribution is also applied to a

real data set on Reliability analysis and verified that the new distribution is a better model than the other five models.

Keywords: Kumaraswamy distribution, Harris Extended distribution, Beta Kumaraswamy Weibull distribution, survival analysis, financial modeling.

ON A PRODUCTION INVENTORY WITH RETRIAL AND SELF-GENERATION OF PRIORITIES OF CUSTOMERS

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Abstract. Consider a single server production inventory system in which customer arrives according to a Markovian arrival process. If the arriving customer finds the server busy or the inventory level zero moves to an orbit of infinite capacity with a certain probability or exits the system with complementary probability. Customers in the orbit independently retry to access the free server at exponentially distributed time intervals. A retrial customer returns to the orbit with a certain probability if the server is busy or the inventory is dry or exits the system with complementary probability. Also, each customer in orbit, independently of others, generates priority with exponentially distributed inter-occurrence time and a priority-generated customer will get immediate service if the server is free. Otherwise, such a customer has to wait in the space of capacity one, which is reserved only for priority-generated customers. A second priority-generated customer is forced to leave the system and search for emergency service elsewhere if the previously generated priority customer is waiting. The service times of ordinary and priority-generated customers follow phase-type distributions with appropriate representations. The production process is governed by (s, S) policy, and the time between two successive addition of items to the inventory is exponentially distributed. The model is a level-dependent quasi-birth-death (LDQBD) process, and the Matrix-Analytic Method is used to obtain a solution.

Keywords: Markovian Arrival Process, Phase type distribution, Self-generation of priorities, Retrial queues, Matrix Analytic Method.

UNRELIABLE THREE PHASE BATCH ARRIVAL RETRIAL QUEUEING SYSTEM WITH CUSTOMER IMPATIENCE, FEEDBACK, BERNOULLI VACATION AND SEARCH OF CUSTOMERS

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Abstract. In this article, an unreliable batch arrival retrial queueing system with three phases of service, customer impatience, feedback, Bernoulli vacation and search of customers are analyzed. Customers arrive according to Poisson process and the server provides service in accordance with first in, first out. If the server is free, then one of the customers from the arriving batch join the service and others join the retrial group. Impatient customers balk

and renege at the system. The server breakdown in all three phases of service and repair starts immediately. After completion of repair, the server continues to serve the interrupted customer in the system. After completing each phases of service, the customer either opts for optional service or joins the retrial group as a feedback customer or leaves the system. Upon completion of first essential service and second optional service, the server either searches for customer in the retrial group with certain probability or remains idle. After completion of third optional service, the server goes for a vacation with certain probability. The retrial times, service times, repair times and vacation times are arbitrarily distributed. Performance measures such as the mean system size, retrial group size, expected size of retrial group during various server states like when the server idle, busy in all three phases of service, under repair in all three services, the server is on vacation and reliability measures are derived using supplementary variable technique.

Keywords: Batch arrival, feedback, repair, balking, reneging, vacation, orbital search.

ANALYSIS OF SINGLE SERVER QUEUEING SYSTEM WITH DIFFERENTIATED VACATIONS AND DIFFERENTIATED BREAKDOWNS

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Abstract. This research work considers a single server queuing model with differentiated vacations. In addition there is a possibility of two types of failures when the server is in a busy period; namely hard and soft failure. In the time of hard failure server may work with a slow service rate. The steady state probabilities are derived and the numerical analysis is given to validate our theoretical results. Cost analysis is also given to the proposed model.

Keywords: Single server, Differentiated breakdown, Differentiated vacation, Cost analysis
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A SINGLE PRODUCT INVENTORY SYSTEM WITH CONSTRAINED PROBABILISTIC MISCLASSIFICATION IN SCREENING PROCESSES AND RE-DO FACILITY WITH CUSTOMER WARRANTY POLICY

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Abstract.A single product EOQ inventory model is initiated in this article by incorporating the re-do process for the identified screened items in the existing model done by [Mehmood Khan et al. (2011)]. Initially, the inventory starts with a finite number of items. The screening process is initiated immediately after each replenishment and demands are satisfied simultaneously. The level of inventory has been declining up to a certain time point due to both screening and the demand. No perfection is guaranteed even if we employ 100 % screening on the items which are arrived to the inventory due to human error or error in the

screening missionaries which are employed for testing processes or electricity fluctuations or some natural disasters. As a result, errors may occur with some probability. It may cause in two ways either some of the good item may be wrongly identified with some probability as a defective item or some of the defective items may wrongly identified with the complimentary probability as a good item. It is assumed that the misclassification error probabilities are known. Therefore, it is assumed that the statistical type-I error occur by identifying good item as bad and type-II error occur by identifying bad item as good in screening process. Inspection error develops an increase in total system cycle cost and leads to the loss of customer's goodwill. All identified bad items by screening process are sent for re-do process as a single lot after screening. The re-do facility is not guaranteed in put right all the defective items as good. A fraction of items from the lot is made perfect and sent back to the inventory as perfect items in the end of re-do process within the inventory cycle. The retailer grants a warranty period to the customers to return back their purchased item to the inventory once they found it is defective due to the statistical error in screening. It is assumed that the returning of defective items by the customers after their purchase to the inventory only after completion of the screening process. Items are replenished instantaneously once the items in the inventory become zero. In this scenario, it is arrived the explicit expression for the expected total system cost. The model is illustrated with the numerical example with suitable parameter values. Effect of the re-do rate, screening rate, demand rate, and various costs on total cost is sensitively analysed to recommend the retailers the spectrum of their feasibility in taking their decision in the above said environment in which they run their business.

Keywords: Instantaneous Replenishment: Screening process; Re-do process; Warranty Period; Statistical Errors.

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ANALYSUS OF BULK QUEUEING SYSTEM WITH STATE DEPENDENT WORKING VACATION

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Abstract. In this paper bulk arrival queueing system with state dependent working vacation is considered. Server provides service in two service modes as single service and batch service depending on queue length. Service process will be initiated only if the queue length is at least 'a'. In the service completion if the queue length is less than 'a' then the server leaves for working vacation. In the period of working vacation customers are served in lower service rate than the regular service rate with two service modes. If the service process during working vacation ends prior to the working vacation period then the server leaves for non-working vacation. On the other hand if the service process exceeds the working vacation period then the slow service rate will be changed to regular service rate. For the proposed model probability generating function of the queue size at an arbitrary time will be obtained by using supplementary variable technique. Various performance characteristics are also be derived with suitable numerical example. Additionally optimum cost analysis is also established.

Customers are arriving into the system in bulk according to Poisson arrival rate. Server provides service in two service modes as single service and batch service will depending upon the queue length. Service will be initiated only if the queue length is at least 'a'. During single service customers are served individually, whereas batch service customers are served in batches according to general bulk service rule introduced by Neuts. Let ϵ be a queue length. The server provides single service if the queue length is $c < \epsilon \geq a$, where $c > a$. Batch service is possible only if $b \geq \epsilon \geq c$. According to general bulk service rule, on service completion epoch if $b \geq \epsilon \geq c$ then entire batch taken for service. On the other hand if $\epsilon \geq b$ then service will be provided for only 'b' customers, remaining $\epsilon - b$ customers as to wait in the queue for next batch of service. Either single service completion or batch service completion,

depending upon the queue length the server will change the service mode. Change of service mode is possible only at service initiation epoch. Upon service completion if the queue length is less than 'a' then the server leaves for working vacation. In vacation queueing models the server is not providing service. But during working vacation period customers are served in lower service rate than the regular service rate in two service modes. If the slow service ends prior to the working vacation period then the server remains idle until the working vacation period ends. On the other hand if slow service exceeds the working vacation period than the slow service rate will be changed in to regular service rate. On completion of working vacation if the queue size is still zero then the server leaves for non-working vacation.

EXPLORING CONSECUTIVE k-OUT-OF-n : G SYSTEMS WITHIN THE FRAMEWORK OF A RQI MODEL

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Abstract. We consider a Reliability-Queueing-Inventory (RQI) system with two reli ability models, namely linear and circular consecutive k-out-of-n : G systems. The study incorporates the labelling and relabeling of system components. The investigation considers the repair of failed components and the replenishment with $(n-k+1)$ units. The repair initiates when the number of working components drops to L due to failure, and a replenishment order is placed when the component count reaches N. The failures/repair/lead times follow an exponential distribution. Steady-state probabilities are explicitly derived. The key performance measures are computed and analyzed numerically, facilitating a comparative study to identify the most reliable system. Additionally, we derive some relevant distributions for both systems. The research concludes by discussing an optimization problem associated with both models. This work contributes to the field of reliability engineering and presents valuable insights for decision-making in system design and maintenance.

