Determining Bounds for Performance Parameters of an ATM Multiplexer with Homogeneous ON-OFF Sources Using Markov Decision Processes*†

András Pfening[1], Khalid Begain[2], Miklós Telek[1]

[1] Department of Telecommunications
Technical University of Budapest, 1521 Budapest, Hungary
E-mail: {pfening,telek}@hit.bme.hu

[2] Department of Computer Science
Mu'tah University, 61710 Mu'tah, Jordan
E-mail: begain@nets.com.jo

Abstract

The paper addresses the analysis of a single multiplexing node of ATM networks. This problem has been studied in several papers providing both analytical and simulation results, however most of them assume a continuous time or fluid flow model of the system which is an approximation of the real situation. In this paper, a discrete time model based on a finite number of sources and a finite size buffer is introduced from which results on cell loss, average buffer length, and delay are given based on a two dimensional Discrete Time Markov Chain. The accurate analysis of the introduced physical model requires a detailed knowledge on the distribution of the incoming cells in the time slots and it is very hard to evaluate numerically even for small models. Based on the introduced Discrete Time Markov Chain model of the system, a Markov Decision Process is defined, with appropriate cost functions to determine the optimal and the worst cell arrival schedule, which is then used to calculate the bounds of performance measures.

Key words: Discrete time model, Markov decision process, performance bounds.

1 Introduction

Broadband ISDN (B-ISDN) is the network planned to carry different types of information including voice, video, and data. The CCITT has adopted the Asynchronous Transfer Mode (ATM) as the switching technique for the future high speed network due to its flexible and effective utilization of network resources. Since then ATM has become an intensive research area and the main interest has been devoted to the development of methods in order to ensure Quality of Service requirements (throughput, cell loss, delay, etc) for each data type.

The ATM is a packet-like switching and multiplexing technique in which messages are split into short fixed-length (53 Bytes) packets called cells. Cells may be lost or may suffer delay for different reasons, while they are transmitted from the source to the destination. The buffer overflow in an intermediate switching or multiplexing node can be one of the reasons of the loss or delay. The tolerance for cell loss or delay varies with the type of carried traffic. For example,

*This work was performed with the support of the European Community through the contract COPERNICUS-1463. András Pfening and Miklós Telek would like to thank Hungarian OTKA for grant No. T-16637.

†Presented in the *Fourth IFIP Workshop on Performance Modelling and Evaluation of ATM Networks.* Ilkley, July 1996.

packetized voice traffic allows relatively high cell loss probability but it has little tolerance to the delay while data can tolerate some delay but they are very sensitive to the cell loss.

In this paper, the problem of multiplexing is addressed. Namely, the special case of N identical ON-OFF sources with one high speed output. This problem has been studied in many papers providing both analytical and simulation results, however, most of them assume a continuous time or fluid flow model of the system which is an approximation of the real situation.

Li [4] applied a discrete time model, assuming finite number of ON-OFF sources and geometrical distributions for the ON and OFF intervals. He fixed that in one time unit only one ON and/or OFF source can change state. The channel capacity was assumed to be an integer number of sources and the buffer size could be either zero or infinite.

Hübner and Tran-Gia [3] used similar model, but in their model the server capacity was given as a non-integer number of information units and the buffer size was finite. They examined three cases. First they determined steady-state probabilities and cell blocking probabilities for fixed number of ON sources, gave approximations for the case of fixed number of ON-OFF sources, finally studied a call admission control scheme based on blocking probabilities.

In this paper an arbitrary number of sources can change its state, contrary to [4], and the output link speed is and integer multiple of the input link speed, contrary to [3], which are rather realistic assumptions.

The paper is organized as follows. In Section 2 we overview the analyzed configuration and the system model and the derivation method of the considered performance parameters, based on the paper of Begain et al. [1], since their study served as a basis of the new results. In Section 3 the application of Markov Decision Processes is described. In Section 4 the proposed method is demonstrated through the analysis of a concrete ATM multiplexer configuration. Finally the paper is concluded in Section 5.

