Blocking Probability Bounds in Multi-service CDMA Networks

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Abstract. Several previous contributions have proposed calculation methods that can be used to determine the steady state (and from it the blocking probabilities) of Code Division Multiple Access (CDMA) systems. This present work extends the classical Kaufman-Roberts formula such that it becomes applicable in CDMA systems in which elastic services with state dependent instantaneous bit rate and average bit rate dependent residency time are supported. Our model captures the effect of *soft blocking*, that is an arriving session may be blocked in virtually all system states but with a state dependent probability. The core of this method is to approximate the original irreversible Markov chain with a reversible one and to give lower and upper bounds on the so called partially blocking macro states of the state space. We employ this extended formula to establish lower and upper bounds on the steady state and the class-wise blocking probabilities.
Keywords — CDMA networks, Soft blocking, Kaufman-Roberts formula, reversible Markov chains

1 Introduction

The Kaufman-Roberts formula for a fixed transmission link carrying multi-rate traffic establishes a recursive relation between the macro states of the system[3]. This formula has been popular to calculate the steady state of systems with large state spaces, such as the transmission links of circuit switched and asynchronous transfer mode (ATM) networks [1]. A fundamental assumption of the Kaufman-Roberts formula is that the system is Markovian and that the so called local balance equations hold, which is the case in reversible Markov chains [3], [4].

The works by Stamatelos *et al.* and Rácz *et al.* extend this formula such that it includes elastic and adaptive traffic [5], [6]. The core idea in these papers is to construct a reversible Markov chain that well approximates the non-reversible system that supports elastic traffic. These papers continue to assume a fixed capacity transmission link.

Along another line, the seminal paper by Altman proposes a Shannon like capacity measure called the best effort capacity for CDMA networks supporting elastic services [7], [8]. This model and the calculation method makes use of some assumptions that lead to a reversible system and the steady state can be determined using

[3] This paper builds on the model and observation of an earlier work that has appeared at the International Teletraffic Congress, ITC 2005, Beijing, China, September 2005 [11].

product forms. The extension of this model proposed by Fodor and Telek in [10] uses matrix inversion to determine the steady state. The Kaufman-Roberts equation is attractive, because it does not suffer from the problem of state space explosion of realistic systems. It turns out to be especially useful when the state dependent soft blocking probabilities are only dependent on the macro states, which is a widely accepted assumption, see for instance [12] and [13]. We note that these latter papers do not model elastic traffic.

A series of works by T. Bonald and A. Proutiére are related to the present paper. These papers take explicitly account that sessions arrive dynamically and their sojourn time depends on the received bit rates. Reference [15] takes explicitly account the impact of the intra-cell interference cancellation principle that is used in High Data Rate CDMA systems. In such systems there is a time division multiple access (TDMA) component that can maximize the overall throughput. Among other aspects, the paper studies the blocking probabilities in such systems when the admission control algorithm is based on the number of active users or when it is based on the minimum data rate. Reference [16] presents a model and computational technique for a CDMA system where arriving calls can be blocked and interrupted due to *outages* and varying bit rate requirements of the mobiles. This model is applied to both the uplink and the downlink of CDMA systems. Finally, reference [17] presents a general model for the analysis of the traffic capacity of cellular data networks based on information theoretic considerations. This work is primarily applicable for time division and frequency division systems and a comparison is made with the information theoretical maximum capacity. This model describes useful relationships between the session-wise data rates and the used amount of resources (e.g. power levels) in TDMA and FDMA systems and presents results that highlight the importance of modeling the dynamic session arrivals and departures. The contribution of the present paper to these works is the development of a recursive relationship between the system states and based on this relationship the establishment of lower and upper bounds on the blocking probabilities when admission control is based on the minimum data rates.

In this paper we consider a multiple access interference (MAI) limited CDMA network basically as modeled in [7] and [10]. Soft blocking is modeled similarly to what has been done in [12] and [13]. This system is conveniently modeled by a Markov chain. Next, in order to reduce the number of states and to establish a recursive relationship, we define the system macro states. The macro states are defined much the same way as in the classical Kaufman-Roberts case, that is, the set of micro states in which the overall (CDMA) resource consumption is the same. A major difference compared to the classical case is that the system is (1) irreversible and (2) the macro states are heterogeneous in the sense that there are macro states that consist of a mixture of blocking and non-blocking states. As we shall see, this second feature is due to the non-linear relation between the resource consumption of the elastic sessions and their bit rates. This structure (the combination of these two features) requires some effort in terms of establishing the balance equations on

which the Kaufman-Roberts recursion can be built. Specifically, we distinguish non-blocking, *partially blocking* and fully blocking macro states. Depending on how the partially blocking macro states are treated and how the class-wise blocking probabilities are calculated from the steady state, we arrive at lower an upper bounds of the steady state probabilities and the class-wise blocking probabilities. We discuss the details of these bounds and present promising numerical results for a CDMA system supporting two elastic service classes.

The paper is organized as follows. Section 2 presents our basic model for elastic traffic in CDMA. In the subsequent section we develop the reversible approximation of this system and based on this approximation we propose an extension of the Kaufman-Roberts formula. Section 4 builds on this formula and proposes a method that can be used to bound the class-wise blocking probabilities. Section 5 presents numerical results. We conclude the paper in Section 6.

2 CDMA Uplink Equations and State Space Structure

The CDMA uplink model is similar to the single transmission link model in that sessions belonging to different service classes share a common resource. In CDMA however, a fast explicit rate control algorithm allows the system to slow down elastic sessions and thereby reduce the required power. In this section we extend the classical multi-rate model and put the CDMA uplink model into a multi-rate context. This model is described in details in [20] and has also been used in [7].

2.1 Basic CDMA Equations

We consider a single CDMA cell at which sessions belonging to one of I service classes arrive according to a Poisson arrival process of intensity λ_i ($i = 1, \ldots, I$). Each class is characterized by a peak bit-rate requirement $\hat{R}_i$ and an exponentially distributed nominal holding time with parameter μ_i. That is, an arriving session is assumed to have a fixed amount of data to send, so the transmission rate assigned to it determines the residency time of the session. This fixed amount of data is assumed to be an exponentially distributed random variable with mean value $\hat{R}_i/\mu_i$. (We note that although this assumption ignores the heavy tailed distribution of file sizes [21], it is an accepted assumption for the purpose of studying the trade-off between the assigned bit rate, the session holding time and the class-wise blocking probabilities, see [7], [18], [19].) When sending with the peak rate for a session, the required target ratio of the received power from the mobile terminal to the total interference energy at the base station is calculated as follows:

$$\tilde{\Delta}_i = \frac{E_i}{WN_0} \cdot \hat{R}_i, \quad i = 1, \ldots, I, \tag{1}$$

where E_i/N_0 is the signal energy per bit divided by the noise spectral density that is required to meet a predefined QoS (e.g. bit error rate, BER); noise includes both

thermal noise and interference. This required E_i/N_0 can be derived from link level simulations and from measurements. $\hat{R}_i$ is the peak bit rate of the session of class i and W is the spread spectrum bandwidth. As a first step, we ignore the interference caused by neighbor cells. (We will incorporate the neighbor cell interference into the model in the next subsection.)