Keywords: Reliability-queueing-inventory system, COLD system, Linear and circular-consecutive k-out-of-n: G system, System repair, Lead time.

CONTRACTIVE MAPS IN THE HYPERSPACE OF MENGER PROBABLILSTIC METRIC SPACE

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Abstract. In this paper we consider probabilistic metric space in compact sets. Here we define Menger probabilistic metric space in the collection of all compact sets, which is the space of fractals. Fixed point theorem in a chainable Menger probabilistic metric space is proved in the paper titled 'Extension of contractive maps in the Menger probabilis tic metric space' by Abdolrahman Razani and Kaveh Fouladgar in 2006. Here we proved contractive maps in the hyperspace of Menger probabilistic metric space.Existence and uniqueness of fixed point in the complete chainable probabilistic metric space is also proved in this paper.

OPTICAL BURST SWITCHING NETWORKS WITH COMBINED RETRANSMISSION AND DEFLECTION ROUTING WITH PRIORITY AND SERVER FAILURE

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Abstract. The best switching technology which combines the benefits of Optical Circuit Switching (OCS) and Optical Packet Switching (OPS) is Optical Burst Switching (OBS) technology. Optical bursts can be transmitted end-to-end without leaving the optical domain, which is advantageous in terms of energy consumption and processing delay. Burst contention is the major challenging problem related with OBS networks when compared to other factors. Various proactive and reactive methods like buffering, retransmission, deflection routing, wavelength conversion and segmentation are available to resolve the contention. Of the important factors, most of the studies focussed on server failures. Server failure do not destroy the system; instead, all the bursts must wait for the wavelength to be activated again. In this study a combined retransmission and deflection routing method is used to increase the number of bursts being processed in the system. In this paper pre emptive priority, feedback and buffer search are also considered and numerical results are developed to enhance the performance of the system.The proposed algorithm is simulated in MATLAB. The main application of OBS network is at the backbone of Internet due to which the usage of Internet is easy and faster.The simulation results show that the proposed method is more efficient than the traditional OBS networks.

Key Words: Optical Burst Switching, Retransmission, Burst priority, Buffer Search, Failure.

ANALYZING THE ACADEMIC TRAJECTORY: A CASE STUDY OF COMMERCE UNDERGRADUATES UTILIZING MARKOV CHAIN MODELING FOR ASSESSMENT AND PERFORMANCE EVALUATION

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Abstract. Academic shaping of learners during schooling and college environment plays the pivotal role of quality education in a nation's progress and development. Specifically focusing on higher education, the study delves into the importance of accurate enrolment forecasting for universities. This forecasting not only aids in shaping educational frameworks and budgeting but also facilitates the provision of necessary facilities, aligning with both short- and long-term goals. The research is centered on analyzing patterns in student assessment and academic performance within the commerce stream of an Autonomous college affiliated with the University of Mumbai. The target population encompasses all undergraduate enrollments spanning from the academic year 2020 to 2023. To achieve this, a Markov Chain model is employed to scrutinize the absorption, retention, and repetition rates of students,

taking into account academic programs and gender as variables. The construction of a fundamental matrix plays a crucial role in determining the expected duration of a student's college journey before graduation. Additionally, the research endeavours to estimate enrolment projections, shedding light on the probability of students' performances in the long run. In essence, this study intricately explores the application of a Markov chain model to elucidate the stochastic nature of student enrolment and assessment, providing **valuable** insights into the dynamics of educational progress.

Keywords: Enrollment forecasting, Budgeting, Student assessment, Markov Chain model, Absorption rates, Retention rates, Repetitive rates, Probability analysis, Stochastic pattern.

COMPARATIVE RATING OF DUAL-CLASS RETRIAL ORBIT WITH THE FEATURE OF RESERVATION AND GUARD BANDWIDTH POLICY

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Abstract. We analyze a novel two-station retrial system with egalitarian processor sharing to model the fluidity (mobility) of users among switching over networks to transfer elastic calls managed by two service providers. The work focuses on a small region containing a single access point (AP) per provider which admits a limited number of users, say, handover users, fresh users, and switch users. We extend the queueing model with comparative ratings among the users to check the quality of service and age of information via rate of information. Thus, in each AP, the service rate depends on the type and the number of users and on the state of the random environment, whereas a reservation policy to prioritize the handover users is employed. Both APs cooperate by allowing the seamless transition of an ongoing file transfer to the fastest service provider. The stationary distribution of the number of users of each type in each AP, and the comparative analysis with the validation of result using numerical illustration.

Keywords: Retrial queue, Comparative ratings, Reservation policy, Switch users, Handover user, Fresh users, Stationary distribution.

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FINITE CAPACITY QUEUES WITH ENERGY REQUIRED SERVICE AND GENERAL VACATION TIMES

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Abstract. We consider an ordinary $M/M/1$ queue with finite waiting line capacity in which each customer needs a random amount of fluid for its service. Like the ordinary fluid flow model, fluid inflow is governed by a Markov process and the server consumes fluid at a fixed rate during the service time. At each time the fluid buffer become empty, the server wait for a random amount of time to re-start the customer service. The analytical framework resem bles fluid vacation models with exhaustive discipline, in which the fluid is removed at a constant rate during the service period. We adapt the level crossing method used in fluid vacation models with exhaustive discipline for finding the stationary analysis of fluid level. The Laplace transform of the steady-state vector and the mean fluid level at an arbitrary epoch are de rived. Additionally, we computed performance measures related to queue size. Finally, we presented the computability of the analytical results using some examples.

ON AN INFINITE SERVER QUEUE BOILING DOWN TO A FINITE SERVER QUEUE

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Abstract. Classical infinite server problems are characterised by a very large number of servers coupled with customer arrivals. Each customer starts getting service the moment he joins the system. This type of model includes M/M/infinity; M/G/infinity and several variants of them. An interesting variant of such queues is considered by Liu, Templeton, and Kashyap, where customers arrive at a system with a very large number of servers. Some customers prefer to be served in singles. On arrival, they go to an idle server and start getting service. On the other hand, a few others may not be able to afford to have a single service; they wait in a queue. When the number of customers in that queue reaches the value R, they together take service. When R = 1, we have the classical infinite server queue. In infinite server queues, the moment a customer gets his service completed, leaves the system. However, we see several real-life scenarios in which the customers who complete the service in the infinite server part, join a waiting line of infinite capacity. Nevertheless, these customers are served by a finite number of servers. Therefore they are forced to wait. A typical example is a mall where a large number of customers can enter, and take service by picking up items they wish to purchase. This is stage 1 of the process which is the infinite

server part. Those who complete the service, proceed for billing of the purchased items. Upon the billing process of each one getting completed, the customers pay for the items purchased and leave the system. So departure rate is much slower than that at stage 1. The purpose of this paper is to analyse such systems completely. Here we restrict ourselves to simple assumptions such as Poisson arrival to the infinite server stage 1. The service time of all customers has the same exponential distribution and distinct services are independent. In stage 2, customer service times also follow an exponential distribution. The queue discipline here is first in, first taken for service. The system size distribution will be completed which, hopefully, is in closed form. Then several performance metrics of the system will be derived. Finally, an optimization problem to minimize the waiting time of customers in stage 2, is analysed. Here the objective is to find the optimal number of servers to be deployed to minimize the expected cost per unit time of the system subject to an upper threshold on the waiting time of customers.

Keywords: Queueing Theory, Infinite Server Queues, Finite Server Queues, Performance Evaluation.