2 System Model Description

2.1 Model Assumptions

Physical model

Consider a multiplexing node with the following features (Figure 1):

- N identical sources with two states (ON, OFF).
- Sources in the ON state generate cells with rate v_s, where the time unit is taken so that v_s=1 [cell/time unit] holds.
- Sources in the OFF state do not generate cells.
- There is one output transmission link with transmission rate $v_l = C$ [cells/time unit].
- If more cells arrive than the output link capacity, the extra cells are stored in a buffer of length L.
- Cells arriving when the buffer is full are lost.

The system is studied in order to find analytical results on the expected cell loss, the cell delay, and the average buffer content.

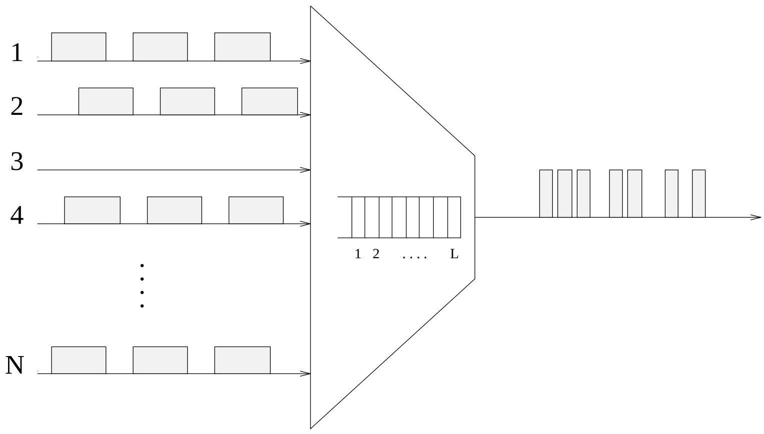

Figure 1: The analyzed multiplexer configuration

Source process

Assume that the behaviour of a source can be described by a discrete-time Markov chain (DTMC) with two states (ON-OFF). The distribution of the length of the ON periods is assumed to be of geometrical with parameter β, while the OFF periods are also geometrical with parameter α. The transition probabilities of the DTMC are :

$$\begin{array}{ll} \Pr\{OFF \to ON\} = \alpha & \Pr\{OFF \to OFF\} = 1-\alpha \\ \Pr\{ON \to OFF\} = \beta & \Pr\{ON \to ON\} = 1-\beta \end{array}$$

Let us define now ξ_n denoting the number of sources in ON state at time n. It is obvious that this process is also a DTMC with state space $\Omega = \{0, 1, ..., N\}$ and the state transition probabilities can be written as:

$$p_{ij} = \sum_{k=\max(i+j-N,0)}^{\min(i,j)} \left(\begin{array}{c} i \\ k \end{array} \right) (1-\beta)^k \beta^{i-k} \left(\begin{array}{c} N-i \\ j-k \end{array} \right) \alpha^{j-k} (1-\alpha)^{N-i-j+k}$$

This expression of the transition probabilities takes into account that the transition from state i to state j may occur if k out of the i ON sources ($0 \leq k \leq i$) stay in the ON state and $(j-k)$ other sources turn from the OFF to the ON state.

Let $\mathbf{p} = \{p_i\}, i = 1, \ldots, N$ denote the steady state probability vector of ξ_n.

Buffer content

The process describing the number of cells in the buffer plays an essential role in evaluating the performance parameters mentioned before. Let η_n denote this process with state space $\Phi = \{0, 1, \ldots, L\}$, where L is the size of the buffer. The state transition probabilities of η_n are dependent on the state of process ξ_n, therefore we study the two processes together.

The global model

By these assumptions, we define the compound process (ξ_n, η_n) with the states (i, j), where $i \in \Omega$ and $j \in \Phi$ and the state transition probabilities are as follows:

$$p_{i,j,u,v} = Pr[u \text{ ON source}, v \text{ cells in buffer at time } (n+1) | i, j \text{ at time } n],$$

with the steady-state probability matrix denoted by $\pi(i,j)$.

