Waiting time analysis of BMAP vacation queue and its application to IEEE 802.16e sleep mode

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ABSTRACT

The paper deals with the continuous-time $BMAP/G/1$ queue with multiple vacations and with its application to IEEE 802.16e sleep mode. The lengths of the vacation periods have general distribution and they depend on the number of preceding vacations (dependent multiple vacation). We obtain new formulas for the vector Laplace-Stieljes transform of the stationary virtual waiting time and for its first two moments in case of First-Come First-Serve scheduling. Finally the application of this vacation model to IEEE 802.16e sleep mode mechanism is demonstrated.

Categories and Subject Descriptors

G.3 [**Mathematics of Computing**]: Probability and Statistics—*Queueing theory*

General Terms

Theory, Performance

Keywords

Queueing theory, multiple vacation model, BMAP, waiting time

1. INTRODUCTION

Queueing models with server vacation are effective instruments in analysis of telecommunication models. For more details on vacation models we refer to the excellent book of Takagi [1] and the survey of (Doshi [2]).

Since the introduction of batch Markovian arrival process ($BMAP$) by Lucantoni [3] many authors investigated queueing models with $BMAP$. The reason is that $BMAP$ enables more realistic and more accurate traffic modeling. Most of these works apply the standard matrix analytic-method pioneered by Neuts [4] and further extended by many others (see e.g., [5]). However only a few works are available on $BMAP$ queueing models with server vacation.

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Chang and Takine [6] considered a class of $BMAP$ queues with generalized vacation and determined the vector probability generating function (vector GF) of the stationary queue length and its factorial moments for models with exhaustive discipline. Turck et al. [7] investigated a discrete-time $BMAP/G/1$ queue with multiple vacations, exhaustive discipline and First-Come First-Serve (FCFS) scheduling. This time-slotted model allows that the fixed lengths vacation periods depend on the number of preceding vacations. They applied this model to the IEEE 802.16e sleep mode mechanism in wireless access networks.

In this paper we investigate a generalized continuous-time counterpart of the model of [7], in which the lengths of the vacation periods can have a general distribution. In this model the vacation periods depend on the number of preceding vacations. We call this vacation strategy as *dependent multiple vacation*. To the best knowledge of the authors, no waiting time results are available for this continuous-time vacation model.

The motivation of this work is to give a general queueing model for the analyzing the performance of the sleep mode mechanisms in wireless networks. The model can take the traffic correlation into account, since the arrival process is $BMAP$. The analytical results of the model can be used to study the first two moments of the packet delay as a function of a traffic intensity or a traffic correlation parameter. Furthermore the model can predict the influence of the sleep mode parameters on the trade-off between the mean packet delay and the mean power consumption, which facilitates the tuning of these parameters to the requirements of the actual application scenario.

The queueing theoretic contribution of this paper is the new formulas for the vector Laplace-Stieljes transform (vector LST) of the stationary virtual waiting time and for its first two moments. The derivation of the vector LST of the stationary virtual waiting time is based on determination of two joint transforms: the joint transform of the stationary number of customers in the system and the forward recurrence vacation time, as well as the joint transform of the stationary number of customers in the system and the forward recurrence customer service time at an arbitrary instant in service period.

We demonstrate the application of this vacation model to the IEEE 802.16e sleep mode mechanism [8] by establishing the formulas for determining the mean packet delay and the mean power consumption.

The rest of this paper is organized as follows. In section 2 we introduce the model and the notations. The derivation of the joint transforms follows in Section 3. The new formulas of vector LST of the stationary virtual waiting time and its first two moments are derived in section 4. In section 5 we determine the stationary probability vector at start of the whole vacation. The application to IEEE 802.16e sleep mode mechanism is discussed in section 6. Final remarks close the paper in section 7.

2. MODEL AND NOTATION

2.1 BMAP process

The details of $BMAP$ related definitions and notations can be found in [3]. Here we summarize only the parts, which are needed for our analysis.

The $BMAP$ batch arrival process is characterized by $\{(\Lambda(t), J(t))\,; t \geq 0\}$ bivariate continuous-time Markov chain (CTMC) on the state space $(\Lambda(t), J(t))$; where $(\Lambda(t) \in \{0, 1, \ldots\})$ denotes the number of arrivals in $(0, t]$ and $(J(t) \in \{1, 2, \ldots, L\})$ is the phase, the state of a background CTMC (phase process), at time t. The infinitesimal generator of BMAP is given as

$$\begin{pmatrix} \mathbf{D}_0 & \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{D}_3 & \ldots \\ \mathbf{0} & \mathbf{D}_0 & \mathbf{D}_1 & \mathbf{D}_2 & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_0 & \mathbf{D}_1 & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where $\mathbf{0}$ and $\{\mathbf{D}_k; k \geq 0\}$ are $L \times L$ matrices.

$\mathbf{D}_0$ and $\{\mathbf{D}_k; k \geq 1\}$ govern the transitions corresponding to no arrivals and to batch arrivals with size k, respectively. The irreducible infinitesimal generator of the phase process is $\mathbf{D} = \sum_{k=0}^{\infty} \mathbf{D}_k$. Let $\boldsymbol{\pi}$ be the stationary probability vector of the phase process. Then $\boldsymbol{\pi}\mathbf{D} = \mathbf{0}$ and $\boldsymbol{\pi}\mathbf{e} = 1$ uniquely determine $\boldsymbol{\pi}$, where $\mathbf{e}$ is the column vector having all elements equal to one. $\widehat{\mathbf{D}}(z)$, the matrix generating function (matrix GF) of $\mathbf{D}_k$ is defined as

$$\widehat{\mathbf{D}}(z) = \sum_{k=0}^{\infty} \mathbf{D}_k z^k, \quad |z| \leq 1. \tag{1}$$

The stationary arrival rate of the BMAP,

$$\lambda = \boldsymbol{\pi} \left. \frac{d}{dz} \widehat{\mathbf{D}}(z) \right|_{z=1} \mathbf{e} = \boldsymbol{\pi} \sum_{k=0}^{\infty} k \mathbf{D}_k \mathbf{e}, \tag{2}$$

is supposed to be positive and finite.