Let n_i be the number of ongoing sessions of class i. We will refer to vector $\underline{n} = \{n_i\}, \quad i = 1, \ldots, I$ as the *state* of the system. We now assume that arriving sessions are blocked by a suitable admission control algorithm that prevents the system to reach the state in which the power that should be received at the base station would go to infinity. In other words, a suitable admission control algorithm must prevent the system to reach its *pole capacity* (as defined by Equation (8.10) of [20] and (5) of [7]). According to these definitions, the pole capacity is the polyhedron $\underline{n}^*$ that is defined by:

$$\underline{n}^* = \{\underline{n} : 1 = \sum_{i=1}^{I} n_i \Delta_i\}, \quad \text{where} \quad \Delta_i = \frac{\tilde{\Delta}_i}{1 + \tilde{\Delta}_i}, \quad i = 1, \ldots, I.$$

In order to model this, we use the standard equations (8.3)-(8.12) from [20] as follows. The power P_i that is received at the base station from the mobile terminal for session i must fulfill (assuming that the terminal can control the power level for each session separately):

$$\frac{P_i}{P_N + Y_{own} - P_i} = \tilde{\Delta}_i, \quad i = 1, \ldots, I; \quad Y_{own} = \sum_{i=1}^{I} n_i P_i. \tag{2}$$

where Y_{own} is the total power received by the base station from within its own cell (or sector), and P_N is the background noise power.

Rewriting (2), we get:

$$\frac{P_i}{P_N + \sum_{l=1}^{I} n_l P_l} = \Delta_i. \tag{3}$$

Multiplying with n_i and summing over i:

$$\sum_{i=1}^{I} P_i n_i = \frac{P_N \sum_{l=1}^{I} n_l \Delta_l}{1 - \sum_{l=1}^{I} n_l \Delta_l}. \tag{4}$$

Substituting (4) into (3) and denoting $\Psi = \Psi(\underline{n}) = \sum_{l=1}^{I} n_l \cdot \Delta_l$:

$$P_i = \left(P_N + \frac{P_N \cdot \Psi}{1 - \Psi}\right) \cdot \Delta_i = \frac{P_N \cdot \Delta_i}{1 - \Psi}, \quad i = 1, \ldots, I. \tag{5}$$

From (5) it is clear that the admission control algorithm must prevent that Ψ (often referred to as the *load factor*) becomes larger $\hat{\Psi} \triangleq 1-\epsilon;\ \epsilon > 0$. In the single class case, it means that the number of admitted sessions must fulfill: $n \leq \lfloor \hat{\Psi}/\Delta \rfloor$ (where we now let $n = n_1$ and $\Delta = \Delta_1$). In the rest of the paper we assume that such admission control algorithm operates in the system. We note that because of the relation expressed by Equation (3) and the definition of $\Psi(\underline{n})$, one can consider $\Psi(\underline{n})$ as the overall used "resource" in state $\underline{n}$ in the multi-rate CDMA system, while $\hat{\Psi}$ corresponds to the "total available capacity". (The value of $\epsilon > 0$ determines the normal operation region of the CDMA system and is usually determined such that the noise rise in the CDMA cell - defined as $\frac{Y_{own}+P_N}{P_N}$=$\frac{1}{1-\Psi}$ - always remains below 5-7 dB.) This observation can be seen as an analogy between the multi-rate CDMA model and the multi-rate loss models developed in the 80's and 90's [1]. We also mention that the non linear relation between the assigned resource and the rate requirement is well known, see for instance [2] for an early reference.

2.2 The Impact of Slow Down

Recall that the required target ratio (Δ_i) depends on the required bit-rate. Explicit rate controlled elastic services tolerate a certain slow down of their peak bit-rate ($\hat{R}_i$) as long as the actual instantaneous bit rate remains greater than the minimum required $\hat{R}_i/\hat{a}_i$. When the bit rate of a class-i session is slowed down to $\hat{R}_i/a_i$, ($0 < a_i \leq \hat{a}_i$) its required Δ_{a_i} value becomes:

$$\Delta_{a_i} = \frac{\tilde{\Delta}_i}{a_i + \tilde{\Delta}_i} = \frac{\Delta_i}{a_i \cdot (1-\Delta_i) + \Delta_i}, \quad i = 1, \ldots, I, \tag{6}$$

which increases the number of sessions that can be admitted into the system, since now all $\underline{n}$ states are feasible for which

$$\Psi_a = \sum_{i=1}^{I} n_i \cdot \Delta_{a_i} < \hat{\Psi}.$$

We use the notation $\Delta_{min,i} = \Delta_{\hat{a}_i}$ to denote the class-wise minimum target ratios (can be seen as the minimum resource requirement), that is when the session bit-rates of class-i are slowed down to that class' minimum value. The smallest of these $\Delta_{min,i}$ values $\Delta = \min_i \Delta_{min,i}$ can be thought of as the finest "granularity" with which the overall CDMA resource is partitioned between competing sessions.

It is the task of the bandwidth sharing policy to determine the $\Delta_{a_i} \geq \Delta_{min,i}$ values (and consequently the $a_i \leq \hat{a}_i$ class-wise instantaneous slow down factors) for each state of the system such that $\Psi_a \leq \hat{\Psi}$. Because of the admission control assumption, such a resource assignment is always possible in feasible states. The set of (micro) states $\underline{n}$ in which the overall resource consumption ($\Psi_a(\underline{n})$) is the same composes a macro state of the system. We number these macro states with $j = 1 \ldots J$ and denote the resource consumption in macro state with index j by ν_j.

Specifically, the jth macro state of the system ($\Omega(j)$) is conveniently defined as the set of micro states in which the overall resource consumption is ν_j, that is:

$$\Omega(j) = \{\underline{n} | \nu_j(\underline{n}) = \Psi_a(\underline{n}) = \sum_{i=1}^{I} n_i \Delta_{a_i}\}.$$

With a slight abuse of notation and terminology, where it is not confusing, we sometimes refer to *both* the index j and $\Omega(j)$ as "macro state j". Also, it will be useful to introduce the operator $\mathcal{J}(x)$ that returns the index of the macro state in which the used CDMA resource is x if such a macro state exists. This implies that specifically for ν_j: $\mathcal{J}(\nu_j) = j$.

The model input parameters are summarized in Table 1.

2.3 Modeling the Interference from Neighbor Cells

The interference contribution from other cells is typically quite high (around 30-40%). This is taken into account as follows. We think of the CDMA system as one that has a maximum of $\hat{n} = \frac{\hat{\Psi}}{\Delta}$ number of (virtual) channels. The neighbor cell interference ξ is a random variable of log-normal distribution. That is, its distribution function, mean and standard deviation respectively are given by:

$$D_\xi(x) = \frac{1}{2}\Big(1 + \text{erf}(\frac{\ln x - N}{\sqrt{2}S})\Big); \quad \alpha = \frac{\varphi}{\varphi + 1} \cdot \hat{n} \quad \text{and} \quad \sigma = \alpha, \tag{7}$$

where N and S are the corresponding parameters of the normal distribution and are obtained from α and σ as follows:

$$N = \ln\left(\frac{\alpha^2}{\sqrt{\alpha^2 + \sigma^2}}\right); \quad S = \sqrt{\ln\left(1 + \frac{\sigma^2}{\alpha^2}\right)};$$

$\text{erf}(x)$ is the error function defined as the integral of the Gaussian distribution: $\text{erf}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$; and φ is a factor characterizing the neighbor cell interference and is an input parameter of the model (Table 1). The mean value α is equal to the average capacity loss in the cell due to the neighbor cell interference and σ is chosen to be equal to α as proposed by [14] and also adopted by [12]. (When $\varphi = 0$, the neighbor cell interference is ignored in the model.)

