A New Method for Spectral Shaping Coding

Peter Vámos[†] László Osváth[†] Miklós Telek[‡]

Technical University of Budapest, H-1521 Budapest, Hungary

I. Introduction

In many applications (e.g. digital optical or magnetic recording/transmission) it is important the spectral shaping of data to match it to the physical properties of the channel/storage medium. This can be carried out by using block codes [1, 2] or some adaptive method [4, 5]. In this paper we introduce a feed back controlled adaptive coder structure, together with some simulation results and a detailed analysis of its application in the most important DC suppressed code generation [2, 3, 6].

II. The Coder Structure

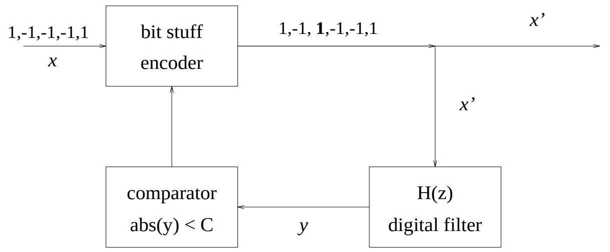

Figure 1: Feed back controlled bit stuff encoder

The bit stuff encoder has two states: either transmits a bit from the input to the output or inserts a redundant bit. The bit stuffing is controlled by the feed back loop. Whenever the level at the filter's output reaches a given threshold c the bit stuff encoder inserts a bit with opposite sign to the filter's output signal:

$$x'_{n+1} = \begin{cases} x_{m+1} & \text{if} \quad |y_n| < c \\ -\mathrm{sgn}(y_n) & \text{if} \quad |y_n| \geq c \end{cases}$$

The above structure works actually as a negative feed back with loop filter. The spectral components enhanced by the filter $\widetilde{H}(z) = 1 + z^{-1}H(z)$ will be dominant in the control of the bit stuff encoder, so the coder's interventions are diminishing their energy. Thus the output signal's spectrum will be suppressed where the $\widetilde{H}(z)$ filter is enhancing, and will be enhanced where the filter is suppressing. So the spectrum of the output signal can be described roughly as

$$S_{x'}(\omega) \approx \frac{1}{1 + |\widetilde{H}^2(e^{j\omega/F_0})|}. \qquad (1)$$

The decoding can be performed with a similar, but feed forward structure. To avoid the infinite error propagation the filter's pulse response should be finite in the following sense: $\lim_{n\to\infty} h(nT) = 0$. It implies, that any suppression in the spectrum can have only a finite, however arbitrarily large value. In exchange, we can use the information conveying the charge state, so the code will have a higher efficiency.

[†]Department of Telecommunications and Telematics, e-mail: vamos@ttt-202.ttt.bme.hu, osvath@ttt-202.ttt.bme.hu

[‡]Department of Communications, e-mail: telek@hit.bme.hu

III. Coding with Low-Pass FIR Filter

When $H(z) = \sum_{i=0}^{w-1} z^{-i}$ is inserted, then $y_n = \sum_{i=0}^{w-1} x'_{n-i}$. In accordance with (1) it is supposed to result in a DC suppressed code spectrum. The code itself will comply with the following constraint: the charge in a sliding window of length $w+1$ will never exceed $c-1$. The above constraint is same as in [7].

Definition 1: The states of the finite state machine describing the coder are the states of the FIR filter:

$$S(i) = (s_0(i), \ldots, s_{w-1}(i)).$$

The least significant bit $s_0(i)$ is first out, the most significant bit $s_{w-1}(i)$ was last in. The states are lexicographically ordered: $S(0) = (-1, \ldots, -1), \ldots, S(2^w - 1) = (1, \ldots, 1)$.

Definition 2: The state $S(j)$ is a child of $S(i)$ if they are contiguous: $j = \lfloor \frac{i}{2} \rfloor$ or $j = \lfloor \frac{i}{2} \rfloor + 2^{w-1}$

Definition 3: The weight of a state is defined as

$$\mathrm{W}\{S(i)\} = \sum_{j=0}^{w-1} s_j(i).$$

Definition 4 (Classification of states):

1 Light: $|\mathrm{W}\{S(i)\}| < c$, $N_L = \sum_{j=\frac{w-c}{2}+1}^{\frac{w+c}{2}-1} \binom{w}{j}$;

2 Heavy: $|\mathrm{W}\{S(i)\}| = c$, $N_H = 2\binom{w}{\frac{w-c}{2}}$;

3 Too heavy: $|\mathrm{W}\{S(i)\}| > c$, $N_T = 2\sum_{j=0}^{\frac{w-c}{2}-1} \binom{w}{j}$.

The too heavy states are invalid. The $q_{i,j} = \mathbb{P}(S(i) \to S(j))$ transition probabilities can be given by the coding rules:

$$q_{i,j} = \begin{cases} 1/2 & \text{if} \quad S(i) \text{ is light} \quad \text{and } S(j) \text{ is a child of } S(i) \\ 1 & \text{if} \quad S(i) \text{ is heavy and } S(j) \text{ is a child of } S(i) \\ & \qquad \text{and} \quad s_{w-1}(j) = -\mathrm{sgn}(\mathrm{W}\{S(i)\}) \\ 0 & \qquad \text{otherwise} \end{cases}$$

The states with same children can be joined. It implies the following theorem for state reduction:

Theorem 1: All states between $S(n2^j)$ and $S((n+1)2^j - 1)$ can be joined into one single state, if $S(n2^j)$ and $S((n+1)2^j - 1)$ are both light.