2.2 The BMAP/G/1 queue with dependent multiple vacation and exhaustive service

Batch of customers arrive to the infinite buffer queue according to a $BMAP$ process defined by $\widehat{\mathbf{D}}(z)$. The service times are independent and identically distributed. B, $B(t)$, $\widetilde{B}(s)$, b, $b^{(2)}$, $b^{(3)}$ denote the service time r.v., its cumulated distribution function, its LST and its first three moments, respectively. The mean service time is positive and finite, $0 < b < \infty$. Due to the exhaustive service the customers are served until the queue becomes empty. Then the server takes the first vacation period. If the server, upon return from the r-th ($r \geq 1$) vacation period, finds the queue empty then it immediately takes the next vacation period, whose length depends on the number of preceding vacation periods. We call the model with this vacation strategy as *dependent multiple vacation model.* We define the *total vacation period* as the sum of all vacation periods until the next service. In addition we define the *cycle time* as a service period and the total vacation period together. The server utilization is $\rho = \lambda b$.

For every $r \geq 1$ the consecutive r-th vacation periods are independent and identically distributed. Thus let V_r, $V_r(t)$, v_r denote the length of the r-th ($r \geq 1$) vacation period, its cumulated distribution function and its mean, respectively. $\widetilde{V}_r(s)$ denotes the LST of V_r, which is defined as $\widetilde{V}_r(s) = \int_{t=0}^{\infty} e^{-st} dV_r(t)$. The arrival process, the customer service times and the vacation periods are mutually independent. The service is nonpreemtive. The FCFS scheduling is applied.

Although the length of total vacation period depends only on the phase of the $BMAP$ process at the start of total vacation period, the length of an interval until an arbitrary instant in total vacation period also depends on the whole arrival process. However the length of any interval inside of the r-th vacation period ($r \geq 1$) is independent of the arrival process, as r already implicitly includes a condition on the arrival process. Therefore, in order to utilize this independency, the description of the internal structure of the total vacation period is necessary.

In the following $[Y]_{i,j}$ stands for the i,j-th element of matrix $\mathbf{Y}$. Similarly $[y]_j$ denotes the j-th element of vector $\mathbf{y}$.

We define matrix $\mathbf{A}_k$, whose (i,j)-th element denotes the conditional probability that during a customer service time the number of arrivals is k and the initial and final phases of the $BMAP$ are i and j, respectively. That is, for $k \geq 0$, $1 \leq i, j \leq L$,

$$[\mathbf{A}_k]_{i,j} = P\left\{\Lambda(B) = k, J(B) = j | J(0) = i\right\}.$$

The matrix GF $\widehat{\mathbf{A}}(z)$ is defined as $\widehat{\mathbf{A}}(z) = \sum_{k=1}^{\infty} \mathbf{A}_k z^k$. $\widehat{\mathbf{A}}(z)$ can be expressed explicitly as ([3])

$$\widehat{\mathbf{A}}(z) = \int_{t=0}^{\infty} e^{\widehat{\mathbf{D}}(z)t} dB(t). \tag{3}$$

Since matrix $\widehat{\mathbf{A}}(1)$ is stochastic, we assume that $\widehat{\mathbf{A}}(z)$ can be inverted for $|z| \leq 1$.

To describe the arrivals during the r-th vacation period, for $r \geq 1$, we define matrices $\mathbf{U}_{r,k}$, whose (i,j)-th element, for $k \geq 0$, $1 \leq i, j \leq L$, is given as $[\mathbf{U}_{r,k}]_{i,j} = P\left\{\Lambda(V_r) = k, J(V_r) = j | J(0) = i\right\}$. The matrix GFs, $\widehat{\mathbf{U}}_r(z) = \sum_{k=0}^{\infty} \mathbf{U}_{r,k} z^k$, are given as

$$\widehat{\mathbf{U}}_r(z) = \int_{t=0}^{\infty} e^{\widehat{\mathbf{D}}(z)t} dV_r(t). \tag{4}$$

Similarly to describe the arrivals during the total vacation period, we define matrices $\mathbf{U}_{(k)}$. Let V denote the length of the total vacation period. The matrices $\mathbf{U}_{(k)}$ are defined by their (i,j)-th elements, for $k \geq 1$, $1 \leq i, j \leq L$, as $[\mathbf{U}_{(k)}]_{i,j} = P\left\{\Lambda(V) = k, J(V) = j | J(0) = i\right\}$. Using them the matrix GF of the number of arriving customers during the total vacation period is defined as

$$\widehat{\mathbf{U}}(z) = \sum_{k=1}^{\infty} \mathbf{U}_{(k)} z^k. \tag{5}$$

The case when the r-th vacation period occurs is described by means of matrix $\prod_{k=1}^{r-1} \widehat{\mathbf{U}}_k(0)$, whose (i,j)-th element denotes the conditional probability that during the first $r-1$ vacation periods no arrivals occur and the phases of the $BMAP$ at the start of the first vacation and at the end of the $r-1$-th vacation are i and j, respectively. We remark here that the empty product of matrices equals the unity matrix, which is denoted by $\mathbf{I}$. The model implies that in case of exactly r vacation periods definitely there is at least one arrival in the last vacation period and it is the only vacation period having arrival. Consequently the partial matrix GF of the number of customers arriving during the total vacation period consisting of exactly r vacation periods can be expressed by $\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)(\widehat{\mathbf{U}}_r(z)-\widehat{\mathbf{U}}_r(0))$. Summing up over r results in the matrix GF of the number of customers arriving during the total vacation period as

$$\widehat{\mathbf{U}}(z)=\sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)(\widehat{\mathbf{U}}_r(z)-\widehat{\mathbf{U}}_r(0)). \quad (6)$$

We define matrix $\widetilde{\boldsymbol{\mathcal{V}}}(s)$, which is related to the LST of the last vacation period, as

$$\widetilde{\boldsymbol{\mathcal{V}}}(s)=\sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\left(\widetilde{V}_r(s)\mathbf{I}-\widehat{\mathbf{U}}_r(0)\right), \quad (7)$$

Note that $\widetilde{\boldsymbol{\mathcal{V}}}(0)=\mathbf{I}$, since $\widetilde{V}_r(0)=1$ and $\prod_{k=1}^{\infty}\widehat{\mathbf{U}}_k(0)=\mathbf{0}$, because matrices $\widehat{\mathbf{U}}_k(1)$ are stochastic for $k\geq 1$.