Recall that we think of Ψ as the used resource in state $\underline{n}$. Then in a given state $\underline{n}$ let $b_\Psi(\underline{n})$ denote the probability that the neighbor cell interference is greater than the available capacity in the current cell (i.e. $(\hat{\Psi} - \Psi)$):

$$b_\Psi(\underline{n}) = Pr\{\xi > \hat{\Psi} - \Psi\} = 1 - Pr\{\xi < \hat{\Psi} - \Psi\} = 1 - D_\xi(\hat{\Psi} - \Psi). \tag{8}$$

The impact of state dependent soft blocking resulted, e.g. by the neighbor cell interference, can conveniently be taken into account by modifying the λ_i arrival rates in each state by the (state dependent) so called passage factor: $\sigma_i(\underline{n}) = g_i(1 - b_\Psi(\underline{n})) =$

Table 1. Model (Input) Parameters

I	Number of service classes
$\hat{R}_i$	Peak bit rate associated with class-i sessions
λ_i	Arrival intensity of sessions belonging to class-i
$1/\mu_i$	Mean (nominal) holding time of sessions belonging to class-i
$\hat{a}_i$	Maximum slow down (using the terminology of [7]) of $\hat{R}_i$
φ	Parameter of the other cell (sector) interference (see Equation (7))
E_i/N_0	Normalized signal energy per bit requirement of class-i

$g_i(D(\hat{\Psi} - \Psi(\underline{n})))$. The passage factor is the probability that a class-i session is not blocked by the admission control algorithm when such a session arrives in system state $\underline{n}$ [12]. Obviously, the passage factor of the hard blocking states is zero. [4] When $g_i(x) = x \ \forall k$, the passage factor only depends on the macro state of the system and is the same for all classes. This is the assumption of the current paper. As we shall also see in the Markov model, the hard blocking states will determine the "edges" of the state space, while the passage factors will affect the transition rates between states.

2.4 Determining the System State Space

Recall that in each $\underline{n}$ state of the system, the inequality $\sum_i n_i \cdot \Delta_{a_i} \leq \hat{\Psi}$ must hold. It follows that the maximum number of sessions from each class can be calculated as follows:

$$n_{max,i} = \lfloor \frac{\hat{\Psi}}{\Delta_{\hat{a}_i}} \rfloor, \quad i = 1, \ldots, I.$$

The states that satisfy the above inequality are the *feasible states* and constitute the state space of the system (Θ). The feasible states, in which the acceptance of an additional class-i session would result in a state outside of the state space are the class-i *blocking states*. The set of the class-i blocking states is denoted by Θ_i. Due to the "Poisson Arrivals See Time Averages" (PASTA) property, the sum of the class-i blocking state probabilities gives the (overall) class-i blocking probability [1].

In each feasible state, it is the task of the bandwidth sharing policy to determine the $\Delta_{a_i}(\underline{n})$ class-wise target ratios for each class. $\Delta_{a_i}(\underline{n})$ reflect the fairness criterion that is implemented in the resource sharing policy and is out of the scope of this paper. From these, the class-wise slow down factors and the instantaneous bit-rates of the individual sessions can be calculated as follows:

$$a_i(\underline{n}) = \frac{\Delta_i \cdot (1 - \Delta_{a_i}(\underline{n}))}{\Delta_{a_i}(\underline{n}) \cdot (1 - \Delta_i)}; \quad R_{a_i}(\underline{n}) = R_i / a_i(\underline{n}) \tag{9}$$

For ease of presentation, in the rest of the paper we will not indicate the dependence of a_i, Δ_{a_i} and R_{a_i} on the micro state $\underline{n}$. We note that the above mentioned

[4] From this point we somewhat casually use the term *blocking* to refer to *hard* blocking, while we explicitly spell out *soft blocking* when this casual usage is not confusing.

bandwidth sharing policy and the associated fairness issue using the same modeling assumptions as in this paper is the topic of [22].

2.5 An Example of the State Space

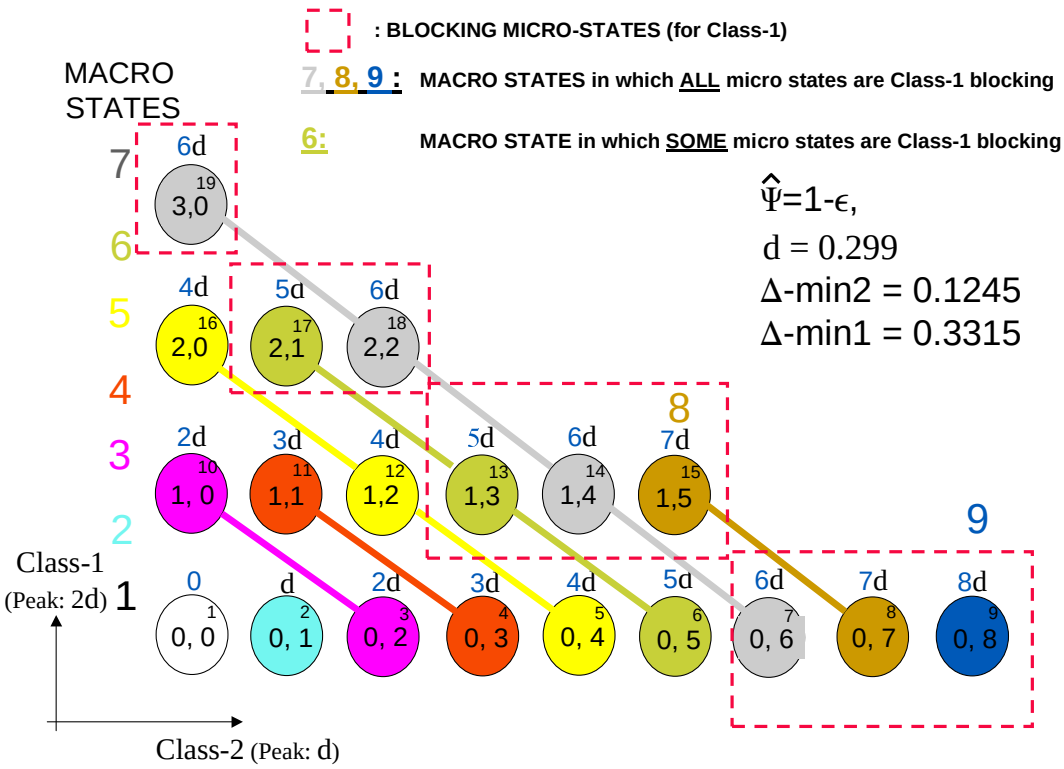

Fig. 1. An example of a state transition diagram with two service classes. There are 19 micro states and 9 macro states. For instance, macro state 6 comprises the Class-1 blocking micro states (2,1), (1,3) and the non-blocking micro state (0,5).

Figure 1 depicts an example of the state space. In this example we neglect the other-cell interference ($\varphi = 0$). There are two traffic classes with peak resource requirements $\Delta_1 = 2d$ and $\Delta_2 = d$ respectively (where $d = 0.299$). Both traffic classes are elastic and tolerate a slow down of their bit rates to one third of their respective peak bit rates (i.e. $\hat{a}_1 = \hat{a}_2 = 3$), which corresponds to $\Delta_{min,1} = 0.3315$ and $\Delta_{min,2} = 0.1245$.

This system can be in one of the 19 feasible micro states as illustrated in Figure 1. The 9 class-1 blocking micro states are indicated in dotted squares. There are 9 macro states, out of which macro state 6 (consisting of micro states: (2,1), (1,3) and (0,5)) deserves attention. In this macro state the overall peak resource consumption is $5d > \hat{\Psi}$ and therefore there is a need for slowing down some of the sessions. On the other hand, all three micro states are feasible states, because if all sessions are slowed down to their minimum rates, the resource consumptions become $2\Delta_{min,1} +$

$\Delta_{min,2} = 0.7875 < \hat{\Psi} = 1$, $\Delta_{min,1} + 3\Delta_{min,2} = 0.705$ and $5\Delta_{min,2} = 0.6225$. What is noteworthy is that micro states (2,1) and (1,3) are class-1 blocking states (micro states (3,1) and (2,3) are certainly not feasible), but micro state (0,5) is non-blocking, since micro state (1,5) is part of the state space ($\Delta_{min,1} + 5\Delta_{min,2} = 0.954$).