ANALYSIS OF NON-MARKOVIAN BATCH ARRIVAL RETRIAL QUEUE WITH PRIORITY SERVICES, DISCOURAGEMENT, RESERVICE, DIFFERENTIATE BREAKDOWN, RESTORATION, DELAYED VACATION

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Abstract. In this paper, We consider Non-Markovian batch arrival retrial queue with priority services, discouragement, re-service, differentiate breakdown, resto
ration, delayed vacation. Priority and ordinary customers arrive according to Poisson processes, and their service time based on the general distribution. The server constantly offers a single service for both priority and ordinary customers. If the server is unavailable, ordinary customer may balk the system. Server failure may happen at any time during normal busy period. The two types of system failure are hard and soft failures. Hard failure can be characterised as an equipment breakdown which demands the availability of a skilled repair person, which is an extensive process. Soft failure is described as breakdown based on by circumstances as instead of mechanical components, and it can be generally resolved by restarting the system. Customer may re-enter the system as a feedback customer for receiving normal service due to inadequate quality of service after every priority service is completed. The server goes on vacation after priority services are completed; the time it takes the server to go on vacation is known as delay time. We compute the Laplace transforms of the time-dependent probabilities of system states using the probability generating function and supplementary variable technique. Numerical results are obtained which are also examined to facilitate the sensitivity analysis of system descriptions.

Keywords: Batch Arrivals; Priority Queues; Discouragement; Re-service; Differentiate Breakdown; Restoration.

PERFORMANCE ANALYSIS OF WIRELESS SENSOR NETWORKS – A QUEUEING THEORETIC APPROACH

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Abstract. Across the globe, calamities occur frequently, both naturally and man-made, where humans and wildlife have to face tremendous impacts. The damages can be reduced by environmental monitoring with the Wireless Sensor Network (WSN), which gathers, processes, and transmits physical occurrences using the Internet of Things (IoT). The primary obstacle is the failure of the vital disaster management sensors, which can be reduced without completely stopping the system by scheduling a repair window and a backup sensor.

MODELING AND ANALYSIS OF DYNAMIC SPECTRUM ACCESS SCHEME IN COGNITIVE RADIO NETWORK UNDER IMPERFECT SENSING

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Abstract. The explosive growth of wireless services and applications over the past few years has resulted in a massive surge in the demand for radio spectrum. Radio spectrum is in short supply as a result of the widespread use of wireless devices and applications. Over the time, these technologies have improved and progressed, making them more useful for a variety of applications in fields such as the military, finance, healthcare, social inter action, mobile communications, Internet of Things (IoT) devices, data communication, video conferencing, and intelligent transportation systems. In today's wireless networks, efficient spectrum usage has become essential due to spectrum scarcity and the growing demand for internet services and applications with faster data rates. Furthermore, a large portion of the spectrum remains underutilized. Cognitive Radio Network (CRN) has been proposed as a promising solution to fulfill the demand of increasing radio spectrum. In CRN, primary users (PUs) or licensed users have priority over secondary users (SUs) or unlicensed users while accessing and idle channel. SUs can use the channel opportunistically i.e. whenever PUs are not using the channel. While using the channel opportunistically, SUs perform spectrum sensing to find the idle channels. Two kinds of errors are prone to occur in spectrum sensing (i) miss-detection and (ii) false alarm. Miss-detection is the error which occur when a newly arriving SU senses the channel occupied by PU as idle channel and starts using that channel and this kind of error results in collision between PU and SU which leads to an increase in dropping probability. False alarm occurs when a newly arriving SU senses the idle channel as busy channel and do not use that channel due to sensing errors and this leads to wastage of available resources. The SU flows are divided into two categories: non-real-time (NRSU) and real time (RSU) as per their delay requirements. RSUs are delay sensitive while NRSUs are delay tolerant. In order to develop an analytical model, sensing errors must be taken into consideration. Without considering the effect of sensing errors, the performance of the

system will be overestimated. The system is modeled using Continuous-time Markov chain (CTMC) by considering the heterogeneity of SUs under the effect of imperfect sensing. A four-dimensional CTMC model is used in the development of the analytical model. To analyze the performance of the system, several performance measures like channel availability, dropping probability, forced termination probability, retainability and spectrum utilization are calculated and the obtained results are compared with perfect sensing to show the influence of errors on the performance of the system. Numerical results are plotted under various traffic loads to show the efficacy of the proposed model.

Keywords: Cognitive radio network, spectrum sensing, quality of service, Markov chain.

A STUDY OF PACKET TRANSMISSION DELAY IN SOFTWARE DEFINED NETWORKS

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Abstract. Software-Defined Networking (SDN) revolutionizes network management through dynamic configuration and enhanced performance. Controllers, acting as the net work's brain, guide switches on packet routing using the OpenFlow protocol. Exam ining the switch-controller interaction reveals two modes: reactive and proactive. In reactive mode, the controller instructs switches on packet forwarding, while proactive mode allows independent rule changes over time, ensuring predictable traffic flow and congestion avoidance. Existing research highlights challenges in representing real-time SDN traffic within probability distributions, impacting re sult reliability. This study addresses the limited exploration of SDN potential, emphasizing the need for tailored methodologies due to the unique nature of SDNs. Delays in packet transmission is a crucial parameter for network efficiency, involve switch processing, controller interaction, and packet return. Analyzing these de lays provides insights into network performance improvement. Our study aims to simulate the SDN to predict average packet delays under varying probabilities of new incoming packets. This simulation model accommodates the recurring patterns in packet arrival times, offering valuable insights for optimizing network efficiency amid structural changes. This research provides a foundation for understanding and enhancing SDN performance in dynamic environments. In our investigation of SDN performance, we have designed a simulation frame work using cost-effective and easily deployable software-based tools. While recog nizing the potential for processing overhead due to reliance on the system OS, these tools offer practical advantages in terms of ease of setup and affordability. For traffic generation and analysis, we employ Scapy as a versatile tool, capable of simulating various mathematical distributions and serving dual roles in packet generation and capturing. The network environment is simulated using Mininet, a powerful emulator that allows us to create a virtual network with customizable parameters such as latencies and bandwidths, replicating realistic network topologies. As the central decision-making entity, Ryu is selected as the controller, offering flexibility in application development and effective management of the simulated network. This chosen setup provides a reliable and cost-efficient platform for a comprehensive exploration of SDN performance under diverse traffic scenarios, paving the way for valuable insights into the dynamic behavior of software-defined network. The average delays were calculated for different packet sizes and different prob abilities of an unknown packet arriving at the network. The delay keeps getting higher as the probability of new packets entering the network increases.

Even though there is no clearly observable pattern when it comes to comparing delays for packets with different sizes within the same probability, it can be seen that the packet delay in general increases as more new packets enter the system. This is because while known packets can be forwarded directly to the desired destination, unknown packets need to be examined by the controller to determine how they are to be forwarded by a switch. The plot of average delay against new packet probability revealed a linear relationship. Utilizing observed delays, a concise mathematical equation was formulated to quantitatively express the connection between average delay and the probability of new packet arrivals. This equation serves as a predictive tool, offering insights into how changes in probability impact average delay within the SDN environment. This concise model enhances our ability to forecast and optimize SDN performance under diverse traffic scenarios.

Keywords: Software-Defined Network, Average packet transfer delay, Simulation, Mininet, Ryu controller.

CHARACTERISATION OF KRISHSUPRA-P DISTRIBUTION AND ITS APPLICATION

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Abstract. Here an innovative generalization of Sauleh distribution is considered and named it as KrishSupra-P Distribution (KSPD). Its several statistical properties were discussed and the model parameters are predicted using the Maximum Likelihood Estimation (MLE) approach. The model's biomedical application using a real data set is discussed and contrasted with some other famous distributions.

Keywords: Weighted distribution, Sauleh distribution, Order statistics, Survival analysis, estimates.

ON MAP/G ^{b}/1/N QUEUE WITH BATCH SIZE DEPENDENT SECOND OPTIONAL SERVICE

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Abstract. The current article is about a finite buffer, fixed batch size bulk service queue, in which a single server renders two type of services, first essential service (FES) and second optional service (SoS).