Let t_ℓ^m denote the start of total vacation period in the ℓ-th cycle. The probability vector $\mathbf{m}$, is defined by its elements as

$$[\mathbf{m}]_j=\lim_{\ell\to\infty}P\left\{J(t_\ell^m)=j\right\}.$$

$\mathbf{m}$ is interpreted as the stationary probability vector of the phase process at starts of total vacation periods.

We define the vectors $\boldsymbol{p}_r$, $r\geq 1$, by their i-th entry, which is the probability that during a total vacation period, there are at least m vacation periods, and the phase of BMAP at the start of the r-th vacation period is i. The vectors $\boldsymbol{p}_r$ are given as

$$\boldsymbol{p}_r=\mathbf{m}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0). \quad (8)$$

The mean total vacation period, which is denoted by v, can be expressed by the help of $\boldsymbol{p}_r$, $r\geq 1$ as

$$v=\sum_{r=1}^{\infty}v_r\boldsymbol{p}_r\mathbf{e}=\mathbf{m}\sum_{r=1}^{\infty}v_r\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\,\mathbf{e}. \quad (9)$$

The stability of the model requires that the mean cycle time is finite. This directly implies that also the mean total vacation period must be finite. This leads to

$$v=\mathbf{m}\sum_{r=1}^{\infty}v_r\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\,\mathbf{e}<\infty. \quad (10)$$

Under this condition the model is stable if and only if $\rho<1$.

3. THE JOINT TRANSFORMS

In this section we derive expressions of joint transforms, which are needed to get the LST of the stationary virtual waiting time.

Let $N(t)$ be the number of customers in the system at time t. We introduce $F^v(t)$, which is the forward recurrence vacation time at time t in total vacation period, given that there is a virtual arrival at time t. It is defined as the interval from time t until the end of the total vacation period. The vector joint transform, $\widehat{\mathbf{q}}^v(z,s)$, is defined by its elements as

$$[\widehat{\mathbf{q}}^v(z,s)]_j=\lim_{t\to\infty}\sum_{n=0}^{\infty}\int_{\tau=0}^{\infty}e^{-s\tau}dP\big\{\ F^v(t)\leq\tau,N(t)=n,$$
$$J(t)=j\mid t\in t.v.p.\big\}z^n,\quad |z|\leq 1,\quad Re(s)\geq 0,$$

where t.v.p. stands for total vacation period. The $\widehat{\mathbf{q}}^v(z,s)$ is interpreted as the vector joint transform of the number of customers in the system and the forward recurrence vacation time at an arbitrary instant in total vacation period.

Similarly we introduce $F^c(t)$, which is the forward recurrence customer service time at time t in a service period, given that there is a virtual arrival at time t. It is defined as the interval from time t until the end of the service of the customer, which is under service at time t. The vector joint transform, $\widehat{\mathbf{q}}^c(z,s)$, is defined by its elements as

$$[\widehat{\mathbf{q}}^c(z,s)]_j=\lim_{t\to\infty}\sum_{n=0}^{\infty}\int_{\tau=0}^{\infty}e^{-s\tau}dP\big\{\ F^c(t)\leq\tau,N(t)=n,$$
$$J(t)=j\mid t\in s.p.\big\}z^n,\quad |z|\leq 1,\quad Re(s)\geq 0,$$

where s.p. stands for service period. The $\widehat{\mathbf{q}}^c(z,s)$ is interpreted as the vector joint transform of the number of customers in the system and the forward recurrence customer service time at an arbitrary instant in service period.

3.1 Joint transform in total vacation period

THEOREM 1. *The vector joint transform of the number of customers in the system and the forward recurrence vacation time at an arbitrary instant in total vacation period is given as*

$$\widehat{\mathbf{q}}^v(z,s)\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)=\frac{\mathbf{m}\left(\widehat{\mathbf{U}}(z)-\widetilde{\boldsymbol{\mathcal{V}}}(s)\right)}{v}. \quad (11)$$

PROOF. We introduce the vectors $\boldsymbol{p}_r^*$, $r\geq 1$, by their i-th entry, which is the probability that a random epoch in total vacation period (consisting of at least r vacation periods) belongs to r-th vacation period and the phase of BMAP at the start of r-th vacation period is i. Let us consider the Semi-Markov process in total vacation period ($t\geq 0$), whose state at time t composes from the phase of BMAP at the start of current vacation period (e.g. the r-th) and the index of the this vacation period (in that case r). Then $[\boldsymbol{p}_r]_i$ describes the probability of state (i,r) of the Markov chain embedded at starts of vacation periods and $\boldsymbol{p}_r^*$ is exactly the equilibrium distribution of the Semi-Markov process. Therefore vectors $\boldsymbol{p}_m^*$ can be expressed as

$$\boldsymbol{p}_r^*=\frac{v_r\boldsymbol{p}_r}{v}=\frac{v_r\mathbf{m}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)}{v}. \quad (12)$$

First we express $\widehat{\mathbf{q}}_r^v(z,s)$, which is the partial vector joint transform of the number of customers in the system and the forward recurrence vacation time at an arbitrary instant in the r-th vacation period for $r \geq 1$.