2.6 System Behavior

We now make use of the assumptions that the arrival processes are Poisson and the nominal holding times are exponentially distributed (see Subsection 2.1). The transitions between states are due to an arrival or a departure of a session of class-i. The arrival rates are given by the intensity of the Poisson arrival processes. Due to the memoryless property of the exponential distribution, the departure rates from each state depend on the nominal holding time of the in-progress sessions and on the slow down factor in that state. Specifically, when the slow down factor of a session of class-i is $a_i(\underline{n})$, its departure rate is $\mu_i/a_i(\underline{n})$.

The Markovian property for such systems was observed and formally proven by Altman *et al.* [9], and Nunez Queija *et al.* [19]. It is also used by Massoulie and Roberts in [18], where the departure rates of the birth-death process are modulated by the actual instantaneous bandwidth of elastic traffic. Thus, the system under these assumptions is a continuous time Markov chain (CTMC) whose state is uniquely characterized by the state vector $\underline{n}$.

2.7 Assigning a Blocking Measure to the Macro States

Let $\Pi(\underline{n})$ denote the stationary probability distribution of the CTMC representing our system. Also, recall from Subsection 2.2 that $\Omega(j)$ is the set of micro states forming macro state j, and let $\Omega_i(j)$ be the Class-i-blocking subset of $\Omega(j)$.

In contrast to the classical Kaufman-Roberts case, a CDMA macro state may consist of non-blocking micro states only, a "mixture" of non-blocking and blocking micro states or blocking micro states only. In order to define the recursive relationship between macro states, there is a need to assign a blocking measure to each macro state. To this end, we first introduce the following micro state blocking measure. In each micro state $\underline{n}$:

$$\beta_i(\underline{n}) = \begin{cases} 0 & \text{if } \underline{n} \text{ is not a class-}i \text{ blocking state} \\ \Pi(\underline{n}) & \text{if } \underline{n} \text{ is a class-}i \text{ blocking state.} \end{cases} \tag{10}$$

Consider now the $\Omega(j)$ micro states of the macro state j where $|\Omega(j)| > 0$. Then:

- Macro state j is a non-blocking macro state with respect to class-i if $|\Omega_i(j)| = 0$. In these case: $\beta_i(j) = 0$.
- Macro state j is a fully blocking macro state with respect to class-i if $\Omega_i(j) = \Omega(j)$. That is, all micro states of this macro state are blocking with respect to class-i. In these case: $\beta_i(j) = \sum_{\Omega(j)} \Pi(\underline{n})$.

– Macro state j is a partially blocking macro state with respect to class-i if $|\Omega_i(j)| > 0$ and $\Omega_i(j) \neq \Omega(j)$. In these case: $\beta_i(j) = \sum_{\Omega_i(j)} \Pi(\underline{n})$. Clearly, the calculation of $\beta_i(j)$ in this case requires that the blocking micro states within macro state j (that is the set $\Omega_i(j)$) are known.

3 Recursive Equation for Elastic Traffic in CDMA

3.1 Constructing the Reversible System

As a first step, we devise a method that can be used to construct a reversible Markov chain for virtually all work conserving bandwidth sharing policies. Recall that in system state $\underline{n}$ all class-i sessions are slowed down by a factor of $a_i(\underline{n})$. This implies that the class-i session departure (death) rate becomes $\frac{\mu_i}{a_i(\underline{n})}$. Since the system is not reversible, the local balance equations in general do not apply:

$$n_i \cdot \frac{\mu_i}{a_i(\underline{n})} \cdot \Pi(\underline{n}) \neq \lambda_i \sigma(\underline{n}_i^-) \Pi(\underline{n}_i^-); \qquad i = 1, \ldots, I, \tag{11}$$

where $\underline{n}_i^- = \underline{n} - \underline{e}_i$ and $\underline{e}_i$ is vector whose elements are zero, except for its i-th element which is 1.

We may think of the quantity $\dfrac{\mu_i}{a_i(\underline{n})}$ as the state dependent departure rate in the original Markov chain. Similarly to what had been done in [5] and [6], we look for an approximating Markov chain with the same Ω state space in the form that is defined by (12).

$$\Phi_i(\underline{n}) n_i \mu_i \Pi(\underline{n}) = \lambda_i \sigma(\underline{n}_i^-) \Pi(\underline{n}_i^-); \qquad i = 1, \ldots, I. \tag{12}$$

A Markov chain constructed like this is reversible if and only if the values of $\Phi_i(\underline{n})$ for every state is derived from another positive state dependent variable $x(\underline{n})$ as described by the following equation (see also [1]):

$$\Phi_i(\underline{n}) = \frac{x(\underline{n}_i^-)}{x(\underline{n})}; \qquad x(\underline{0}) = 1. \tag{13}$$

We select the class dependent function $\Phi_i(\underline{n})$ in such a way that the modified model possesses the following properties (without further motivating these properties):

– The resource sharing discipline keeps its work conserving property, i.e. either all flows get their respective required peak resource (and are served with their peak rate R_i) or the system is serving at its full capacity.
– Any flow may transfer the same amount of data in state $\underline{n}$ of the modified Markov chain as it would do in state $\underline{n}$ of the original Markov chain.

Table 2 compares the properties of the original (irreversible) and the modified (reversible) Markov chains. The first row corresponds to the holding times and show that they needed to be changed to get a reversible Markov chain. The second row shows the *traffic demand* of the flows. We may think of the traffic demand of a class-i session as its peak resource requirement multiplied by its mean nominal holding

Table 2. Comparison of the Original and the Modified Markov Chains

Properties in state $\underline{n}$		
	Original Markov chain	Reversible Markov Chain
Departure rate	$(\mu_i)/a_i(\underline{n})$	$\Phi_i(\underline{n})\mu_i$
Traffic demand	$\Delta_i/(\mu_i)$	$\Delta_i/(\mu_i)$
Used resource	Δ_{a_i}	$\Phi_i(\underline{n})\Delta_i$

time, (that is its mean holding time when served at its peak rate). This product corresponds to the number of bits that the flow would transfer if it was served at its peak bit-rate. The key observation is that according to the second property and the reasoning above, these values need to be the same in state $\underline{n}$ of both Markov chains. The third row shows the actual resource consumed by class-i flows in state $\underline{n}$, which we may think of as a "bandwidth like" quantity in the multi-rate model. Thus, as shown in the last row of Table 2, the second property determines the resource shares ($\delta_i(\underline{n})$) of the flows in the modified Markov chain.