- Customers arrive at the system following Markovian Arrival Process (MAP) which is an 'm' state Markov chain. The MAP is described by two parameters (square) matrices, D_0 and D_1, of dimension m, where D_0 denotes no arrival of customer in the system and D_1 denotes customers arrival in the system. The transitions are governed

by irreducible generator $D = D_0 + D_1$, i.e., if δ is the stationary vector then, $\delta D = 0$ and $\delta e = l$, where e stands for a column vector of 1's of dimension m. (For more detail on MAP see the article [1], [2])

- The FES is compulsory for all the joining customers to the system and is rendered by a server in groups of size b on first come first serve (FCFS) basis. If there are less than b ($1 \leq b \leq N$) customers are waiting in the queue for service then the server will remain idle, otherwise, a batch of b customers will be taken by the server for providing FES (if the server is free).
- Once the FES of a group of b customers is over a part of batch, i.e., k ($1 \leq k \leq b$) customers may require to join SoS (offered by the same server with batch size dependent service rate) with some probability α_k ($1 \leq k \leq b$) and remaining will leave the system immediately. It may also happen that no customer requires SoS and the entire batch, served in FES, leave the system with probability α_k. This is the main contribution of our current study that we have allowed a portion of the batch served in FES to join SoS rather than making it compulsory for the entire batch to join or leave SoS, as present in the existing literatures (see, e.g., [3], [4] and the references therein).
- Once the server is busy in SoS the FES for the waiting customers in the queue cannot be started until the SoS of the batch of customers with the server is over.
- The service time distribution is considered to be generally distributed for both the cases FES and SoS.
- The density function (distribution function) [mean] of the service time distribution for FES is considered to be $h(.)(H(.))[h]$ and that for SoS of a batch of size x ($l \leq x \leq b$) is considered to be $\tilde{h}_x(.)$, $(\tilde{H}(.))[\tilde{h}_x]$.
- The buffer size is considered to be finite (N)
- For mathematical analysis the Kolmogorov equations of the model is constructed and by solving them using supplementary variable technique (SVT) and embedded Markov chain technique (EMCT) we obtained the steady state queue length distribution for FES and steady state joint distribution of the number of customers in the queue and with the server for SoS. Finally, several numerical studies are done by considering PH type service time distributions to establish the usefulness of the study of our current model.

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PRICING POWER EXCHANGE OPTION WITH STOCHASTIC LIQUIDITY RISK

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Abstract. Power exchange options are a generalization of power options and exchange options. An exchange option is a contract that offers the option holder the right, but not the obligation, to exchange the value of one asset for the value of another asset at some specified maturity time. These derivatives were created so that investors can reduce the risk of adverse relative changes between the two assets. Power options are exotic options whose payoff at maturity is given by some specified positive power of the underlying asset price. They allow investors to implement a high-leverage strategy and hedge against nonlinear price risks. Additionally, for an investor with a strong view of the market, writing a power option will yield significantly more premium income than writing an ordinary option. The payoff of power exchange options is given by $(\lambda_1 {S_1}^{\alpha_1}(T) - \lambda_2 {S_2}^{\alpha_2}(T))$

Market liquidity refers to the ease with which an asset, such as a financial instru ment or security, can be bought or sold in the market without causing a significant impact on its price. In a liquid market, there are enough buyers and sellers, and trad ing activity is sufficient to ensure that transactions can be executed quickly and at relatively stable prices. Conversely, in an illiquid market, there may be fewer partic ipants, and lower trading volumes, making it more challenging to buy or sell assets without affecting their prices.

While the existing research on power exchange options provides excellent insights into their significance in reducing risks, there is a notable absence in the discussion of liquidity risk - an important element frequently overlooked. Existing models are based on unrealistic assumptions of completely competitive and frictionless markets, which are uncommon even in today's well-established marketplaces. In reality, almost every asset in the market contends with varying degrees of liquidity issues.

In this article, we will explore the pricing of power exchange options in the pres ence of market liquidity. Market liquidity is modeled as a mean-reverting stochastic process in this case. We will analyze the combined impact of market liquidity and the sensitivity of each asset to market liquidity on asset price.

STOCHASTIC MODELLING OF HAPS-SMBS SYSTEMS IN 6G NETWORKS FOR ENERGY SAVING: AN ANALYTICAL APPROACH

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Abstract. High Altitude Platform Station (HAPS) systems have recently regained significant attention due to their inherent quasi-stationarity and smaller footprint compared to Low Earth Orbits (LEOs). The envisioned HAPS-mounted Super Macro-Base Station (SMBS) stands out as a robust platform to enhance HAPS capacity in 6G networks. The HAPS-SMBS wireless network architecture facilitates data acquisition, computing, caching, and

processing across various application domains. Unlike terrestrial gateways, HAPS-SMBS boasts a wide upper footprint, enabling simultaneous coverage of multiple LEO satellites. Consequently, HAPS-SMBS serves as an interface for satellite networks, ensuring seamless communication with aerial and terrestrial networks. These wireless networks are well-equipped to address high capacity, low latency, and computing needs, especially in densely populated metropolitan areas. However, energy consumption in HAPS-SMBS for 6G networks remains a concern. To address this energy challenge, we propose a stochastic model with four states: active, stand-by, beam-searching, and sleep state. In this model, upon the arrival of a User Request (UR), HAP-SMBS immediately enters the active state and provides the required service. After serving each UR, if the system remains idle, HAPS-SMBS transitions to the stand-by state. During stand-by, the system waits for new arrivals. If no arrivals occur, it shifts to the beam-searching state. In this state, HAPS-SMBS sends multiple dynamically steered beams focused on desired spots, forming up to 100 cells over the served area. If signals are detected, it re-enters the active state; otherwise, it enters sleep mode. During sleep, the system conserves energy with significantly lower consumption compared to stand-by and beam-searching states. HAPS-SMBS remains in sleep mode until a new URL arrives. If the sleep cycle completes without arrivals, the system re-enters the beam-searching state. We analyze this stochastic model using the Markov-Regenerative Process (MRGP). The study demonstrates the trade-off between energy consumption, energy savings, and throughput to achieve energy efficiency in HAPS-SMBS for 6G networks. Our numerical results highlight the significant energy-saving potential of the sleep state, emphasizing its effective contribution to enhancing the overall energy efficiency of the HAPS system.

OPTIMIZING ENERGY EFFICIENCY IN HIGH ALTITUDE PLATFORMS: A MARKOV REGENERATIVE PROCESS APPROACH FOR ADAPTIVE ENERGY SAVING STATES

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Abstract. High Altitude Platforms (HAPs) are airborne communication systems designed to operate in the stratosphere, offering immense potential for various applications, from disaster management to remote sensing. However, the sustainable operation of HAPs critically depends on efficient energy management, especially in the context of the unpredictable and challenging environments they operate in. This study delves into the intricate dynamics of energy efficiency in HAPs and introduces a sophisticated approach utilizing a Markov Regenerative Process (MRP) to model and optimize energy-saving protocols.

In our proposed framework, we define distinct energy saving states that encapsulate the nuanced operational modes of HAPs. During high-demand periods, the HAP operates at full capacity, providing services and maintaining active communication links. In low-traffic intervals or periods of inactivity, the platform minimizes energy consumption by shutting down non-essential systems, essentially entering a sleep mode to conserve energy. To balance energy conservation with basic communication functionality, the HAP operates in a low-power mode, ensuring essential systems remain operational while significantly reducing power consumption. Optimized communication state involves dynamically adjusting communication protocols and parameters based on real-time assessments of link quality and communication demands.

The MRP framework incorporates regenerative epochs, capturing cyclic behaviors in energy usage patterns. An objective function is formulated to minimize overall energy consumption over time, guiding the system toward efficient energy usage strategies. By solving the MRP, steady-state probabilities of different states are computed, enabling the analysis of long-term energy efficiency. This research provides a robust foundation for developing intelligent energy management strategies, ensuring the sustainable operation of HAPs in diverse and unpredictable environments.

EQUILIBRIUM AND OPTIMAL BEHAVIOR OF CUSTOMERS IN MARKOVIAN QUEUE WITH WORKING VACATION AND DELAYED OBSERVATION

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Abstract. Queuing theory traditionally addresses classical queues, involving the analysis of queue performance, their characteristics, and various methodologies. Classic queueing theory posits that entities within queues, such as customers, servers, and administrators, are inactive and devoid of decision-making, their behavior being exogenous. However, for profit motives, both customers and server administrators employ strategic approaches, giving rise to "Strategic Queueing" ([3], [4]). This evolution underscores the importance of understanding the strategies employed by queue entities and their economic impact. Strategic behavior in queues can be examined from two perspectives: **those initiated by the customer** (e.g., joining/balking, reneging [12], retrial [5], or threshold strategies, queue scalping [11]) and **those initiated by the server** (e.g., vacation [2], working vacation [1], unreliable server [9], or heterogeneous service).

In an evolving trend, the literature emphasizes an economic perspective on queueing systems, assuming customers have a reward-cost structure reflecting service preference and aversion to waiting. Rational queueing involves interactions between at least two decision-makers, such as profit-maximizing servers and customers. Noteworthy references include works by Hassin and Haviv [7], 'Rational Queueing' [6] by Refael Hassin, and monographs like [10] by Stidham. Given the strategic nature of queueing systems, this study focuses on customer strategies informed by information and categorizes queues into three main areas: Unobservable queues, Observable queues, and Partially observable queues.