The vector GF of the stationary number of customers in the system at instant, when time τ ellapsed in r-th vacation period, is $\boldsymbol{p}_r^* \, e^{\widehat{\mathbf{D}}(z)\tau}$. The first term captures that a random epoch belongs to the r-th vacation period and the phase probability vector at the beginning of the r-th vacation period. The second term stands for the number of customers arriving in the $(0,\tau)$ interval of the r-th vacation period. The forward recurrence vacation time at instant τ equals $t-\tau$, where t is the length of the r-th vacation period. This is because the definition of forward recurrence vacation time includes a virtual arrival at time τ. To obtain the partial vector joint transform $\widehat{\mathbf{q}}_r^v(z,s)$ we need to take the LST of forward recurrence vacation time over the range of τ and to average the generating function of the stationary number of customers in the system over the duration of the r-th vacation period. This yields

$$\widehat{\mathbf{q}}_r^v(z,s) = \frac{\boldsymbol{p}_r^* \int_{t=0}^{\infty}\int_{\tau=0}^{t} e^{-s(t-\tau)} e^{\widehat{\mathbf{D}}(z)\tau} d\tau \; dV_r(t)}{v_r}. \quad (13)$$

Multiplying both sides of (13) by $\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)$ we have

$$\widehat{\mathbf{q}}_r^v(z,s) \;\; \left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) = \quad (14)$$
$$\frac{\boldsymbol{p}_r^*}{v_r}\int_{t=0}^{\infty} e^{-st}\int_{\tau=0}^{t} e^{\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\tau}\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) d\tau \; dV_r(t).$$

The internal integral term can be rewritten as

$$\begin{aligned}
&\int_{\tau=0}^{t} e^{\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\tau}\;\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\; d\tau = \\
&\int_{\tau=0}^{t}\;\sum_{k=0}^{\infty}\frac{\tau^k\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)^k}{k!}\;\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\; d\tau = \\
&\sum_{k=0}^{\infty}\int_{\tau=0}^{t}\tau^k\; d\tau\;\frac{\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)^{k+1}}{k!} = \qquad (15)\\
&\sum_{k=0}^{\infty}\frac{t^{k+1}}{k+1}\;\frac{\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)^{k+1}}{k!} = e^{\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)t}-\mathbf{I}.
\end{aligned}$$

Substituting (15) into (14), applying (4) and rearranging yields

$$\begin{aligned}
\widehat{\mathbf{q}}_r^v(z,s)\;\;\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) &= \frac{\boldsymbol{p}_r^*}{v_r}\int_{t=0}^{\infty}\left(e^{\widehat{\mathbf{D}}(z)t}-e^{-st}\mathbf{I}\right)\; dV_r(t) \\
&= \frac{\boldsymbol{p}_r^*\left(\widehat{\mathbf{U}}_r(z)-\widetilde{V}_r(s)\,\mathbf{I}\right)}{v_r}. \qquad (16)
\end{aligned}$$

The joint transform $\widehat{\mathbf{q}}^v(z,s)$ is given as $\widehat{\mathbf{q}}^v(z,s) = \sum_{r=1}^{\infty}\widehat{\mathbf{q}}_r^v(z,s)$, from which

$$\widehat{\mathbf{q}}^v(z,s)\;\;\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) = \sum_{r=1}^{\infty}\frac{\boldsymbol{p}_r^*\left(\widehat{\mathbf{U}}_r(z)-\widetilde{V}_r(s)\,\mathbf{I}\right)}{v_r}. \quad (17)$$

Applying (12) and rearranging results in

$$\begin{aligned}
&\widehat{\mathbf{q}}^v(z,s)\;\;\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) = \qquad (18)\\
&\frac{\mathbf{m}\sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\left(\widehat{\mathbf{U}}_r(z)-\widetilde{V}_r(s)\,\mathbf{I}\right)}{v} = \\
&\frac{\mathbf{m}\sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\left(\widehat{\mathbf{U}}_r(z)-\widehat{\mathbf{U}}_r(0)\right)}{v} - \\
&\frac{\mathbf{m}\sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\left(\widetilde{V}_r(s)\,\mathbf{I}-\widehat{\mathbf{U}}_r(0)\right)}{v}.
\end{aligned}$$

The statement comes by applying (6) and (7) in (18). $\square$

3.2 Joint transform in service period

Let $G(\ell)$ denote the number of customer services during the ℓ-th cycle, for $\ell \geq 1$. Additionally $t^s(\ell,r)$ denotes the instants of service start of the r-th customer in the ℓ-th cycle, for $\ell \geq 1$ and $1 \leq r \leq G(\ell)$. We define the vector GF of the stationary number of customers at service start epochs $\widehat{\mathbf{q}}^s(z)$ by its elements as

$$[\widehat{\mathbf{q}}^s(z)]_j =$$
$$\lim_{\ell\to\infty}\sum_{n=0}^{\infty}\frac{\sum_{r=1}^{G(\ell)} P\left\{N(t^s(\ell,r))=n, J(t^s(\ell,r))=j\right\}}{E[G(\ell)]} z^n, \quad |z| \leq 1.$$

THEOREM 2. *The vector joint transform of the number of customers in the system and the forward recurrence customer service time at an arbitrary instant in service period is given as*

$$\widehat{\mathbf{q}}^c(z,s)\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right) = \frac{\widehat{\mathbf{q}}^s(z)\left(\widehat{\mathbf{A}}(z)-\widetilde{\mathbf{B}}(s)\,\mathbf{I}\right)}{b}. \quad (19)$$

PROOF. To get the expression of $\widehat{\mathbf{q}}^c(z,s)$ the same line of argument can be applied as for obtaining (16) to express the partial vector joint transform $\widehat{\mathbf{q}}_r^v(z,s)$. We have to replace $\boldsymbol{p}_r^*$ by $\widehat{\mathbf{q}}^s(z)$, $\widehat{\mathbf{U}}_r(z)$ by $\widehat{\mathbf{A}}(z)$, $\widetilde{\mathbf{V}}_{\mathbf{r}}(s)$ by $\widetilde{\mathbf{B}}(s)$ and v_r by b and it results in the statement. $\square$

In the next proposition we give the expression of $\widehat{\mathbf{q}}^s(z)$, the only unknown in (19).

