

# A distribution estimation method for bounding the reward measures of large MRMs\*

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## ABSTRACT

There is an alternative approach for the numerical analysis of large Markov reward modes. Instead of the direct calculation of the distribution of reward measures, one can apply a two-step method, where the first step is the analysis of the moments of required reward measures and the second step is the distribution estimation based on these moments [6].

The advantage of this approach is that the moments of the reward measures are still computable for such large Markov reward models which can not be analyzed by any direct reward analysis method (e.g., [1, 2, 4, 3]).

The disadvantage of this approach is that the approximation of the distribution of reward measures is rather loose at around the mean of the distribution, but it provides reasonable tight bounds for extreme values. These features makes the two step approach applicable e.g., for the analysis of large safety critical systems.

In previous works we presented effective numerical analysis techniques for the calculation of moments of reward measures [9, 7, 8], and we also developed a numerical procedure for approximating the distribution of reward measures based on their moments [5]. Unfortunately, we could not prove that the numerical procedure presented in [5] and applied in the MRMSolve tool [6] provides valid bounds of the distribution of the reward measures till now.

This missing piece of proof, which makes the overall approach formally valid, is presented this time.

## References

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\*This work was partially supported by Hungarian Scientific Research Fund (OTKA) under Grant No. T-34972.

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