Our research aims to build upon the existing studies mentioned earlier by establishing a connection between the observable and unobservable versions of the queue with a working vacation. In this context, customers determine their decision to either join the queue or balk without access to any information regarding the system's current state (busy or on working vacation) and the queue length. However, later on, they are informed about their current state and position with same rates in both states and may renege. We are considering an $M/M/1$ queue, where the system transitions to a working vacation state when the system becomes empty. The system administrator periodically announces information to customers regarding their current position at a rate of δ according to Poisson process in both states. After an announcement, the customer reevaluates their expected benefit of staying in the system and will renege if it is negative. The arrival, service, announcement, and working vacation processes are assumed to be independent.

The model is inspired by real-world scenarios such as the COVID-19 pandemic, where capac-

ity restrictions are imposed to ensure safety. In these situations, practices like waiting lines and periodic announcements align with crowd management norms. Administrators often enforce limits on facility occupancy such as a threshold to uphold social distancing and regulate capacity. This technique mitigates overcrowding risks and enhances safety for customers and staff. Such measures serve multiple purposes: capacity control, information distribution, fairness, and transparency. For instance, the Ombori Grid SaaS platform [8] demonstrates this concept in healthcare. Using a Virtual Queue (VQ), hospitals utilize virtual pre-booking to manage on-site crowding, enhancing patient experience through mobile-based queueing and timely notifications. Following registration, they receive text notifications indicating the anticipated wait time. Subsequently, when their turn arises, another text message alerts them to return to the facility.

In brief, **we investigates strategic customer behavior in a single-server Markovian queue with working vacation and infinite waiting space. A delayed announcement mechanism is introduced, establishing a connection between observable and unobservable queue states during busy and working vacation periods. Customers, upon arrival, face an unobservable system but receive periodic announcements about their current state and position, enabling them to make strategic decisions to renege if advantageous. We define the customers' strategic choices using three parameters:** q, **representing the probability of joining upon arrival, and the reneging thresholds** n_0 **and** n_1, **applicable during the working vacation and busy periods, respectively. We characterize and compute both equilibrium strategies** $({n_0}^e, {n_1}^e, q^e)$ **of customers and socially optimal strategies** $({n_0}^{soc}, {n_1}^{soc}, q^{soc})$.

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EXTENDED UMA DISTRIBUTION AND ITS APPLICATIONS ON DIABETES MELLITUS

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Abstract. Area Biased Uma distribution is a new type of Uma distribution. The newly created distribution has been analysed with regard to several statistical characteristics, and the method of maximum likelihood estimation is used to estimate its parameter. In order to assess the superiority and flexibility of the suggested new distribution, a real lifetime dataset has been evaluated and examined.

The concept of weighted distribution has remained crucial in probability and statistics for simulating complex lifetime data sets that come from a variety of applied sciences, including engineering, medical sciences, finance, insurance, etc. Fisher (1934) was the first to suggest using weighted distributions to model ascertainment bias. Rao (1965) then standardised the concept in a comprehensive way in relation to modelling statistical data after finding that the usual method of using standard distributions for this purpose was insufficient. When samples can be collected from both the original and the developed distribution, the weighted distribution provides a method for fitting the model to the unknown weight function. The weighted distribution offers a group strategy to address conceptualization and data representation issues related to model integration. When the weight function solely takes into account the length of the units of interest, the weighted distribution becomes a length biased distribution. Cox and Zelen (1969) established the idea of length biased sampling, and the term length biased refers to sample data where the likelihood of recording an observation relies on the size of the observation. Cox (1962) was the first to introduce the idea of a length biased distribution, and it has since been used in a number of biomedical applications including survival analysis, geological sciences, family history, disease, intermediate events, reliability analysis, and population studies where a suitable sampling frame is lacking. Several authors made substantial contributions in the past to the creation of several notable area biased probability models as well as their applications in diverse field.

The present paper introduces area biased Uma distribution as a new generalization of Uma distribution. The different statistical properties of new distribution such as moments, order statistics, survival analysis, bonferroni and Lorenz curves have been studied and investigated. The parameters of proposed new distribution are estimated by using the technique of maximum likelihood estimator and also its Fisher's information matrix have been discussed. Finally a new distribution has been fitted with real data set for examining its superiority.

Keywords: Uma distribution, Area biased distribution, Reliability analysis, Order statistics, Maximum likelihood estimation.

FLEXIBLE MANUFACTURING SYSTEM WITH TWO PRODUCTS AND ZERO SWITCHOVER TIME

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Abstract.A flexible manufacturing system refers to a production method designed to adjust deviations in the product quantity and type being manufactured. The system aims to lower the production cost of a company by enhancing efficiency. Here we consider a flexible manufacturing machine with two different products served by a single server. There are two different queues , Queue 1 and Queue 2. Customers join the queue according to the choice of items they selected. Queue 1 is of finite size b_1 and Queue 2 is of infinite capacity. A customer on arrival ,to an idle server,is immediately taken for service. Customers joins the system with a Poisson process. Arrival rate of customers in the queues are λ_1 and λ_2 respectively. Production time for each product follows exponential distribution. The lone server serves the system by visiting different queues in the order Queue 1 , Queue 2, Queue 1, Queue2, ... The server continues to serve Queue 1 customers until it completely empties.On completion of service in Queue 1 a customer can join Queue 2 for another service and server will provide service to him immediately after giving service in Queue 1 and vice versa. No fresh arrivals will be served in this visit. Switch over time (the duration of the time that it takes a server to move from one queue to another) is assumed to be zero. Server who is serving in Queue 2 will move to Queue 1 on completion of b_1 number of customers. No arrival will be allowed to any of queues during the time the server providing service in the respective queues.

The service times of customers in each queue are exponentially distributed with rate $\mu_i, i = 1, 2$. When inventory level reaches zero the production starts. From then the production will be according to the demand of customers and service time is service time plus production time. Production will be switched off when the queue empties. We investigate this queuing system. . Several performance measures are evaluated. Numerical illustrations of the system behavior is also provided.

A BULK SERVICE QUEUEING MODEL IN FIBC BAG MANUFACTURING INDUSTRY WITH UNRELIABLE SERVER, INSPECTION, REWORK AND MULTIPLE VACATION

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Abstract. In this paper, we studied the behavior of an $M^{[X]}/G(a,b)/1$ queueing system with unreliable server, inspection, rework and multiple vacation. We implement this type of model in the FIBC bag manufacturing industry. The supplementary variable technique is used to implement the suggested model. It is feasible to collect performance metrics and stability requirements. To assess the reliability of analytical conclusions, numerical examples

are developed.

SINGLE SERVER INVENTORY WITH MAP AND PHASE TYPE DISTRIBUTED SERVICE TIME

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Abstract. In this paper, we analyze an MAP/PH/1 inventory model with positive lead time and service time. We include local purchase of items during lead time with a pre-defined probability. The arrivals are according to a MAP and the service time follows a phase type distribution. We establish the stability condition. Some performance measures are studied numerically. The special case of M/M/1 inventory model is analyzed in which case, the product-form solution is attained.

Key words: Local purchase, product-form solution, (S,Q) inventory system.

STUDY OF THE DELAY AT TRAFFIC SIGNALS USING AN $M_t/M_t/1$ QUEUEING MODEL WITH COMPETITION BETWEEN TWO TYPES OF CUSTOMERS

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Abstract. We studied the vehicle traffic at a signalized intersection as an $M_t/M_t/1$ queueing model with two types of customers, who compete for obtaining service from the single server. Arrival of vehicles is according to a Poisson process with time dependent parameter $\lambda(t)$ and their service time follows an exponential distribution with time dependent parameter $\mu(t)$. If an approaching customer has to queue, it may join a queue of negative customers instead of the main queue of positive customers. Formation of the negative customers occurs from the main Poisson stream according to the following rule: a customer while entering the system decides to join the queue of negative customers with probability δ, if it finds the number of negative customers as less than or equal to that of positive customers. When the server is active, at each service completion epoch a negative customer may forcefully get service with probability β_1. The negative customers may thus extend a positive customer's waiting time and increase congestion further. The bad effect caused by negative customers is studied on the basis of the delay experienced by different type of customers. We conduct an extensive numerical study based on real-world data.