PROPOSITION 1. *The vector GF of the stationary number of customers at customer service start epochs can be expressed as*

$$\lambda\widehat{\mathbf{q}}^s(z)\left(z\mathbf{I}-\widehat{\mathbf{A}}(z)\right) = (1-\rho)\,z\frac{\mathbf{m}\left(\widehat{\mathbf{U}}(z)-\mathbf{I}\right)}{v}. \quad (20)$$

PROOF. Let $t^d(\ell,r)$ denote the instants at the departure of the r-th customer in the ℓ-th cycle, for $\ell \geq 1$ and $1 \leq r \leq G(\ell)$. Similar to $\widehat{\mathbf{q}}^s(z)$ we also define the vector GF of the stationary number of customers at customer departure epochs $\widehat{\mathbf{q}}^d(z)$ by its elements as

$$[\widehat{\mathbf{q}}^d(z)]_j =$$
$$\lim_{\ell\to\infty}\sum_{n=0}^{\infty}\frac{\sum_{r=1}^{G(\ell)} P\left\{N(t^d(\ell,r))=n, J(t^d(\ell,r))=j\right\}}{E[G(\ell)]} z^n, \quad |z| \leq 1.$$

Now we relate $\widehat{\mathbf{q}}^d(z)$ to $\widehat{\mathbf{q}}^s(z)$. The number of customers just before an arbitrary departure epoch equals the number of customers at previous customer service start plus the number of customers arriving during that service. This leads to the following $BMAP$ specific relation:

$$z\widehat{\mathbf{q}}^d(z) = \widehat{\mathbf{q}}^s(z)\,\widehat{\mathbf{A}}(z). \qquad (21)$$

We also define the vector GF of the stationary number of customers at an arbitrary instant $\widehat{\mathbf{q}}(z)$ by its elements as

$$[\widehat{\mathbf{q}}(z)]_j = \lim_{t\to\infty}\sum_{n=0}^{\infty} P\{N(t)=n, J(t)=j\}\, z^n, |z|\le 1.$$

Takine and Takahashi proved a stationary relationship between $\widehat{\mathbf{q}}(z)$ and $\widehat{\mathbf{q}}^d(z)$ [9], which is given as

$$\widehat{\mathbf{q}}(z)\,\widehat{\mathbf{D}}(z) = \lambda(z-1)\,\widehat{\mathbf{q}}^d(z). \qquad (22)$$

Finally we also need the factorization formula of Chang et al. [10], which is written as

$$\widehat{\mathbf{q}}(z)\left(z\mathbf{I}-\widehat{\mathbf{A}}(z)\right) = \widehat{\mathbf{q}}^v(z)\,(1-\rho)\,(z-1)\,\widehat{\mathbf{A}}(z). \qquad (23)$$

Post-multiplying (23) by $z\widehat{\mathbf{D}}(z)$, utilizing that $\widehat{\mathbf{A}}(z)$ and $\widehat{\mathbf{D}}(z)$ commute as well as applying (22) and (21) leads to

$$\lambda(z-1)\,\widehat{\mathbf{q}}^s(z)\left(z\mathbf{I}-\widehat{\mathbf{A}}(z)\right)\widehat{\mathbf{A}}(z) = (1-\rho)\,(z-1)\,z\widehat{\mathbf{q}}^v(z)\,\widehat{\mathbf{D}}(z)\,\widehat{\mathbf{A}}(z). \qquad (24)$$

Utilizing that the quantities occurring in (24) are continuous at $z=1$ and post-multiplying both sides by $\left(\widehat{\mathbf{A}}(z)\right)^{-1}$ for $|z|\le 1$ yields

$$\lambda\widehat{\mathbf{q}}^s(z)\left(z\mathbf{I}-\widehat{\mathbf{A}}(z)\right) = (1-\rho)\,z\,\widehat{\mathbf{q}}^v(z)\,\widehat{\mathbf{D}}(z). \qquad (25)$$

Setting $s=0$ in (11) gives $\widehat{\mathbf{q}}^v(z)$ as

$$\widehat{\mathbf{q}}^v(z)\,\widehat{\mathbf{D}}(z) = \frac{\mathbf{m}\left(\widehat{\mathbf{U}}(z)-\mathbf{I}\right)}{v}. \qquad (26)$$

Applying (26) in (25) results in the statement. □

4. STATIONARY VIRTUAL WAITING TIME

The virtual waiting time is the time period, which an arriving customer would experience at a time t until the start of its service. Note that there is not necessarily an arrival at time t, that is why it is called as virtual.

The virtual waiting time depends on the phase of the $BMAP$. Let $W(\tau)$ be the virtual waiting time in the system at time τ. We define the vector cumulated distribution function of the stationary virtual waiting time, $\mathbf{w}(t)$, by its elements as

$$[\mathbf{w}(t)]_j = \lim_{\tau\to\infty} P\{W(\tau)\le t, J(\tau)=j\}.$$

The vector LST of the stationary virtual waiting time is defined as

$$\mathbf{w}(s) = \int_{t=0}^{\infty} e^{-st}d\mathbf{w}(t), \quad Re(s)\ge 0.$$

4.1 LST of stationary virtual waiting time

THEOREM 3. *The vector LST of stationary virtual waiting time can be expressed as*

$$\mathbf{w}(s)\left(\widehat{\mathbf{D}}(\widetilde{\mathbf{B}}(s))+s\mathbf{I}\right) = (1-\rho)\,\frac{\mathbf{m}}{v}\left(\mathbf{I}-\widetilde{\boldsymbol{\mathcal{V}}}(s)\right). \qquad (27)$$

PROOF. Our argument to get the LST of stationary virtual waiting time is based on the unfinished work in the system.

An arriving customer sees the system with probability ρ in service period and with probability $1-\rho$ in vacation period.

Due to FCFS scheduling the waiting time at arrival during the service period is exactly the unfinished work, i.e. it consists of the forward recurrence customer service time of the customer currently under service and the customer service times of the customers, who are already present in the queue at virtual arrival. Note that the number of those customers is one less than the number of customers in the system. Similarly the waiting time at virtual arrival during the vacation period consists of the forward recurrence vacation time and the customer service times of the customers, who are already present at virtual arrival (unfinished work). This yields

$$\mathbf{w}(s) = \left(\rho\,\frac{\widehat{\mathbf{q}}^c(z,s)}{z} + (1-\rho)\;\widehat{\mathbf{q}}^v(z,s)\right)\Bigg|_{z=\widetilde{\mathbf{B}}(s)}. \qquad (28)$$