We now focus on the work conserving property. This consists of two parts. If the system is not over-booked, that is $\nu_j = \underline{n} \cdot \underline{\Delta} \leq \hat{\Psi}$, then the sum of the bandwidth shares must exactly match the total amount of resource occupied. This relation is shown in (14). We note here that this part is handled the same way as it was done in [3], [5] and [6].

$$\sum_{i=1}^{I} n_i \, \Delta_i \, \Phi_i(\underline{n}) = \nu_j \qquad \forall \, \underline{n} : j \leq \hat{\Psi} \tag{14}$$

Equation (14) implies that when $\nu_j \leq \hat{\Psi}$, then $\Phi(\underline{n}) = 1$. On the other hand, when the resource is over-booked, the bandwidth shares of the flows must sum up to the link capacity. Equation (15) quantifies this:

$$\sum_{i=1}^{I} n_i \, \Delta_i \, \Phi_i(\underline{n}) = \hat{\Psi} \qquad \forall \, \underline{n} : \nu_j > \hat{\Psi} \tag{15}$$

Equations (14) and (15) assure that the modified model remains work conserving and describes the system load well. Substituting (13) into (14) and (15) and taking into account the condition that $x(\underline{0})$ must be equal to 1, the values of $x(\underline{n})$ can be determined recursively as follows:

$$x(\underline{n}) = \frac{\sum_{i=1}^{I} n_i \, \Delta_i \, x(\underline{n}_i^-)}{\text{Min}(\hat{\Psi}, \nu_j)}. \tag{16}$$

3.2 Steady State Analysis

The state space of the reversible Markov chain is identical with that of the original system, but the state transitions are modified. We can therefore immediately establish the local balance equations in this modified system (recall the blocking measure

β introduced in Subsection 2.7):

$$n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \rho_i \sigma_i(\underline{n}_i^-) \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right), \tag{17}$$

where $\rho_i \triangleq \lambda_i / \mu_i$. Subtracting $\beta_i(\underline{n}_i^-)$ on the right hand side of this equation reflects that there is no Class-i state transition from state $(\underline{n}_i^-)$ to state $\underline{n}$ if the former is a class-i blocking state. Summing over i, we get for micro states $\underline{n}$ in which $\nu_j \leq \hat{\Psi}$:

$$\sum_{i=1}^{I} n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \nu_j \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \sigma_i(\underline{n}_i^-) \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right).$$

For micro states, in which $\nu_j > \hat{\Psi}$:

$$\sum_{i=1}^{I} n_i \Delta_i \Phi_i(\underline{n}) \Pi(\underline{n}) = \hat{\Psi} \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \sigma_i(\underline{n}_i^-) \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right).$$

Summarizing:

$$\text{Min}[\nu_j, \hat{\Psi}] \cdot \Pi(\underline{n}) = \sum_{i=1}^{I} \rho_i \sigma_i(\underline{n}_i^-) \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right). \tag{18}$$

Summing over $\Omega(j)$:

$$\text{Min}[\nu_j, \hat{\Psi}] \cdot \sum_{\underline{n}: \underline{n} \cdot \underline{\Delta} = \nu_j} \Pi(\underline{n}) = \sum_{\underline{n}: \underline{n} \cdot \underline{\Delta} = \nu_j} \sum_{i=1}^{I} \rho_i \sigma_i(\underline{n}_i^-) \Delta_i \left(\Pi(\underline{n}_i^-) - \beta_i(\underline{n}_i^-) \right). \tag{19}$$

Combining (18) and (19) and making use of the macro state blocking measures defined in Subsection 2.7 leads to the following result.

Theorem 1. *The $q(j)$ unnormalized macro state probabilities satisfy the following set of recursive equations:*

$$\mathit{Min}[\nu_j, \hat{\Psi}] \cdot q(j) = \sum_{i=1}^{I} \rho_i \cdot \Delta_i \cdot \sigma_i\big(\mathcal{J}(\nu_j - \Delta_i)\big) \cdot \Big(q\big(\mathcal{J}(\nu_j - \Delta_i)\big) - \beta_i\big(\mathcal{J}(\nu_j - \Delta_i)\big) \Big). \tag{20}$$

Equation (20) establishes a recursive relationship between macro state j and the $\mathcal{J}(j - \Delta_i)$-s that can be seen as an analogy with the relationship expressed by the classical Kaufman-Roberts formula. Note that (20) yields the relative (unnormalized) macro state probabilities when setting $q(0)$ to some convenient value (typically to 1). It will be useful to note that the unnormalized macro state probabilities are monotone decreasing functions of the β_i macro state blocking measures.

In order to arrive at the $Q(j)$ macro state probabilities, a normalization is necessary. In order to arrive at performance measures of interest and specifically the class-wise blocking probabilities, the issue is to determine the $\beta_i(\mathcal{J}(\nu_j - \Delta_i))$ class-wise blocking measures for each macro state. The issue of normalization and finding the β_i-s is investigated in the next Section.

3.3 A Generalization of the Kaufman-Roberts Formula Based on Theorem 1

Equation (20) can be seen as a generalization of the Kaufman-Roberts formula in the following sense. Both the classical Kaufman-Roberts formula and (20) have the following general structure:

$$q(1) = 1, \quad q(j) = \sum_{k=1}^{j-1} q(k) p_{kj} \;\; j > 1, \tag{21}$$

where the $p_{kj} \geq 0$ coefficients express the "weight" with which the lower indexed macro state probabilities contribute to $q(j)$. For finite capacity systems in which the resource utilization of all admitted sessions have a positive lower bound, the p_{kj} coefficients become zero at a given finite j. For the classical case

$$p_{kj} = \sum_{i:\Delta_i = j-k} \frac{\rho_i \Delta_i}{j}, \tag{22}$$

where the summation goes through the traffic classes whose resource need is $j - k$. In our CDMA system:

$$p_{kj} = \sum_{i:\Delta_i = j-k} \frac{\rho_i \Delta_i \sigma(j - \Delta_i)(q(\mathcal{J}(j - \Delta_i)) - \beta(\mathcal{J}(j - \Delta_i)))}{\min(j, \hat{\Psi})\; q(\mathcal{J}(j - \Delta_i))} =$$

$$= \sum_{i:\Delta_i = j-k} \frac{\rho_i \Delta_i \sigma(k)(q(\mathcal{J}(k)) - \beta(\mathcal{J}(k)))}{\min(j, \hat{\Psi})\; q(\mathcal{J}(k))}. \tag{23}$$

The most important property of (22) is that it contains only macro state dependent input parameters. In contrast, in (23) the $\beta(k)$ parameters can only be determined by means of the micro state probabilities, which are unknown.

We note that the structure of the system state space and the deduction of the p_{kj} coefficients rely on the equations (5) and (1). The relationship between the required resource and the assigned bit rate may be different, which is the case in the model proposed by [17]. In such a case, the issue becomes to establish the p_{kj} coefficients and thereby a recursive relationship as in (21). Once the coefficients are determined, the results of the next section can be generalized to that particular type of system as well.

4 Macro State, Accumulated Macro State and Blocking Probabilities

In order to apply the recursive equations, three further considerations are necessary. First, the value of $q(0)$ is unknown, and it is usually set to 1. Equation (20) can thereafter be used to determine the unnormalized macro state probabilities. Then, in order to arrive at the absolute macro state probabilities, the unnormalized probabilities have to be normalized. Secondly, the class-wise macro state blocking measures

($\beta_i(j)$) need to be determined. This task is non-trivial, because it would require the exact knowledge of the steady state probabilities of the blocking micro states in macro state j. Finally, from the macro state probabilities we need to calculate or estimate the class-wise blocking probabilities. This task is similar to the previous one in that we need to decide on how to take into account partially blocking macro states. These three tasks are considered in the subsections below.

4.1 The Unnormalized and Normalized Macro State Probabilities

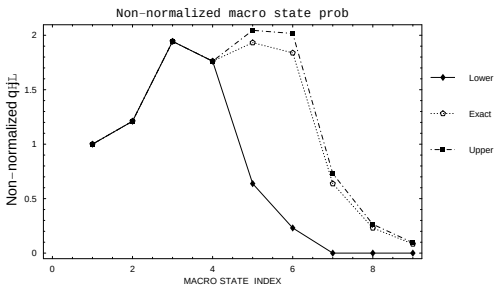

Fig. 2. The unnormalized macro state probabilities ($q(j)$).