In the applied discrete time model there are two time scales associated to the input and the output link speed. The base (i.e. the time of a cell transmission) of the first one is referred to as the macro slot and the base of the second one is referred to as the micro slot. A macro slot is composed by C (integer) micro slots. The time unit was chosen the macro slot. All the above mentioned state transition probabilities are defined on the macro slot scale, since both ξ_n and (ξ_n, η_n) enjoys the Markov property.

The fact that i cells are generated during a macro slot when i sources are in the ON state imply that the state transition probabilities of the compound process (ξ_n, η_n) vary depending on the arrival process.

2.2 Cell arrival models

The accurate analysis of the introduced physical model requires a detailed knowledge on the distribution of the incoming cells in the macro slots and numerically it is very hard to evaluate even for small models. In [1] the authors studied three different special situations of the arrival process for which the performance parameters are easy to evaluate and provides information on the range of the performance measures.

Model 1: Arrivals occur at the beginning of the time slot

In this case it is assumed that one cell arrives from every ON source at the beginning of any time slot, so that the buffer content will be $\min(i+j, L)$ cells, where i is the number of ON sources and j is the number of cells in the buffer at the end of the previous time slot. Thus, the number of cells that will be found in the buffer at the end of the time slot can be written as:

$$\eta_{n+1} = \max(\min(j+i, L) - C, 0)$$

Using the above approach, the number of lost cells $c_{i,j}$ and the total delay of cells $d_{i,j}$ in state (i,j) can be expressed in the following form:

$$c_{i,j} = \max(i+j-L, 0)$$

$$d_{i,j} = \sum_{l=j}^{\min(i+j,L)-1} l,$$

where the delay is measured in the micro slot unit.

Model 2: Cells arrive one-by-one in the micro slot starting when the buffer becomes empty, and the remaining cells (if any) arrive at the end of time slot

For state (i,j)

$$\eta_{n+1} = \min(\max(j+i-C, 0), L),$$

$$c_{i,j} = \max(i+j-C-L, 0),$$

and

$$d_{i,j} = \sum_{l=0}^{\min(i+j-C+1,L)-1} l$$

hold, where $\max(C-j,0)$ is the number of empty micro slots after all the cells being served when $C>j$ and $\max(j-C,0)$ gives the number of cell remaining at the end of the slot. It is obvious, that only one of the above quantities can take positive value at the same time.

Model 3: Cells arrive in batch in the micro slot after the buffer becomes empty or at the end of macro time slot

In this case we assume that cells arrives in batch either immediately after the buffer becomes empty or at the end of macro time slot if the buffer is not empty in the macro time slot.

For state (i,j)

$$\eta_{n+1} = \max(\min(\max(j-C+1,0)+i,L)-\max(C-j,1),0)$$

$$c_{i,j} = \max(\max(j-C+1,0)+i-L,0),$$

and

$$d_{i,j} = \sum_{l=\max(j-C+1,0)}^{\min(\max(j-C+1,0)+i,L)-1} l.$$

2.3 Performance Parameters

Taking into account the model alternatives used to describe the arrival procedure for process (ξ_n,η_n), it can be seen that, for any time instant n, (ξ_{n+1},η_{n+1}) depends only on (ξ_n,η_n), which means it is a DTMC with transition probabilities $p_{i,j,u,v}$ defined as follows:

$$p_{i,j,u,v} = \begin{cases} p_{i,u} & \text{if } \eta_n = j \text{ and } \eta_{n+1} = v \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $p_{i,u}$ is the transition probability of process ξ_n and v is calculated based on the above model alternatives.

With these transition probabilities, the steady-state probabilities $\pi = \{\pi(i,j)\}$ of the compound process (ξ_n,η_n) can be obtained from the well-known DTMC equations [2]. Then, the main performance parameters for the system can be given as follows:

- **The average cell loss**

$$Cl = \frac{\sum_{i=0}^{N}\sum_{j=0}^{L}\pi(i,j)\cdot c_{i,j}}{\sum_{i=0}^{N} i\cdot p_i}$$

  where p_i denotes the steady state probability of state i of process ξ_n, and the denominator gives the average number of the arrived cells.