Setting $z=\widetilde{\mathbf{B}}(s)$ in (19) and (20) gives

$$\widehat{\mathbf{q}}^c(z,s)\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\Big|_{z=\widetilde{\mathbf{B}}(s)} = \frac{\widehat{\mathbf{q}}^s\left(\widetilde{\mathbf{B}}(s)\right)\left(\widehat{\mathbf{A}}(\widetilde{\mathbf{B}}(s))-\widetilde{\mathbf{B}}(s)\,\mathbf{I}\right)}{b}, \qquad (29)$$

$$\lambda\widehat{\mathbf{q}}^s\left(\widetilde{\mathbf{B}}(s)\right)\left(\widetilde{\mathbf{B}}(s)\,\mathbf{I}-\widehat{\mathbf{A}}(\widetilde{\mathbf{B}}(s))\right) = (1-\rho)\,\widetilde{\mathbf{B}}(s)\,\frac{\mathbf{m}\left(\widehat{\mathbf{U}}(\widetilde{\mathbf{B}}(s))-\mathbf{I}\right)}{v}, \qquad (30)$$

respectively. Combining them leads to

$$\rho\frac{\widehat{\mathbf{q}}^c(z,s)}{z}\left(\widehat{\mathbf{D}}(z)+s\mathbf{I}\right)\Big|_{z=\widetilde{\mathbf{B}}(s)} = -(1-\rho)\,\frac{\mathbf{m}\left(\widehat{\mathbf{U}}(\widetilde{\mathbf{B}}(s))-\mathbf{I}\right)}{v}. \qquad (31)$$

Multiplying (28) by $\left(\widehat{\mathbf{D}}(\widetilde{\mathbf{B}}(s))+s\mathbf{I}\right)$ and applying (31) as well as (11) yields

$$\mathbf{w}(s)\left(\widehat{\mathbf{D}}(\widetilde{\mathbf{B}}(s))+s\mathbf{I}\right) = -(1-\rho)\,\frac{\mathbf{m}}{v}\left(\widehat{\mathbf{U}}(\widetilde{\mathbf{B}}(s))-\mathbf{I}\right) + (1-\rho)\,\frac{\mathbf{m}}{v}\left(\widehat{\mathbf{U}}(\widetilde{\mathbf{B}}(s))-\widetilde{\boldsymbol{\mathcal{V}}}(s)\right). \qquad (32)$$

Rearranging (32) results in the statement. □

4.2 First two moments of stationary virtual waiting time

For matrix GF $\widehat{\mathbf{D}}(z)$, for $|z| \leq 1$, $\mathbf{D}^{(k)}$ denotes its k-th ($k \geq 1$) factorial moment. In addition $\mathbf{D}$ denotes its value at $z = 1$. Thus $\mathbf{D}^{(k)} = \frac{d^k}{dz^k}\widehat{\mathbf{D}}(z)|_{z=1}$ and $\mathbf{D} = \widehat{\mathbf{D}}(1)$.

Similarly for LSTs $\mathbf{w}(s)$ and $\widetilde{\boldsymbol{\mathcal{V}}}(s)$, for $Re(s) \geq 0$, $\mathbf{w}^{(k)}$ and $\boldsymbol{\mathcal{V}}^{(k)}$ denote their k-th ($k \geq 1$) moment, respectively. Thus $\mathbf{w}^{(k)} = (-1)^k \frac{d^k}{ds^k}\mathbf{w}(s)|_{s=0}$ and $\boldsymbol{\mathcal{V}}^{(k)} = (-1)^k \frac{d^k}{ds^k}\widetilde{\boldsymbol{\mathcal{V}}}(s)|_{s=0}$.

THEOREM 4. *The first two vector moments of the stationary virtual waiting time can be expressed as*

$$\mathbf{w}^{(1)} = \frac{\mathbf{m}}{v}\frac{\boldsymbol{\mathcal{V}}^{(2)}\mathbf{e}\boldsymbol{\pi}}{2} - (1-\rho)\frac{\mathbf{m}}{v}\boldsymbol{\mathcal{V}}^{(1)}\mathbf{C}_1 + \boldsymbol{\pi}\mathbf{C}_2, \tag{33}$$

$$\begin{aligned} \mathbf{w}^{(2)} &= \frac{\mathbf{m}}{v}\frac{\boldsymbol{\mathcal{V}}^{(3)}\mathbf{e}\boldsymbol{\pi}}{3} \\ &+ \frac{\mathbf{m}}{v}\left(\boldsymbol{\mathcal{V}}^{(2)}\mathbf{e}\boldsymbol{\pi}\mathbf{C}_2 - (1-\rho)\boldsymbol{\mathcal{V}}^{(2)}\mathbf{C}_1\right) \\ &- 2(1-\rho)\frac{\mathbf{m}}{v}\boldsymbol{\mathcal{V}}^{(1)}\mathbf{C}_1\mathbf{C}_2 \\ &+ \boldsymbol{\pi}\left(2\mathbf{C}_2\mathbf{C}_2 - \left(b^2\mathbf{D}^{(2)} + b^{(2)}\mathbf{D}^{(1)}\right)\mathbf{C}_1\right) \\ &+ \boldsymbol{\pi}\frac{b^3\mathbf{D}^{(3)}\mathbf{e}\boldsymbol{\pi} + 3bb^{(2)}\mathbf{D}^{(2)}\mathbf{e}\boldsymbol{\pi} + b^{(3)}\mathbf{D}^{(1)}\mathbf{e}\boldsymbol{\pi}}{3(1-\rho)}, \end{aligned} \tag{34}$$

where matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ are defined as

$$\begin{aligned} \mathbf{C}_1 &= (\mathbf{D} + \mathbf{e}\boldsymbol{\pi})^{-1}\left(\mathbf{I} - \frac{\left(\mathbf{I} - b\mathbf{D}^{(1)}\right)\mathbf{e}\boldsymbol{\pi}}{(1-\rho)}\right), \\ \mathbf{C}_2 &= \frac{\left(b^2\mathbf{D}^{(2)} + b^{(2)}\mathbf{D}^{(1)}\right)\mathbf{e}\boldsymbol{\pi}}{2(1-\rho)} + \left(\mathbf{I} - b\mathbf{D}^{(1)}\right)\mathbf{C}_1. \end{aligned}$$

PROOF. Since $\left(\widehat{\mathbf{D}}(\widetilde{\mathbf{B}}(s)) + s\mathbf{I}\right)\Big|_{s=0}$ in (27) is singular we use the method used by Lucantoni in [3] and Neuts in [11], which utilizes that $(\mathbf{D} + \mathbf{e}\boldsymbol{\pi})$ is nonsingular. The details of such a derivation can be found in [12]. Starting from relation (27) and applying a similar line of arguments gives the theorem. □

5. COMPUTATION OF THE STATIONARY PROBABILITY VECTOR OF THE PHASE PROCESS AT START OF T.V.P.

In this section we give a computation method to determine the unknown $\mathbf{m}$ in (33) and (34).