Since $q_{i,j} = q_{n-i,n-j}$, the state transition probability matrix $\mathbf{Q} = \|q_{i,j}\|_0^n$ is centralsymmetric, so it can be decomposed into four quadrants:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2\mathbf{T} \\ \mathbf{T}\mathbf{Q}_2 & \mathbf{T}\mathbf{Q}_1\mathbf{T} \end{bmatrix} \text{ where } \mathbf{T} = \begin{bmatrix} & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & \end{bmatrix} \qquad (2)$$

For the further reduction of the above matrix there is

Theorem 2: A matrix is centralsymmetric, iif it has two invariant subspaces, an even ($\mathcal{E}$), consisting of $\mathbf{v}_e^* = (\mathbf{v}^*, \mathbf{T}\mathbf{v}^*)$ even, and an odd ($\mathcal{O}$), consisting of $\mathbf{v}_o^* = (\mathbf{v}^*, -\mathbf{T}\mathbf{v}^*)$ odd vectors:

$$\mathbf{Q}\mathbf{v}_e = \begin{bmatrix} \mathbf{Q}_e\mathbf{v} \\ \mathbf{T}\mathbf{Q}_e\mathbf{v} \end{bmatrix}, \qquad \mathbf{Q}\mathbf{v}_o = \begin{bmatrix} \mathbf{Q}_o\mathbf{v} \\ -\mathbf{T}\mathbf{Q}_o\mathbf{v} \end{bmatrix};$$

where $\mathbf{Q}_e = \mathbf{Q}_1 + \mathbf{Q}_2$ and $\mathbf{Q}_o = \mathbf{Q}_1 - \mathbf{Q}_2$.

The $\mathbf{p} \in \mathcal{E}$ stationary distribution vector can be determined straight from the transition probabilities:

$$p_i = \begin{cases} 2p & \text{if} \quad S(i) \text{ is light} \\ p & \text{if} \quad S(i) \text{ is heavy} \end{cases}, \quad \text{where} \quad p = \frac{1}{2N_L + N_H}.$$

Knowing the stationary distribution we can express the rate:

$$R = 1 - \mathbb{P}(\text{stuffing}) = 1 - pN_H = \frac{2N_L}{2N_L + N_H}.$$

One can get the constrained channel's connection mx. from the state transition prob. mx. by substituting ones in places of its nonzero elements: $\mathbf{D} = [\mathbf{Q} \neq 0]$. Since $\mathbf{D}$ is centralsymmetric too, and $\mathbf{D}_e = [\mathbf{Q}_e \neq 0]$ is positive, so it inherits the maximal eigenvalue λ_{max} from $\mathbf{D}$, which is associated with the channel capacity: $C = \log_2 \lambda_{max}$.

IV. Mixing the Constraints

In most applications there is important to limit the run-length too [1, 2]. In addition to the above described window-charge (w, c) constraint, now we are applying a d, k constraint too: no runs can be shorter than $d+1$ and longer than $k+1$ bits. ($d < c$ and $d < k$ is required.)

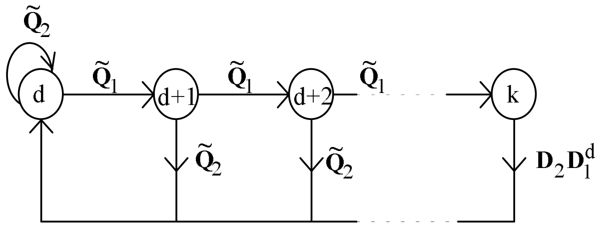

Figure 2: The run-length process

The channel capacity can be determined by using the variable length symbol representation, and the fact, that in the (2) decomposition of the state transition prob. mx. $\mathbf{Q}_1$ stands for continuing the current run, and $\mathbf{Q}_2$ stands for closing it, starting a new run with opposite sign. Introducing the notation $\mathbf{D}_1 = [\mathbf{Q}_1 \neq 0]$ and $\mathbf{D}_2 = [\mathbf{Q}_2 \neq 0]$, the connection matrix describing the run-length process reads as

$$\mathbf{D}_{d,k}(z) = \sum_{i=0}^{k-d} z^{-(d+1+i)} \mathbf{D}_1^i \mathbf{D}_2 \mathbf{D}_1^d.$$

Then the channel capacity is given as the base two logarithm of the $\det[\mathbf{D}_{d,k}(\lambda) - \mathbf{I}]$ characteristic polynomial's largest root.