- **The average cell delay**

$$D = \frac{\sum_{i=0}^{N}\sum_{j=0}^{L}\pi(i,j)\cdot d_{i,j}}{\sum_{i=0}^{N}\sum_{j=0}^{L}(i-c_{i,j})\cdot\pi(i,j)}$$

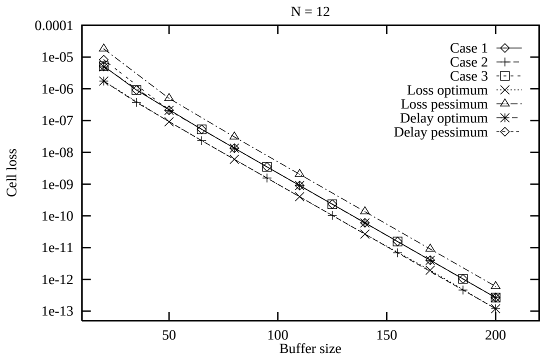

Figure 2: Cell loss probability, $N = 12$

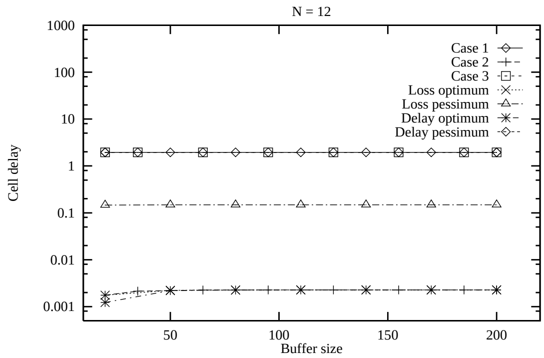

Figure 3: Average delay, $N = 12$

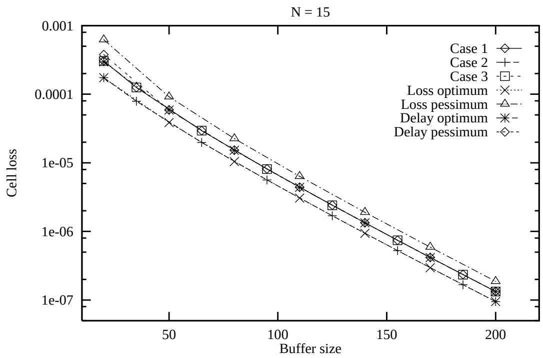

Figure 4: Cell loss probability, $N = 15$

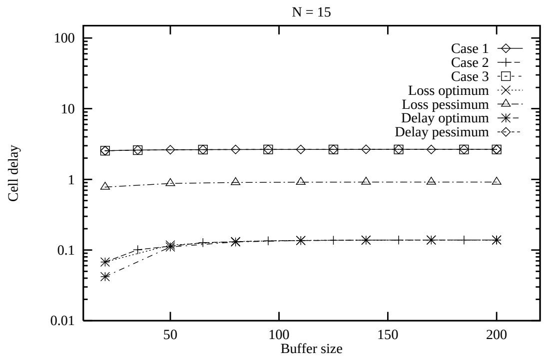

Figure 5: Average delay, $N = 15$

where the denominator gives the average number of transmitted cells.

3 Application of Markov Decision Processes

The behaviour of the system is modeled by a Discrete Time Markov Chain, as it is described in Section 2. The state transition probabilities are given by equation (1), that are visibly dependent on the cell arrival schedule.

In the sequel we address the problem of determining upper and lower bounds for the average cell loss probability and for the average delay. For this purpose we should determine the optimal and the worst cell arrival schedule, from the perspective of the studied performance parameter. We use the technique of Markov Decision Processes (MDP).

3.1 Summary of the concerning MDP results

The idea of the application of MDP consists in representing an action by a specific cell arrival schedule. Applying the appropriate cost function, the optimal and pessimal cell arrival schedule can be determined in each of the system states. The actions can be interpreted as the control of the input lines in a way to achieve the worst/best performance of the multiplexer.