ANALYSIS OF MAP/PH/1 AN INVENTORY RETRIAL QUEUEING SYSTEM WITH PHASE TYPE VACATION, WORKING BREAKDOWN, REPAIR, IMMEDIATE FEEDBACK, CLOSE-DOWN AND BALKING

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Abstract. In this study, an inventory retrial queueing system with a single vacation, working breakdown, repair, immediate feedback, close-down and balking is investigated. We have assumed that customers arrive through a Markovian arrival process and that the server will offer phase-type services to them. The (s, S) policy is used to replenish the inventory and it is anticipated that the replenishing time will follow an exponential distribution. After completion of service, the customers need feedback with probability p to provide service once again otherwise with probability q close down the system. The server close- down the system if there are no consumers in the orbit, or zero inventory, or both, and then the server goes for a single vacation. When a server breaks down, the server provides slow-mode service for that current customer and then starts the repair process. During the server is not available, the customer may balk. In the steady state, it is possible to determine the number of customers in the orbit, the inventory level, and the status of the servers. The cost analysis is derived, and several significant performance measurements are established. Furthermore, a few numerical demonstrations are given to clarify our mathematical model.

keywords: Queueing-inventory; Markovian Arrival Process; Phase-type distribution; (s, S)-type policy; Single Vacation; Working Breakdown; Repair; Matrix Analytic Method; Immediate feedback; Close-down; Balking.

AMS Subject Classification: 60K25, 68M20, 90B22

EQUILLIBRIUM ANALYSIS OF A CLASSICAL QUEUEING-INVENTORY SYSTEM WITH OPTIONAL PROCESSING TIME

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Abstract. We explore the strategic behavior of a single-server queueing-inventory system with an optional processing time having (s, S) order policy. In this model, the customer demands either processed items (with positive lead time) or unprocessed items (with zero lead time). Upon arrival, customers strategically decide whether to wait for the processed item or immediately purchase the unprocessed item. We assume the arrival of demands follows the Poisson process, and the service time and lead time follow an exponential distribution. We also consider that there is no shortage of items, which leads to an explicit steady-state probability distribution. Our investigation focuses on customers' optimal strategies regarding the decision to "wait or not", and we derive the Nash equilibrium. In the event of inventory shortages, we opt for an emergency replenishment policy with a shorter lead

time and subsequently analyze it as a specific case. Addition- ally, we address optimal pricing problems to maximize the system revenue and discuss socially optimal strategies. Significant performance measures are analyzed, and the investigation involves studying the Price of Anarchy (PoA) to quantify the inefficiencies associated with equilibrium strategies. Finally, numerical illustrations are provided to assess the impact of system characteristics.

keywords: Queueing-inventory, Optional processing time, Lead time, Nash equilibrium, Emergency replenishment, Price of Anarchy

QUEUEING INVENTORY SYSTEMS WITH FINITE BUFFER, BATCH DEMANDS AND EMERGENCY REPLENISHMENT

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Abstract. Researches on queueing inventory systems with positive service time has acquired much attention over the last few decades. Such systems arises naturally in many fields of the world and have been studied extensively during these times. Studies in this area began with the works of Sigman and Simichi-Levi (1992) and Melikov and Molchanov (1992) in 1992. Later, Berman et al. (1993), Berman and Kim (1999), Berman and Sapna (2002) and many other researchers contributed to this area. The introduction of product form solutions in queueing inventory sytems marked a major developmental turning point in this area of research. Schwarz et al. (2006), Schwarz et al.(2007), Krenzler and Daduna (2015) are some authors who derived stationary distributions of joint queue length and inventory process in explicit product form. Krishnamoorthy et al. (2015) extended schwarz et al. (2007) where they assumed that the customers need not buy an item at the time of service completion.They also shown that the stationary distribution has product form solution.

In all the above mentioned works, customers demands for only one item from the inventory at the time of service completion. But there are many situations where customers can demand for one or more items from the inventory. Such demands are called batch demands. Even though, batch demands have several applications in real life, only a few studies are done on this topic. Yue et al. (2018) studied an M/M/1 queueing inventory system with geometric batch demands and lost sales under two models where they derived the stationary conditions for both models using QBD process. Chakravarthy (2019) adressed a queueing inventory model with batch demands and positive service time by considering two models. They performed the steady state analysis of these models using classical matrix analytic method. Later, chakravarthy and Rumyantsev (2020) generalises the above model by assuming MAP arrivals and Phase type service times. They analyse these models in steady state using matrix analytic method in single server case and extended these results to the case of multiserver case by simulation. Recently Chakravarthy and Rao (2021) , Linhong et al. (2023) also contributes to this area of research.

We are going to extend chakravarthy and Rumyantsev (2020) by incorporating the concept of buffer and emergency replenishment. In this paper we study a single server queueing inventory model in which customers demands for a batch of items from the inventory. That is each customer demands for one or more items but a maximum of N items from the inventory where $N < S$.There is a finite space for the inventory that can hold up to S items. We use an (s, S) type replenishment policy. That is, if the inventory level drops to s or below, then a replenishment is placed so that it will bring back the inventory to S. The lead

time is assumed to be exponentially distributed. Here the arrival of customers is according to MAP and service time follows phase type distribution. Assume the customer announces the required amount of items at the arrival epoch. If the required amount is unavailable,the arriving customer has three possibilities

i the customers can leave the system without taking any items from the inventory with probability q_1

ii the customers takes away all the available items in the inventory with probability q_2

iii the customers moves to a buffer of finite capacity, say K, after service completion but without taking any item from the inventory with probability q_3

When the replenishment occurs, priority is given to the buffered customers .That is, buffered customers are instantaneously provided with items when replenishment occurs. We also assume that an emergency replenishment with zero lead time is placed when either buffer is full or the demands in the buffer reaches a predefined (finite) number, say $M = S$.

The paper aims to perform a steady state analysis of the model through classical matrix analytic methods and attempts to obtain some important performance measures like system idle probability, mean number of customers in the queue, mean number of customers in the buffer, mean inventory level, probability of customer loss, waiting time of each customer etc.

We can see many real life situations related to this model. For example, in the case of pharmaceutical items, the customers who purchase for a particular medicine will most of the times demand for more than one items as prescribed by their doctor. Also if the demanded item is unavailable, they can leave without buying any item or can leave with available items or can wait until the items become available.

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LIMIT THEOREMS FOR A CIR PROCESS WITH EXTERNALLY-EXCITING AND SELF-EXCITING JUMPS

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Abstract. The Cox-Ingersoll-Ross (CIR) process is a stochastic process commonly employed in the modeling of short-term interest rates or default intensity in credit risk. Moreover, it has been seen that several events give rise to positive jumps in a default intensity process. Therefore, in order to investigate the occurrence of positive jumps in the default intensity process, a generalized version of the CIR process is used, that incorporates jumps modeled by Poisson process.

However, it has been noted that in the real world, the occurrence of jumps does not always exhibit time homogeneity. The occurrence of default contagion becomes more apparent during financial crisis. Hence, there exists a necessity for a better model than the Poisson process, as it fails to adequately account for the phenomenon of default clustering. The Poisson process is replaced with the simple point process that possesses a self-exciting and clustering property, commonly referred to as the Hawkes process. Furthermore, an important amount of literature suggests that default intensity can also be influenced by several external circumstances. Therefore, a novel generalization of the CIR process is proposed, incorporating both externally-exciting and self-exciting jumps.

In this paper we consider the CIR process with externally-exciting and self- exciting jumps with exponential exciting function. By employing the concept of infinitesimal generator we obtain the limit theorems, including the law of large numbers and central limit theorem.

ACHIEVING OPTIMAL RESOURCE UTILIZATION IN INDUSTRIAL ENVIRONMENTS: COMBINING POLLING METHODS AND AN EXHAUSTIVE SERVICE DISCIPLINE WITH AN UNRELIABLE SERVER

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Abstract. Two types of queues, type I queue with infinite capacity and type II queue with a finite capacity, are both taken into account. While the service time distribution adheres to phase type, each queue's input process operates in line with the Markov arrival process (MAP). The server cycles through the line, starting with the type I queue, and provides standard service to everyone while also offering optional service based on the demands of the client. During the visiting period of each queue, a strict service discipline is observed. Working breakdown and repair are distributed exponentially, whereas many vacations are distributed according to a phase type. The entire number of consumers in the system are investigated using matrix analysis under the steady state probability vector. We analyze busy period analysis, an invariant probability vector, and a few performance indicators in our model. The proposed model's numerical and graphical findings are also examined.