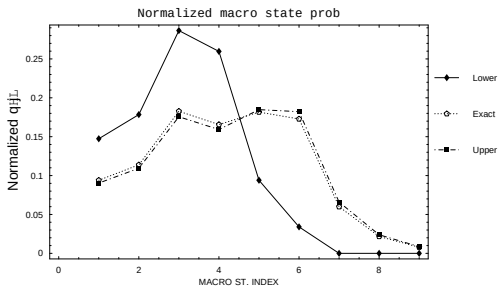

Fig. 3. The normalized macro state probabilities ($Q(j)$).

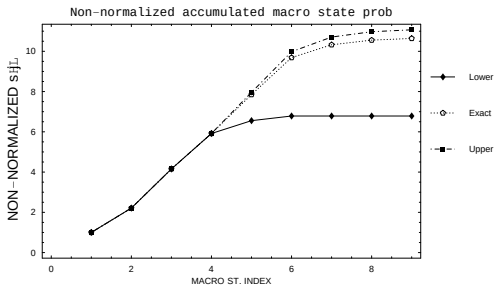

Fig. 4. The unnormalized accumulated macro state probabilities ($s(j)$).

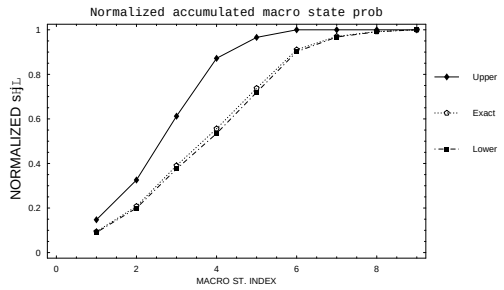

Fig. 5. The normalized accumulated macro state probabilities ($S(j)$).

Recall that in order to calculate the macro state probabilities, we need to know the $\beta_i(j)$ macro state blocking measures. These $\beta_i(j)$ values however are not available, since the micro state probabilities are not available. However, each macro state can be marked as ”non blocking”, ”partially blocking” and ”fully blocking”, since determining whether a certain micro state is Class-i blocking or not does not require that its state probability be established. Therefore, a lower and an upper bound on the real $\beta_i(j) = \sum_{\underline{n} \in \Omega(j)} \beta_i(\underline{n})$ values can be determined as follows. A lower bound

on the class-wise macro state blocking measure is given as follows:

$$\underline{\beta}_i(j) = \begin{cases} 0 & \text{if } \Omega_i(j) \neq \Omega(j) \text{ (non-blocking or partially blocking macro state),} \\ q(j) & \text{if } \Omega_i(j) = \Omega(j) \text{ (fully blocking macro state).} \end{cases} \tag{24}$$

This equation states that if j is fully blocking, the associated $\beta_i(j)$ has to be set to $q(j)$; otherwise to 0. Likewise, the upper bound:

$$\bar{\beta}_i(j) = \begin{cases} 0 & \text{if } |\Omega_i(j)| = 0 \text{ (non-blocking macro state),} \\ q(j) & \text{if } |\Omega_i(j)| > 0 \text{ (partially blocking or fully blocking macro state).} \end{cases} \tag{25}$$

That is, if j is partially or fully blocking, its blocking measure is set to $q(j)$, otherwise to 0.

Consider (20) setting $q(0) = 1$. Let $\underline{q}(j)$ and $\bar{q}(j)$ denote the calculated non-normalized macro state probabilities when substituting $\bar{\beta}_i(j)$ and $\underline{\beta}_i(j)$ into (20) respectively. [5] As we noted after Theorem 1, $q(j)$ is a decreasing function of $\beta_i(j)$. Then, since $\underline{\beta}_i(j) \leq \beta_i(j) \leq \bar{\beta}_i(j)$, $\forall i, j$, it follows that:

$$\underline{q}(j) \leq q(j) \leq \bar{q}(j) \qquad \forall j > 0. \tag{26}$$

This relation is illustrated in Figure 2 in the case of a simple example in which the number of macro states is 9. The relation after normalization does not hold for all j (Figure 3).

4.2 The Unnormalized and Normalized Accumulated Macro State Probabilities

Consider now the non-normalized and normalized *accumulated* macro state probabilities that are defined as follows respectively:

$$s(j) = \sum_{k \leq j} q(k); \qquad \underline{s}(j) = \sum_{k \leq j} \underline{q}(k); \qquad \bar{s}(j) = \sum_{k \leq j} \bar{q}(k). \tag{27}$$

$$S(j) = \frac{s(j)}{s(J)}; \qquad \underline{S}(j) = \frac{\underline{s}(j)}{\underline{s}(J)}; \qquad \bar{S}(j) = \frac{\bar{s}(j)}{\bar{s}(J)}, \tag{28}$$

where J is the largest macro state. Because of (26):

$$\underline{s}(j) \leq s(j) \leq \bar{s}(j) \quad \forall j > 0. \tag{29}$$

[5] Note that when we substitute $\bar{\beta}_i(j)$ and $\underline{\beta}_i(j)$ as defined by (25) and (24) respectively into (20), it iteratively yields the *unnormalized* β_i values, since the $q(j)$-s are the unnormalized macro state probabilities. The *normalized* β'_i values are defined in a similar manner but using the $Q(j)$ normalized macro state probabilities.

4.3 Bounding the Accumulated Macro State Distribution

To upper and lower bound the accumulated macro state distribution - determined by the p_{kj} $(k, j \leq J)$ coefficients in (23) - we introduce a set of coefficients $\bar{p}_{kj}$ and $\underline{p}_{kj}$ $(k, j \leq J)$ such that for $\forall j \leq J$:

$$\frac{\underline{q}(j)}{\underline{s}(j)} \leq \frac{q(j)}{s(j)} \leq \frac{\bar{q}(j)}{\bar{s}(j)}. \tag{30}$$

The following theorem will turn out to be crucial for providing the blocking probability bounds.

Theorem 2. *The accumulated macro state distributions $\underline{S}(j)$ and $\bar{S}(j)$ determined by $\bar{p}_{kj}$ and $\underline{p}_{kj}$ satisfy*

$$\bar{S}(j) \leq S(j) \leq \underline{S}(j) \tag{31}$$

for $\forall j \leq J$.

The proof is provided in Appendix I.

4.4 Calculating the Bounds on the Class-wise Blocking Probabilities

Recall that if we had access to the micro state probabilities, the class-wise blocking probabilities could be calculated as $B_i = \sum_j \beta_i'(j)$. A lower bound can be calculated by using the lower bounds on the normalized macro state probabilities ($\underline{Q}(j)$-s) and summing these for the fully blocking macro states (thus omitting the partially blocking states). In a similar manner, calculating the $\bar{Q}(j)$-s and summing them over both the partially and fully blocking macro states results in an upper bound of the class-wise blocking probability. These intuitively straightforward results are expressed in the following theorem.

We define the limit of partially and fully blocking macro states as $j_i^p = \max(j : |\Omega_i(j)| = 0)$ and $j_i^f = \max(j : \Omega_i(j) \neq \Omega(j))$, and the lower and upper bounds of the class-wise blocking probabilities as $\underline{B}_i = \sum_{j > j_i^f} \underline{Q}(j) = 1 - \underline{S}(j_i^f)$ and $\overline{B}_i = \sum_{j > j_i^p} \bar{Q}(j) = 1 - \bar{S}(j_i^p)$. We note that with this notation the exact blocking probabilities can be expressed as the sum of the (normalized) blocking measures of the partially blocking states and the fully blocking state probabilities:

$$B_i = \sum_{j_i^p < j \leq j_i^f} \beta_i'(j) + \sum_{j_i^f < j} Q(j) = \sum_{j_i^p < j \leq j_i^f} \beta_i'(j) + 1 - S(j_i^f) \tag{32}$$

Considering that $j_i^p \leq j_i^f$ and (29) we have the following useful theorem.