We define the homogenous bivariate Markov chain $\left\{(N(t_k^d), J(t_k^d)); k \in \{1, \ldots\}\right\}$ on the state space $(N(t_k^d), J(t_k^d))$, where t_k^d denotes the k-th customer departure epoch for $k \geq 1$. We define matrix $\mathbf{G}$, whose (i, j)-th elements is given as the probability that starting from state $(n+1, i)$ in the Markov chain the first state visited in level n is (n, j), $n \in 0, 1, 2, \ldots$, $1 \leq i, j \leq L$.

THEOREM 5. *The stationary probability vector of the phase process at starts of total vacation periods is given by*

$$\begin{aligned} \mathbf{m} &= \mathbf{e_L}\left((\mathbf{I} - \mathbf{K}) \,||\, \mathbf{e}\right)^{-1}, \\ \mathbf{K} &= \sum_{r=1}^{\infty}\prod_{k=1}^{r-1}\widehat{\mathbf{U}}_k(0)\left(\widehat{\mathbf{U}}_r(\mathbf{G}) - \widehat{\mathbf{U}}_r(0)\right), \end{aligned} \tag{35}$$

where $\widehat{\mathbf{U}}_r(\mathbf{G})$ stands for $\sum_{k=0}^{\infty}\mathbf{U}_{r,k}\mathbf{G}^k$, for $r \geq 1$.

For computing matrix $\mathbf{G}$, the only unknown in (35), the standard algorithm of Lucantoni [3] can be applied.

PROOF. We define matrix $\mathbf{K}$, whose (i, j)-th element is given as the probability that the Markov chain embedded at the customer departure epochs (defined above), starting from the state $(0, i)$ returns to the level 0 for the first time by hitting the state $(0, j)$.

The unknown vector $\mathbf{m}$ is the invariant probability vector of $\mathbf{K}$ and therefore it satisfies $\mathbf{mK} = \mathbf{m}$. Rearranging yields

$$\mathbf{m}(\mathbf{I} - \mathbf{K}) = 0. \tag{36}$$

Matrix $\mathbf{K}$ is stochastic and hence $(\mathbf{I} - \mathbf{K})$ has rank $L-1$. Therefore an additional relation is required to solve (36) for $\mathbf{m}$. For this we use the normalization condition $\mathbf{me} = 1$. Let $\mathbf{e_i}$ stand for the row vector, whose i-th element equals to 1 and its other elements are 0. In addition let $\mathbf{Y} \,||\, \mathbf{x}$ denote the matrix $\mathbf{Y}$ with the last column replaced by the column vector $\mathbf{x}$. Now combining normalization condition with (36) gives $\mathbf{m}$ as

$$\mathbf{m} = \mathbf{e_L}\left((\mathbf{I} - \mathbf{K}) \,||\, \mathbf{e}\right)^{-1}. \tag{37}$$

Each customer arriving during the total vacation period generates a first passage described by matrix $\mathbf{G}$, and thus for $\mathbf{K}$ we get:

$$\mathbf{K} = \sum_{k=1}^{\infty}\mathbf{U}_{(k)}\mathbf{G}^k. \tag{38}$$

Applying (5), (6) and (38) results in the statement. □

6. APPLICATION TO THE IEEE 802.16E SLEEP MODE MECHANISM

The purpose of the IEEE 802.16e sleep mode mechanism ([8]) is to enable a power consumption reduction at the Mobile Stations (MSs) by utilizing the natural idle periods of the traffic. The MS periodically inserts *sleep intervals*, whose lengths are predetermined and negotiated with the Base Station (BS).

In the *sleep interval* the MS switches off its air interface and enters in the energy saving mode. At the end of the *sleep interval* the MS switches back for a short *listening interval* to check whether packets are waiting at BS for downlink traffic. If not then the MS enters into the next *sleep interval*. However if any packet arrived to the BS for the MS during the last *sleep interval* then the MS remains active and an *awake interval* starts. Thus the price for the MS power reduction is the higher packet delay, since the packets arriving during a *sleep interval* must wait until the end of the next *listening interval*. If packets arrive to the MS for uplink during a *sleep interval*, the MS immediately interrupts the *sleep* *interval* and remains active until all packets are transmitted in both directions.

The standard defines three types of power saving classes. In class type I. starting with the *initial-sleep interval* the size of the next *sleep interval* is always doubled until reaching the *final-sleep interval*, which is then repeated. Class type II. has fixed-length *sleep interval*. Finally in class type III. the *sleep interval* is negotiated only for one occasion.

We apply the presented queueing model for the power saving class of type I. We neglect the uplink traffic, since usually it is small compared to the downlink traffic. Thus the customers of the queueing model correspond to the packets sent from BS to MS. Each vacation period models the actual *sleep interval* together with the *listening interval* following it. Hence V_R, for $r \geq R$, is the sum of the fixed length *final-sleep interval* and the fixed length *listening interval*. However the doubling rule is relaxed. In other words:

$$\begin{aligned} V_r &= V_R, \quad r \geq R, \\ V_r &< V_R, \quad r < R. \end{aligned} \tag{39}$$