Keywords: Markovian Arrival Process-Phase type service-Vacation-Exhaustive discipline-Optional service.

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OPTIMIZING VARIABLE ANNUITY FEES: FINITE HORIZON Q-LEARNING ANALYSIS OF SEMI-MARKOVIAN HEALTH AND VIX-LINKED FEES WITH FAMILY SECURITY BENEFITS

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Abstract. Variable annuities (VAs) and other equity-linked products in life insurance allow policyholders to participate in the financial market while also offering protection against unfavorable investment performance. VAs are linked to the performance of underlying investment options, typically mutual funds. The value of these investments can fluctuate based on market conditions, exposing the policyholder to the risk of potential losses. For example during the COVID-19 pandemic, for instance, if a policyholder allocated funds to a mix of stock and bond mutual funds within the annuity, the value of these investments could have fluctuated significantly in response to the market conditions brought about by the global health crisis. The crisis and its repercussions on insurers have emphasized the necessity for robust risk management techniques.

As regulations shift towards valuing liabilities based on current market prices, the future costs and assets for VAs become more uncertain, particularly during periods of financial

instability. Variable annuity contracts commonly utilize charges deducted directly from policyholders' accounts to fund the financial guarantees they provide. Typically, these charges are established at a constant percentage of the total value of the account and deducted at regular intervals until the account reaches maturity. The current fee structure frequently results in a discrepancy between the income that insurers get and the market value of their obligations. This discrepancy can lead to higher costs for hedging and a decrease in the efficiency of risk-management techniques.

While the most popular guarantee, like GLWB associated with VAs, can be mitigated through the use of long-dated options, this approach may prove costly due to the limited availability of suppliers offering long-term derivatives in the financial market. An alternative method for hedging these long-term guarantees involves establishing a hedging portfolio through the continuous renewal of short-dated options. Short-dated options are more liquid and better suited to safeguard the insurer against short-term fluctuations in implied volatility. However, during periods of financial crisis, this hedging strategy may become expensive, as the acquisition cost of new options can be both high and unpredictable.

Preprint submitted to Insurance: Mathematics and Economics December 19, 2023 In light of these challenges, the Chicago Board of Options Exchange (CBOE) has proposed, in a white paper linking the Variable Annuity charge to the CBOE's trademark Volatility Index (VIX). Furthermore, it is noteworthy that the persistence of a constant fee structure can contribute to an increment in the surrender risk associated with Variable Annuities which is one of the central focus. The stability in fees over time can impact the overall surrender value, providing policyholders with a more predictable and potentially favorable surrender benefit. The VIX, known as the Chicago Board Options Exchange (CBOE) Volatility Index, gauges professional investors' expectations regarding S&P 500 index volatility over the next 30 days. Serving as a marker of market volatility, the VIX enables the fee structure to adapt in real-time to market fluctuations. During periods of heightened volatility, the charge may be adjusted to reflect increased risk, offering a more agile approach to risk management within variable annuities.

Furthermore, a significant gap in the existing literature concerning Variable Annuities (VAs) pertains to the limited exploration of family security benefits. This specific benefit serves as a form of safeguarding for the designated beneficiaries and the offspring of the policyholder in the event of their demise. Family security benefits remain a focal point of oversight in existing research. Notably, most of the literature focuses on exploring social security benefits, leaving family security benefits relatively underexplored and demanding further scholarly attention. This study explores two critical aspects within the financial planning domain: strategic minimization of fees by the insurer through a Q-learning framework and analysis of the equitable fee structure inherent in the contract. Specifically, this study investigates the health and VIX-linked fee structure of variable annuity products with guaranteed lifetime withdrawal benefit (GLWB) rider. Additionally, semi-Markovian health benefits and family security benefits have been incorporated into the analysis, adding another layer of sophistication to the modeling. A mixed discrete- continuous time model is proposed to calculate the fair fee of the product, using equilibrium conditions between premium and benefit. The Finite Horizon Q-Learning technique is used for the optimal reduction of the fee from the insurer's point of view, depending on the health and market returns. In this article, the Heston model has been considered to generate the fund value. Moreover, our model provided family security benefits (FSB) to the policyholder, including the survivors' and the children's benefits. A numerical investigation was carried out to demonstrate the potential of this financial product and explore potential enhancements. It's important to note that the withdrawal amount encompasses the guaranteed amount from the GLWB rider and medi-

cal expenses, excluding the family security benefits. A comprehensive comparative analysis was conducted with an existing model with constant fees and without FSB. The findings highlighted superior benefits in comparison to the fee in our model.

A BIRTH-DEATH PROCESS WITH N- POLICY FOR THE DEATH PROCESS

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Abstract. We study a Birth-death process (BDP), in which the death process is triggered only by the accumulation of the population size to a specific size, N. The birth and death rates are assumed to depend on the population size. Motivation for such processes comes from the fact that certain populations such as that of tumor start decaying after it has grown to some size. A transient analysis of the above BDP is conducted by applying the generating function method. A numerical study of various system performance measures has also been conducted.

Key Words: Birth-death process; death triggering; transient analysis; generating function

A MARKOV MODEL BASED STUDY OF WAITING TIME OF A DYNAMIC DISTRIBUTION AGENT IN AN ONLINE FOOD DELIVERY SYSTEM

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Abstract. Distribution agents (DAs) play a very important role in the online food delivery system. Usually, a DA joins the queue formed in front of a restaurant. The food delivery apps are designed in such a way that the DA near the restaurant will receive the order reaching that restaurant. This paper introduces a dynamic distribution agent (DDA) who never joins a queue in the hope of receiving orders from more than one restaurant. Using continuous-time Markov models, we analyze the expected waiting time of the DDA and hence examine whether the above DDA's strategy is profitable or not. We consider two models: first in which both the inter-order time and the inter-service time (food preparation time + delivery time) follow different exponential distributions; second in which these random times follow distinct phase-type distributions. Our numerical study based on real-world data suggest that the strategy of the DDA will succeed depending on the number of DAs in competition and the number of restaurants available.

Key Words: Continuous-time Markov chain; Exponential distribution; Phase-type distribution; Online food delivery system; Waiting time; Dynamic distribution agent.

ON TWO DISTINCT 2-SERVER QUEUES (I)MAP/ 2-SERVER INTERDEPENDENT SERVICE AND (II) PH/ 2-SERVER INTERDEPENDENT SERVICE

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Abstract. In this paper, we consider two queueing models: Model I and Model II. In Model I, a queueing system with two servers is analysed where the arrival and service processes are independent but service processes themselves are interdependent. Customers arrive according to Markovian arrival process (MAP) with representation $D = (D_0, D_1)$ of order m. The evolution of the service processes is governed by transitions on the product space of two Markov chains which are descriptors of service processes. The transitions in this Markov chain follow a semi-Markov rule, and the sojourn times in states are governed by the exponential distribution. Model II differs from Model I only in arrival process. The arrival process follows phase-type distribution with representation (α, T) of order m. Numerical comparison of the Model I and Model II is presented.

A NEW EXTENDED EXPONENTIAL DISTRIBUTION AND ITS APPLICATIONS

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Abstract. Recently, there has been a great interest among statisticians and applied researchers in constructing flexible distributions for better modeling of non-monotone failure rates. This study introduces a novel two-parameter Nadarajah-Haghighi (Extended Exponential) model based on the Kavya-Manoharan transformation [1], and Nadarajah-Haghighi distribution [2]. The hazard rate function is an important quantity characterizing life phenomena. Its hazard function can be decreasing, increasing, and bathtub-shaped depending on the parameters. This paper provides a comprehensive mathematical treatment of the new distribution and derives explicit expressions for some of its basic mathematical quantities. The method of maximum likelihood is used for estimating the model parameters and a Monte Carlo simulation is conducted. Fitted the proposed model to real data sets to prove empirically its flexibility as compared to other lifetime distributions.

The research addresses the development of a new statistical model, the two-parameter Nadarajah-Haghighi model, within the framework of the Kavya-Manoharan G family. This model offers a unique perspective on probability distributions, and its statistical properties are investigated in depth. Structural properties like the Lorenz curve, linear representation of the density function, survival function, hazard rate function, quantile function, median,

skewness, kurtosis, moments, and moment generating functions are derived. The methodology involves deriving explicit expressions for the statistical characteristics of the new model. Special attention is given to the density and failure rate functions, providing insights into their shapes under different parameter values.