Theorem 3. *For the set of coefficients $\bar{p}_{kj}$ and $\underline{p}_{kj}$ that satisfy (30), the following inequality chain holds:*

$$\underline{B}_i \leq B_i \leq \overline{B}_i \quad \forall i. \tag{33}$$

The proof is given in Appendix II.

An important characteristics of Theorem 3 is that it only relies on the $\bar{p}_{kj}$ and $\underline{p}_{kj}$ coefficient bounds and on the j_i^p and j_i^f macro state indexes. Thus, based on the discussion in Subsection 3.3, Theorem 3 can be generalized for other non-linear systems as well provided that the p_{kj} coefficients of Equation (21) (similarly to (22) and (23)) and the j_i^p and j_i^f indexes can be determined. This observation could be used to extend our results to models proposed for TDMA or FDMA systems in for instance [17].

4.5 A Macro State Level Analysis Method for CDMA

Our goal in this subsection is to provide an analysis method based on Theorem 2 for approximating $S(j)$ such that the bounds depend only on the common properties of macro states, where these common properties include the range of the possible micro state dependent measures in a given macro state. In our CDMA system (specified by (23)), the only micro state dependent measure is $\beta(k)$ when k is a partially blocking macro state. In this case $0 < \beta(k) < q(k)$ defines the valid range of the macro state blocking measure (whose exact value cannot be calculated without knowing the micro state probabilities) and determine the valid range of the p_{kj} coefficients associated with macro state k.

To this end we note that if (30) holds, then we know that (31) holds and Theorem 3 applies. Therefore, starting from $\underline{q}(1) = \bar{q}(1) = 1$ we may iteratively calculate $\underline{q}(j)$ and $\bar{q}(j)$ based on

$$\underline{q}(j) = \sum_{k=1}^{j-1} \underline{q}(k)\underline{p}_{kj} \quad \text{and} \quad \bar{q}(j) = \sum_{k=1}^{j-1} \bar{q}(k)\bar{p}_{kj}, \tag{34}$$

where the $\underline{p}_{kj}$ ($\bar{p}_{kj}$) parameters are the minimum (maximum) of the valid range of the coefficient associated with macro state k (obtainable from (23)). The following theorem provides a non-recursive condition for (30) which can be checked on the fly together with the calculation of the consecutive macro state probabilities.

Theorem 4. *The macro state level analysis provides a valid upper and lower bound of the blocking probabilities if for $\forall j \leq J$:*

$$\begin{aligned} \frac{\underline{q}(j)}{\underline{s}(j)} &= \min\left(\frac{q(j)}{s(j)}\middle|\underline{q}(\ell) \leq q(\ell) \leq \bar{q}(\ell), \forall \ell < j, \underline{p}_{kj} \leq p_{kj} \leq \bar{p}_{kj}, \forall k < j\right) \\ \frac{\bar{q}(j)}{\bar{s}(j)} &= \max\left(\frac{q(j)}{s(j)}\middle|\underline{q}(\ell) \leq q(\ell) \leq \bar{q}(\ell), \forall \ell < j, \underline{p}_{kj} \leq p_{kj} \leq \bar{p}_{kj}, \forall k < j\right) \end{aligned} \tag{35}$$

Proof: Considering a given set of $p_{k\ell}$ parameters, $q(j)$ recursively depends on all previous $p_{k\ell}$ ($\ell < j$) via the $q(\ell)$ non-normalized macro state probabilities. If (35) holds for $\forall k \leq \ell$, the upper and lower limits of this dependence are given by $\underline{q}(\ell)$ and $\bar{q}(\ell)$, since $q(\ell) = \sum_{k=1}^{\ell-1} q(k)p_{k\ell}$ is a monotone function of all $q(k)$ and $p_{k\ell}$ ($\forall k < \ell$) and the minimum and the maximum of $q(\ell)$ are obtained according to (34). Taking the whole range of all previous $q(\ell)$ non-normalized macro state probabilities and the p_{kj} parameters associated with the current macro state we obtain the possible range of $q(j)/s(j)$. If the extreme values of this range are $\underline{q}(j)/\underline{s}(j)$ and $\bar{q}(j)/\bar{s}(j)$ then the blocking probability limits calculated from (34) are valid bounds. □

4.6 Summary: Calculating the Blocking Probability Bounds

Theorem 1, together with (22) and (24)-(25) and (34) provides a means to iteratively calculate $\underline{q}, \bar{q}, \underline{s}$ and $\bar{s}$. Using Theorem 4 as an indicator for the (sufficient) condition (30), the $\underline{B}_i$ and $\bar{B}_i$ values as defined in Theorem 3 establish a lower and upper bound respectively on the class-wise blocking probabilities.

4.7 A Heuristic Estimate on the Class-wise Blocking Probabilities

Recall from Theorem 1 and Section 4 that depending on how the macro state blocking measures are taken into account in (20), we may arrive at a lower or an upper bound on the $Q(j)$ macro state probabilities. Then, depending on how we take into account (the bounds on) the partially blocking macro states, we arrive at lower and upper bounds specifically on the class-wise blocking probabilities. The combination of these 2 choices (1: lower/upper bound on all $Q(j)$-s and 2: consider the partially blocking state only or *both* the partially and fully blocking states) give four options for estimating the blocking probabilities. It is intuitively clear that calculating the lower bounds on each $Q(j)$ and then summing the fully blocking states only results in a lower bound ($\underline{B}_i = \sum_{j:\Omega_i(j)=\Omega(j)} \underline{Q}(j)$), while calculating the upper bounds on each $Q(j)$ and then summing both the partially and fully blocking states results in an upper bound on the blocking probabilities ($\overline{B}_i = \sum_{j:|\Omega_i(j)|>0} \bar{Q}(j)$).

The following heuristic *guess* value takes into account all four options:

$$\gamma_i = \frac{\sum\limits_{j:\Omega_i(j)=\Omega(j)} \underline{Q}(j) + \sum\limits_{j:\Omega_i(j)=\Omega(j)} \bar{Q}(j) + \sum\limits_{j:|\Omega_i(j)|>0} \underline{Q}(j) + \sum\limits_{j:|\Omega_i(j)|>0} \underline{Q}(j)}{4} \tag{36}$$

The intuitive explanation for this guess value is that the four options "average out" their respective estimation errors and can be expected to give a reasonable estimate of the blocking probabilities. This will be discussed further in the numerical section.

Table 3. Input Parameters for Case I and Case II

Parameter	Case I (Figures 6-13)	Case II (Figures 14-19)
Δ_1	$2 \cdot \Delta$	$2 \cdot \Delta$
Δ_2	Δ	Δ
$\hat{a}_1$	$1 \ldots 5$: (Fig.6, 7; ... 12, 13)	$1 \ldots 5$: (Fig.14, 15; ... 18, 19)
$\hat{a}_2$	$1 \ldots 6$ (along the x axis)	$1 \ldots 6$ (along the x axis)
λ_1	λ	$\lambda/2$
λ_2	λ	$2 \cdot \lambda$

Table 4. Common Parameters for Case I and Case II

Parameter	Value
Δ	0.099
λ	87.2613
$\mu_1 = \mu_2$	32.03

5 Numerical Results

5.1 Implementation

We have implemented the method described in Section 3 and Section 4 in a *Mathematica* script. It takes the input parameters of Table 3 and 4 and generates the original (irreversible) and the modified (reversible) state spaces and state transitions, calculates the normalized macro state probabilities and from them the lower and upper bounds and the guess value on the blocking probabilities. For verification purposes, it also calculates the exact blocking probabilities by solving for the steady state with the direct (non-recursive) method as described in [10].