Taking (39) into account the quantities $\widehat{\mathbf{U}}(z)$, $\widetilde{\boldsymbol{\mathcal{V}}}(s)$ and E[#vacation periods per t.v.p.] can be expressed as

$$\begin{aligned} \widehat{\mathbf{U}}(z) &= \sum_{r=1}^{R-1} \prod_{k=1}^{r-1} \widehat{\mathbf{U}}_k(0) \left(\widehat{\mathbf{U}}_r(z) - \widehat{\mathbf{U}}_r(0) \right) \\ &+ \prod_{k=1}^{R-1} \widehat{\mathbf{U}}_k(0) \left(\mathbf{I} - \widehat{\mathbf{U}}_R(0) \right)^{-1} \left(\widehat{\mathbf{U}}_R(z) - \widehat{\mathbf{U}}_R(0) \right), \end{aligned} \tag{40}$$

$$\begin{aligned} \widetilde{\boldsymbol{\mathcal{V}}}(s) &= \sum_{r=1}^{R-1} \prod_{k=1}^{r-1} \widehat{\mathbf{U}}_k(0) \left(\widetilde{V}_r(s)\mathbf{I} - \widehat{\mathbf{U}}_r(0) \right) \\ &+ \prod_{k=1}^{R-1} \widehat{\mathbf{U}}_k(0) \left(\mathbf{I} - \widehat{\mathbf{U}}_R(0) \right)^{-1} \left(\widetilde{V}_R(s)\mathbf{I} - \widehat{\mathbf{U}}_R(0) \right), \end{aligned} \tag{41}$$

$$\begin{aligned} &E[\#\text{vacation periods per t.v.p.}] \\ &= \mathbf{m}\Big[\sum_{r=1}^{R-1} r \prod_{k=1}^{r-1} \widehat{\mathbf{U}}_k(0) \left(\widehat{\mathbf{U}}_r(1) - \widehat{\mathbf{U}}_r(0) \right) \\ &+ \prod_{k=1}^{R-1} \widehat{\mathbf{U}}_k(0) \widehat{\mathbf{U}}_R^2(0) \left(\left(\mathbf{I} - \widehat{\mathbf{U}}_R(0) \right)^{-1} \right)^2 \left(\widehat{\mathbf{U}}_R(1) - \widehat{\mathbf{U}}_R(0) \right) \\ &+ \prod_{k=1}^{R-1} \widehat{\mathbf{U}}_k(0) R \left(\mathbf{I} - \widehat{\mathbf{U}}_R(0) \right)^{-1} \left(\widehat{\mathbf{U}}_R(1) - \widehat{\mathbf{U}}_R(0) \right) \Big] \mathbf{e}. \end{aligned} \tag{42}$$

The primary performance measure in the IEEE 802.16e sleep mode mechanism is the mean packet delay, which can be given by the help of (33). Another object of interest is the saving in the energy consumption due to the *sleep intervals*. For this purpose we use the mean power consumption. Let T_l stand for the length of the *listening interval*. We assume that the power consumption is the same during all active periods, i.e. during transmitting, receiving and listening. $\mathcal{P}_s$, $\mathcal{P}_a$ and $\mathcal{P}$ denote the constant power during the *sleep intervals*, the constant power during the active periods and the power at an arbitrary time, respectively. Let α be the time fraction of the *listening intervals* in the total vacation period, which can be given as

$$\alpha = \frac{E[\#\text{vacation periods per t.v.p.}]T_l}{E[\text{length of t.v.p.}]}. \tag{43}$$

The time fraction of the *sleep intervals* equals the time fraction of the vacation $(1-\rho)$ multiplied by the time fraction of the *sleep intervals* in the total vacation period $(1-\alpha)$. The time fraction of the active period consists of the time fraction of the service period (ρ) and the time fraction of the *listening interval* $((1-\rho)\,\alpha)$. Using these arguments the mean power consumption can be expressed as

$$E[\mathcal{P}] = (\rho + (1-\rho)\alpha)\,\mathcal{P}_a + (1-\rho)(1-\alpha)\mathcal{P}_s. \tag{44}$$

7. FINAL REMARKS

It is a topic of future work to study the performance of the IEEE 802.16e sleep mode mechanism by applying the presented vacation model and to provide numerical examples.

The presented analysis can be extended to express also $\widehat{\mathbf{q}}(z)$ in terms of $\mathbf{m}$ and $\widehat{\mathbf{U}}(z)$.

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9. REFERENCES

[1] H. Takagi. *Queueing Analysis - A Foundation of Performance Evaluation, Vacation and Prority Systems, vol.1.* North-Holland, New York, 1991.

[2] B. T. Doshi. Queueing systems with vacations - a survey. *Queueing Systems*, 1:29–66, 1986.

[3] D. L. Lucantoni. New results on the single server queue with a batch markovian arrival process. *Stochastic Models*, 7:1–46, 1991.

[4] M. F. Neuts. *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach.* The John Hopkins University Press, Baltimore, 1981.

[5] D. L. Lucantoni. *The BMAP/G/1 queue: A tutorial, Models and Techniques for Performance Evaluation of Computer and Communications Systems, L. Donatiello and R. Nelson Editors.* Springer Verlag, 1993.

[6] S. H. Chang and T. Takine. Factorization and stochastic decomposition properties in bulk queues with generalized vacations. *Queueing Systems*, 50:165–183, 2005.

[7] K. D. Turck, S. D. Vuyst, D. Fiems, and S. Wittevrongel. Performance analysis of the IEEE 802.16e sleep mode for correlated downlink traffic. *Telecommunication systems*, 39:145–156, 2008.

[8] *IEEE 802.16e-2005, Part 16: Air interface for fixed and mobile broadband wireless access systems Ů Amendment 2: physical and medium access control layers for combined fixed and mobile operation in licensed bands - Corrigendum 1.* Feb. 2006.

[9] T. Takine and Y. Takahashi. On the relationship between queue lengths at a random instant and at a departure in the stationary queue with bmap arrivals. *Stochastic Models*, 14:601–610, 1998.

[10] S. H. Chang, T. Takine, K. C. Chae, and H. W. Lee. A unified queue length formula for BMAP/G/1 queue with generalized vacations. *Stochastic Models*, 18:369–386, 2002.

[11] M. F. Neuts. *Structured stochastic matrices of $M/G/1$ type and their applications*. Marcel Dekker, New York, 1989.

[12] Zs. Saffer and M. Telek. Analysis of BMAP/G/1 vacation model of non-M/G/1-type. In *LNCS, (EPEW 2008)*, volume 5261, pages 212–226, Sept. 2008.