Since there is no reason for applying discount rate in the cost structure, hence the optimal total cost is going to be infinite. Instead the growing rate of the cost is optimized, i.e. the average cost of the system per unit of time. There are two possibilities of the optimization, the technique called policy iteration and another method making use of the results concerning the

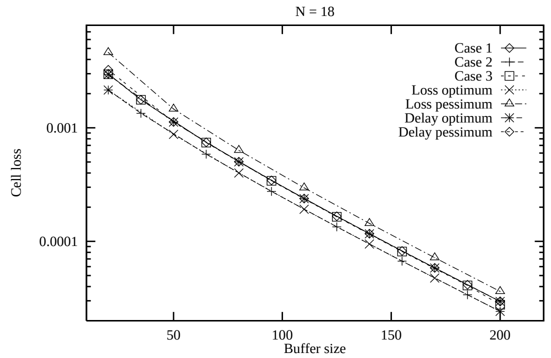

Figure 6: Cell loss probability, $N = 18$

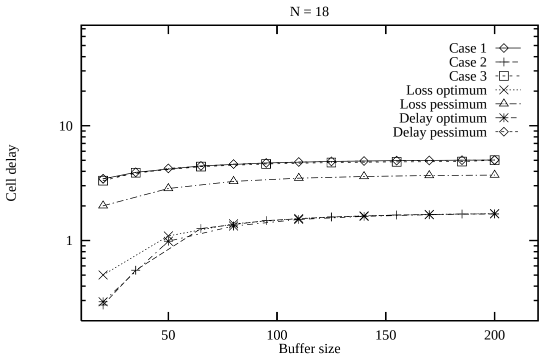

Figure 7: Average delay, $N = 18$

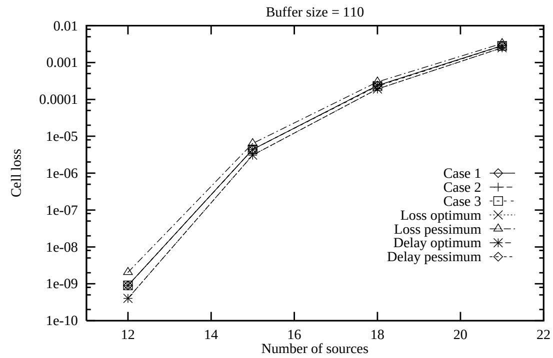

Figure 8: Cell loss probability, $B = 110$

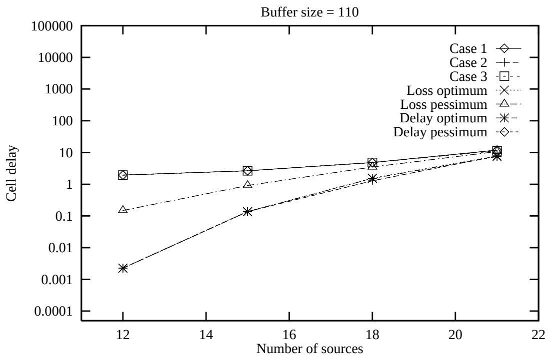

Figure 9: Average delay $B = 110$

discounted systems. In the sequel we briefly summarize these methods.

Let us assume the following notation. We denote by $a \in A$ the actions of the system, in our case a stands for the arrival schedule of the cells. We denote by f a policy, i.e. the mapping that for each state determines the chosen action: $f : (\Omega, \Phi) \to A$. $p_{i,j,u,v}(a)$ is the state transition probability from state (i, j) to state (u, v) when action a is chosen in state (i, j), and $C(i, j, a)$ constant cost is paid. We denote the long term average cost (growing rate) of the system that applies policy f by g_f.