To implement the run-length constraint we should insert an other feed back loop for monitoring the run-length. The coding rule will be the following:

$$x'_{n+1} = \begin{cases} -\text{sgn}(y_n) & \text{if} \quad \text{abs}(y_n) \geq c \\ -x'_n & \text{if} \quad \text{abs}\left(\sum_{i=0}^{k} x'_{n-i}\right) = k+1 \\ x'_n & \text{if} \quad \text{abs}\left(\sum_{i=0}^{d} x'_{n-i}\right) < d+1 \\ x'_n & \text{if} \quad \text{abs}\left(y_n - \sum_{i=0}^{d} x'_{n-w+1+i} - dx'_n\right) > c \\ x_{m+1} \text{ (no stuffing)} & \text{otherwise} \end{cases}$$

The last stuffing case is an auxiliary (a) constraint is applied to avoid the collision of c and d constraints after short runs.

The state transition probability matrix has the form of

$$\mathbf{Q}_{d,k} = \sum_{i=0}^{k-d-1} \widetilde{\mathbf{Q}}_1^i \widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{Q}}_1^{k-d} \mathbf{D}_2 \mathbf{D}_1^d;$$

where $\widetilde{\mathbf{Q}}_1$ stands for continuing the current run, and $\widetilde{\mathbf{Q}}_2$ stands for closing it by inserting $d+1$ bits with opposite sign, so $\widetilde{\mathbf{Q}}_2 = \mathbf{Q}_2 \mathbf{D}_1^d$. To get $\widetilde{\mathbf{Q}}_1$ we should slightly modify $\mathbf{Q}_1$ because of the a constraint. Introducing the $\mathbf{q}_1 = \widetilde{\mathbf{Q}}_1 \mathbf{1}$ and $\mathbf{q}_2 = \widetilde{\mathbf{Q}}_2 \mathbf{1}$ row sum vectors, and using the fact that $\widetilde{\mathbf{Q}}_1 + \widetilde{\mathbf{Q}}_2$ should be stochastic, $\widetilde{\mathbf{Q}}_1 = \text{diag}(\mathbf{q}_1)\mathbf{D}_1 = \text{diag}(\mathbf{1} - \mathbf{q}_2)\mathbf{D}_1$.

To get the coder's rate first we determine the average run-length on the output:

$$N_{out} = d + \mathbf{p}_{d,k}^* \sum_{i=0}^{k-d} \widetilde{\mathbf{Q}}_1^i \mathbf{1};$$

where $\mathbf{p}_{d,k}^* = \mathbf{p}_{d,k}^* \mathbf{Q}_{d,k}$ is the stationary distribution vector. Introducing the indicator vector $\mathbf{i} = [\mathbf{q}_1 == \frac{1}{2}] = [\mathbf{q}_2 == \frac{1}{2}]$, which selects the states where no stuffing is applied, and the coder gets a bit from the input, the average number of input bits per a run reads as

$$N_{in} = \mathbf{p}_{d,k}^* \sum_{i=0}^{k-d-1} \widetilde{\mathbf{Q}}_1^i \mathbf{i}.$$

Then the rate is given as $R = N_{in}/N_{out}$.

V. Coding with Low-Pass IIR Filter

If we are applying the $H(z) = 1/(1 - \alpha z^{-1})$ IIR filter as loop filter, then $y_n = \sum_{i=0}^{\infty} \alpha^i x'_{n-i} = x'_n + \alpha y_{n-1}$. The stationary distribution of y will tend to a continouse distribution if $1/2 \leq \alpha < 1$.

References

[1] H.Kobayashi, "A survey of coding schemes for transmission or recording of digital data," *IEEE Trans. Commun. Technol.*, vol. COM-19, pp. 1067–1100, Dec. 1971.

[2] K.A.S.Immink, *Coding Techniques for Digital Recorders*, Prentice Hall, London, 1991.

[3] K.A.S.Immink, "Performance of simple binary DC-constrained codes," *Philips J. Res.*, vol. 40, No. 1, pp. 1–21, 1985.

[4] J.C.Cavers and R.F.Marchetto, "A new coding technique for spectral shaping of data," *IEEE Trans. Commun.*, vol. COM-40, pp. 1418–1422, Sept. 1992.

[5] P.E.Bender and J.K.Wolf, "A universal algorithm for generating optimal and nearly optimal run-length-limited charge-constrained binary sequences," *Proc. IEEE Symp. on Inform. Theory*, San Antonio, Texas, Jan. 1993.

[6] A.X.Widmer and P.A.Franaszek, "Transmission code for high-speed fiber-optic data networks," *Electronics Letters*, vol. 19, pp. 202–203, 1983.

[7] H.Waldman and C.A.M.Pingarilho, "Spectral shaping codes," *Proc. IEEE Symp. on Inform. Theory*, Trondheim, Norway, June, 1994.