The maximum likelihood estimation method is then applied to estimate the model's unknown parameters. The study yields valuable results, showcasing the versatility of the new model's statistical characteristics. The derived expressions reveal intriguing patterns in the density function and failure rate function, shedding light on the behavior of the model under varying conditions. A comprehensive simulation study is conducted to validate and assess the model's performance. The simulations provide empirical evidence of the model's behavior and help evaluate its robustness across different scenarios.

The practical utility of the proposed model is demonstrated through an application using a dataset from Choulakian and Stephens (2001). The dataset, consisting of 72 exceedances spanning from 1958 to 1984, serves as a real-world case study to illustrate the model's applicability and performance in capturing the underlying statistical patterns. The simulation study and application results are discussed in the context of existing literature, highlighting the novel contributions and potential implications of the proposed new Nadarajah-Haghighi model. Comparisons with other models are drawn to underscore the uniqueness and efficacy of the new model.

In conclusion, the study presents a robust and versatile two-parameter Nadarajah-Haghighi model based on the Kavya-Manoharan G family. The explicit expressions for its statistical characteristics, coupled with the successful application to real-world data, affirm its potential as a valuable addition to the realm of probability distributions.

Keywords Nadarajah-Haghighi model, Kavya - Manoharan G family, maximum likelihood estimation, simulation study, statistical characteristics, real-world application.

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MAP/PH/1 QUEUEING SYSTEM WITH SERVER DETERIORATION AND RECOVERY UNDER VACATION AND BACKUP SERVER

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Abstract. In this paper, we investigate a queueing system with vacation in which the service rate of the server deteriorates during the service period and recovers during the vacation

period. During the recuperation period, the main server is replaced with a backup server with inferior capabilities. The system is analyzed using the matrix analytic method and various performance measures are derived. Numerical examples are presented to illustrate the results and the sensitivity of the model parameters.

FORECASTING OF THE ROAD ACCIDENTS IN INDIA USING TIME SERIES ANALYSIS

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Abstract. Annually, approximately 1.5 lakh people die on Indian roads due to a variety of causes such as over speeding, mobile phone use, drunken driving, overloaded vehicles, poor vehicle conditions, inadequate lighting, running red lights, unsafe overtaking, negligence of civic bodies, weather conditions, driver errors, driving on the wrong side, road defects, vehicle defects, cyclist errors, pedestrian mistakes, and more.

This study aims "to forecast the number of accidents in India for the years 2023 and 2024 using time series analysis". Time series analysis involves examining a sequence of data points collected over time. The Autoregressive Integrated Moving Average (ARIMA) technique is employed for forecasting. ARIMA models are valuable for predicting time series data, capturing patterns, trends, and seasonal variations using past values, differences, and errors.

The ARIMA model is denoted as (p, d, q), where 'p' represents autoregression (AR), 'd' signifies differencing (I for Integrated), and 'q' represents the moving average (MA). The accident data in India was gathered from the National Crime Records Bureau (NCRB) between 2001 and 2022. The NCRB, an Indian government agency, collects and analyzes crime data as per the Indian Penal Code (IPC) and Special and Local Laws (SLL).

Analysis of the data suggests that the ARIMA (2, 4, 1) model, with lower AIC and BIC values, is most suitable for the dataset. The notation indicates that the original data was differenced four times for stationarity, incorporated an autoregressive component with two lagged values, and considered a moving average component with one lagged forecast error. Thus, this model is chosen based on model selection criteria. Moreover, the study reveals an upward trend in the number of road accidents in India

keywords: ARIMA, NCRB, AIC, BIC

PHASE-TYPE MODEL-BASED CLUSTERING

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Abstract. Finite mixtures of distributions belonging to the exponential family have broad utility, including their usage for model-based clustering. There is increasing recognition of mixtures of asymmetric distributions as powerful alternatives to traditional Gaussian and mixtures of t distributions. The current work contributes to that assertion by addressing some facets of estimation and inference for mixtures-of-phase-type distributions, including model-based clustering. Inference regarding the appropriateness of a common-shape mixture-of-phase- type distribution is motivated by theory from gamma-mixture clustering

on infant habituation. We provide extensive simulation results that demonstrate the strong performance of the routines.

ANALYSIS OF MAP/PH/1 RETRIAL QUEUEING INVENTORY SYSTEM WITH CONSTANT RETRIAL RATE, BERNOULLI VACATION, WORKING BREAKDOWN AND REPAIR

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Abstract. In this article, we examine a single server retrial queueing inventory model with Bernoulli vacation.The concept of Bernoulli vacation is that after providing service to the customer, the server may opt to avail vacation with probability p or start the service to subsequent customer with probability $1-p$. During the busy period, the server may get breakdown.As a result the customer in the current state who is receiving service at a slower rate and then the server go to a repair process. After the completion of repair process, the server remains idle. We assumed that the customers arrive according to Markovian arrival process, service and repair times follows phase type distribution. The Inventory is replenished according to an (s, S) policy and the replenishing time is assumed to follow exponential distribution. We discussed the steady state probability vector, busy period, system performance and for a better understanding of our model, numerical results are provided.

keywords: Retrial Queueing Inventory Model, Markovian Arrival Process, Bernoulli Vacation and Working breakdown.

FINITE MARKOVIAN SINGLE SERVER FEEDBACK QUEUE WITH STATE-DEPENDENT RATES

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Abstract. The content of the paper is a single server queue with a finite waiting line. The arrival process and service time distributions are the Poisson process and negative exponential distribution respectively. The arrival and service rates are state-dependent. In addition, after completion of the service, the customer may decide to leave the system or enter the head of the waiting line or feedback into the waiting line for another service. The decision follows a Bernoulli distribution. This model is mathematically defined using an infinitesimal generator matrix and the matrix is solved using the method group generalised inverse. Using the group generalized inverse matrix, the steady- state probabilities are obtained analytically. Some performance measures are derived. Also, some numerical illustrations are provided by a particular healthcare situation.

Keywords: Finite capacity Markovian queue, feedback State-dependent arrival and service rates, Infinitesimal generator matrix, Group generalised inverse, Steady state probabilities,

and Performance measures.

COMPARING NON-PREEMPTIVE PRIORITY QUEUING PERFORMANCE WITH FUZZY QUEUING MODELS INTUITIONISTIC FUZZY QUEUING AND NEUTROSOPHIC QUEUING MODELS WITH VARYING SERVICE RATES

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Abstract. This study provides non-preemptive priority fuzzy, intuitionistic fuzzy and Neutrosophic queuing models with unequal service rates. For Queuing modelling performance evaluations of the supply chain, industrial, stock management, data exchange, workstations, and telecommunications equipment, non-preemptive priority queues are appropriate. The parameters of non-preemptive priority queues can be fuzzy due to unpredictable causes. The primary goal of this research is to compare the performance of a non-pre-emptive queuing model applying fuzzy queuing theory, intuitionistic fuzzy queuing theory and neutrosophic queuing model. The performance metrics in the fuzzy queuing model, intuitionistic fuzzy queuing are given as a range of value, however, the fuzzy neutrosophic queuing model offers a multitude of values. We have offered numerical examples to support our findings, and the comparative analysis with the validation of result.

keywords: Non-Preemptive Priority queue, Triangular Fuzzy and Intuitionistic Number, Neutrosophic Fuzzy Number.

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MULTI-SERVER MARKOVIAN HETEROGENEOUS ARRIVALS QUEUEING SYSTEM WITH MULTIPLE VACATIONS, SECOND OPTIONAL SERVICE AND IMPATIENT CUSTOMERS

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Abstract. This paper discusses a M/M/c heterogeneous arrivals queueing system with multiple vacations, second optional service and impatient customers. Every time a cus- tomer enters the system, a random timer is started. If the service is not finished before the impatient timer expires, the customer may leave the system. The server gives service in two types, the first type service is essential and the second type of service is optional. The server goes for a vacation of random duration if the system is empty. There is a chance that the arriving customers may balk with a certain probability. The Proba- bility Generating Functions (PGFs) approach has been used to derive the steady-state probabilities. Performancemeasures and Numerical illustrations are also discussed.

keywords: Essential and Optional service; Steady state; Impatient customer; Multi- server; Performance measures.

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