Policy iteration

Assume we already have an initial f policy, and we would like to improve it by changing some of its decisions. First we determine the growing rate (g_f) and a set of constants ($v_f(i, j), i \in \Omega, j \in \Phi$) by solving the equation set of $\#\Omega \times \#\Phi$ equations:

$$g_f = C(i, j, f(i, j)) - v_f(i, j) + \sum_{u \in \Omega, v \in \Phi} p_{i,j,u,v}(f(i, j)) v_f(u, v). \tag{2}$$

(Here we have one more variable than equation, however the $v_f(i, j)$ values will differ only in an additive term, while g_f will remain the same. Fixing the value of one of these variables, say $v_f(0, 0)$, the rest of the unknown variables can be calculated.) Then for each (i, j) state and each valid a action we calculate the test quantity

$$t_f(i, j) \hat{=} C(i, j, a) - v_f(i, j) + \sum_{u \in \Omega, v \in \Phi} p_{i,j,u,v}(a) v_f(u, v). \tag{3}$$

The improved f^* policy chooses in state (i,j) the action, for which the test quantity is minimal:

$$f^*(i,j) \,\hat{=}\, \arg\min_a \left\{ C(i,j,a) - v_f(i,j) + \sum_{u\in\Omega, v\in\Phi} p_{i,j,u,v}(a) v_f(u,v) \right\}. \tag{4}$$

Then the procedure is repeated with the new f^* policy. It can be proved that if no improvement can be achieved by changing any the policy's decision, then the optimal policy is found [2].

Solution through discounted systems

We make use of the fact that there exists $\beta_0 < 1$ for which if $1 \geq \beta \geq \beta_0$ then for the systems that differ only in the β discount rate, the same f_β policy will be optimal. As a consequence the optimal policy for the case of no discount can be determined as the limiting policy for systems with β discount rate, if β tends to 1 [5].

For this we should find the optimal policy for systems where β discount rate is involved in the cost structure. We have the methods of policy iteration and successive approximation.

1. policy iteration

   For an initial f policy we calculate the expected total cost function (assumed β discount rate) by solving the equation set

   $$V_f^\beta(i,j) = C(i,j,f(i,j)) + \beta \sum_{u\in\Omega, v\in\Phi} p_{i,j,u,v}(f(i,j)) V_f^\beta(u,v). \tag{5}$$

   Then the improved f^* policy is defined as

   $$f^*(i,j) \,\hat{=}\, \arg\min_a \left\{ C(i,j,a) + \beta \sum_{u\in\Omega, v\in\Phi} p_{i,j,u,v}(a) V_f^\beta(u,v) \right\}. \tag{6}$$

   Then the procedure is repeated with the new f^* policy. It can be proved that if no improvement can be achieved by changing any the policy's decision, then the optimal policy is found [2].

2. successive approximation

   The optimal cost function can be calculated directly in discounted case by defining the mapping M over the set of bivariate bounded functions:

   $$(MV)(i,j) = \min_a \left\{ C(i,j,a) + \beta \sum_{u\in\Omega, v\in\Phi} p_{i,j,u,v}(a) V(u,v) \right\}. \tag{7}$$

   It can be proved, that the repeated application of this mapping results in the optimal cost function, independently of the initial function:

   $$M^n V \to V_{opt}^\beta.$$

   From the optimal cost function the optimal policy is derived as:

   $$f^*(i,j) \,\hat{=}\, \arg\min_a \left\{ C(i,j,a) + \beta \sum_{u\in\Omega, v\in\Phi} p_{i,j,u,v}(a) V_{opt}^\beta(u,v) \right\}. \tag{8}$$

The aim of the analysis is to determine	The applied cost function
minimal average cell loss rate	number of lost cells
maximal average cell loss rate	N - number of lost cells
minimal average delay rate	total delay in the slot
maximal average delay rate	B^2 - total delay in the slot

Table 1: The applied cost functions

3.2 Implementation issues

The cost function values are summarized in Table 1. The minimization of the average cost with these cost functions results in cell arrival schedules representing optimal or pessimal schedule from the perspective of the corresponding performance measures.

In our studies difficulties was initiated by the large number of the possible decisions (arrival schedules). If n cells arrive, when the speed of the output line is C times the speed of an input link, $\binom{n+C-1}{C}$ arrival schedules are possible (if the cells are not distinguished from each other). In the worst case, when all the N sources are active, we get a large number. For instance if $N = 30$, $C = 21$ (the configuration analyzed in [1]) $\binom{50}{21} \approx 6.73 \cdot 10^{13}$.

However we should note that we are only interested in the result of the cost function, which is the same for several arrival schedule, i.e. schedules, where the cost function value and the number of cells remaining in the buffer is the same, respectively. This way we can reduce the number of schedules to study, for instance if we want to determine the schedule causing the minimal cell loss, then the cost in a state is the number of lost cells, in the worst case N, and at the end of the slot at most B cells will remain, thus at most $(N+1)\cdot(B+1)$ different arrival schedule is relevant, which is already a treatable number. However to scan the possible schedules requires the most computation.

4 Numerical example

Begain et al. demonstrated the calculation of the performance parameters on the following example [1]. They examined an ATM multiplexer, to which voice transmission input lines were connected. The mean talkspurt time was chosen to be 352 ms, while the mean silence time 650 ms [6].

In this paper the optimal and pessimal cell arrival schedules and the corresponding performance parameters are calculated for the parameter values $C = 10$, $N = 12, 15, 18, 21$, for various buffer lengths. These values are less than the ones used by Begain et al. because the same parameter value set would have caused much longer calculation times while the conclusions regarding the cell arrival schedules can be made based on the proposed data as well. The presented results were obtained using the method based on discounted systems. The optimal cost functions were approximated by the method of successive approximation.

The results are depicted on Figures 2-8. The cell arrival models studied by Begain et al. are numbered by 1, 2 and 3. These results are shown together with the results derived by applying the cell arrival schedules minimizing and optimizing the average cell loss rate and the average cell delay. It can be seen that Model 2 yields practically the same result as the result corresponding to the optimal cell loss rate arrival schedule, while the worst case represents much worse results (approximately two times more loss rate) than any of the three original models.

The policy that is the worst from the perspective of the average cell loss rate can be described as if the cells in the buffer at the beginning of the slot, plus the cells arriving in the current slot is enough to cause buffer overflow, then the policy orders all the cells to arrive in the first minislot. If the number of the cells is not enough to achieve cell loss, then the cells are scheduled to arrive in the last minislot, raising the chance that in the next slot cells will be lost. Of course if only one cell arrives in the current slot, it does not matter in which minislot it comes, since in the applied model it can leave the multiplexer in the same minislot.

The figures with the graphs of the average cell delay show that the Model 1 corresponds to the arrival schedule causing the worst, and Model 2 corresponds to the arrival schedule causing the optimal average cell delay.

Finally we can conclude that the higher the number of sources are, the difference between the best and the worst performance attributes becomes less, that can be an important argument when the analysis of a multiplexer is necessary (Figures 8 and 9). Another conclusion of the analysis is that if the buffer is dimensioned to ensure the low cell loss probability prescribed in ATM networks, then the delay parameters hardly change.

5 Conclusion

A simple discrete time model of an ATM multiplexer is proposed, however, an exact analysis based on the applied model is a computationally intractable problem. To reduce the computational complexity of the evaluation of performance measures, such as the cell loss probability and the average cell delay, bounds for these measures are calculated. This paper provides the detailed analysis how to determine the performance bounds, which can be calculated in a reasonable response time. The results can serve as a basis for practical ATM multiplexer dimensioning.

References

[1] K. Begain, L. Jereb, M. Telek, and T. V. Do. Simple discrete time models for performance parameters of multiplexer with homogeneous on-off sources. *Journal on Communications*, XLVI:55–58, September 1995.

[2] R.A. Howard. *Dynamic Probabilistic Systems, Volume II: Semi-Markov and Decision Processes.* John Wiley and Sons, New York, 1971.

[3] F. Hübner and P. Tran-Gia. Performance of the finite capacity asynchronous multiplexer with modulated input. *Telecommunication Systems*, 1:263–278, 1993.

[4] S. Q. Li. A new performance measurement for voice transmission in burst and packet switching. *IEEE Transactions on Communications*, COM-35:1083–1094, Nov 1987.

[5] S. M. Ross. *Applied Probability Models with Optimization Applications.* Dover Publications, Inc., New York, 1992.

[6] K. Sriram and W. Whitt. Characterizing superposition arrival processes in packet multiplexers for voice and data. *IEEE J. Selected Areas on Communications*, SAC-4(6):833–846, 1986.