5.2 Input Parameters

The input parameters are summarized in Tables 3 and 4. In both Case I and Case II the class-1 peak resource requirement ($\Delta_1 = 2\Delta$) is twice of that of class-2 ($\Delta_2 = \Delta$). For class-1, the maximum slow down factors are varied between $\hat{a}_1 = 1$ (in Figures 6 and 7 in Case I and 14 and 15 in Case II) up to $\hat{a}_1 = 5$ (in Figures 12 and 13 in Case I and 18 and 19 in Case II). For class-2, $\hat{a}_2$ is varied between $1 \ldots 6$ along the x axis in each figure. The total offered traffic load in both cases is:

$$\Lambda = \sum_i \Lambda_i = \sum_i \lambda_i \cdot 1/\mu_i \cdot \Delta_i = 3\frac{\lambda}{\mu}\Delta.$$

In Case I: $\Lambda_1 = 2\Lambda_2$, in Case II: $\Lambda_2 = 2\Lambda_1$. The other-cell interference ratio (φ) is set to 0 in both cases.

5.3 Numerical Results

Figures 6-13 show the (class-1 and class-2) blocking probabilities as the function of the class-2 maximum slow down factor ($\hat{a}_2$) for Case I, while Figures 14-19 correspond to Case II. In all figures (except for Figure 8 and 9), there are four curves: the lower and upper bounds, the actual ("REAL") value calculated from the reversible Markov chain and the guess value that is the result of the heuristics discussed in the previous section. Figures 8 and 9 compare the blocking probabilities for Case I when using the direct (matrix inversion) method to calculate the exact values from the irreversible Markov chain with those calculated from the reversible

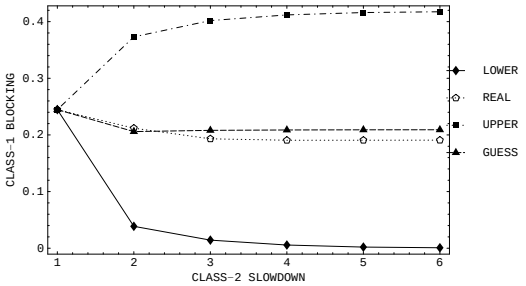

Fig. 6. Case I, $\hat{a}_1 = 1$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

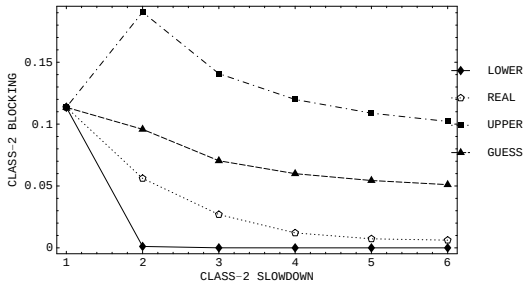

Fig. 7. Case I, $\hat{a}_1 = 1$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$

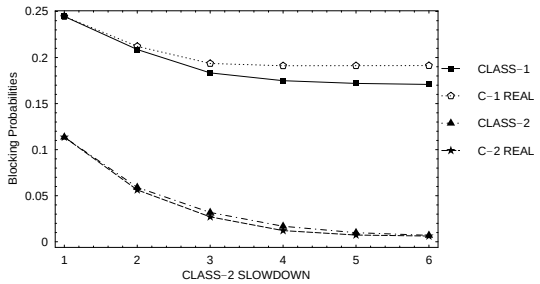

Fig. 8. Irreversible Case I, $\hat{a}_1 = 1$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

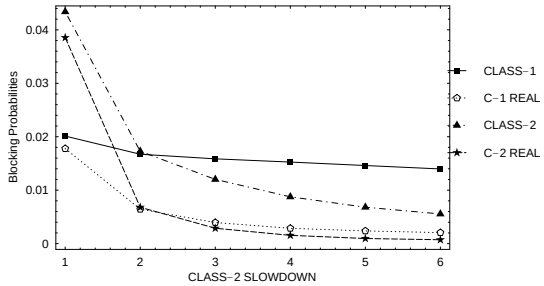

Fig. 9. Irreversible Case I, $\hat{a}_1 = 5$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$

system. These plots illustrate that the reversible approximation works well for both small and greater $\hat{a}$ values.

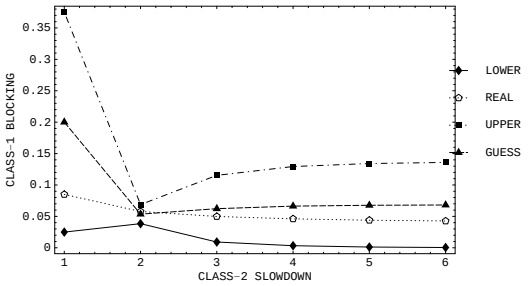

Fig. 10. Case I, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

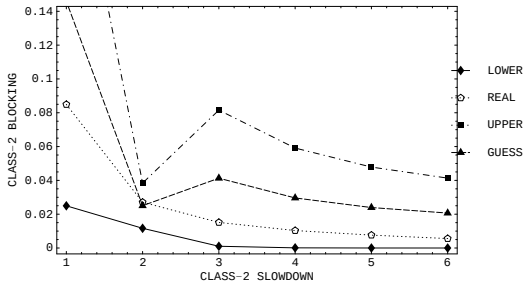

Fig. 11. Case I, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

First we note the impact of allowing for bit rate decrease (i.e. the state dependent slow down of the bit rates). The system is clearly overloaded in both cases with blocking probabilities above 24% and 11% in Case I (Figure 8, $\hat{a}_2 = 1$) and above 26% and 11% in Case II (Figures 14 and 15). Yet, even a small maximum slow down rate (already at $\hat{a}_i = 2...3$) brings the system into a normal operation mode with acceptable blocking probabilities (Figures 10-11 and 16-17.)

In both cases, the class-1 blocking probabilities are higher than the class-2 ones, because the class-1 peak resource demand is twice of that of class-2. It is noteworthy that class-1 blocking is higher in Case II as well, despite that the class-2 arrival intensity is reduced by half. When class-1 sessions are "rigid" ($\hat{a}_1$ is small), the class-1 blocking probabilities are not much affected by varying the maximum slow down rate of class-2. This is especially true in Case I, when the arrival rates are

equal. As class-1 becomes more and more elastic ($\hat{a}_1 = 3 \ldots 5$), especially in Case II, class-1 blocking also decreases as class-2 becomes elastic. For instance, when $\hat{a}_1 = 5$, class-1 blocking decreases from almost 2% close to 0% in Case I and from more than 4% close to 0% in Case II.

Regarding the bounds, they are tightest when the maximum slow down rates are equal or similar ($\hat{a}_1 \approx \hat{a}_2$), especially at small blocking values. An intuitive explanation for this is as follows. Recall that when calculating the lower bound on the blocking probability, a partially blocking macro state is ignored, although it does contribute to overall blocking. This is because we do not know its exact $Q(j)$ (since we do not know the micro state's $\beta_i(\underline{n})$ values). The lower bound therefore becomes "loose" when there are high probability blocking micro states that are part of a macro state, since we miss these micro states from the calculation. When the class-wise $\hat{a}_i$ values are similar, the structure of the Markov chain becomes "more balanced" (in the 2 dimensional case, roughly equal number of states in the x and y directions), and such mixed partially blocking macro states arise less often than when the Markov chain is "unbalanced" (that is when the $\hat{a}_i$ values are much different). Likewise, the tightness of the upper bound depends on the number of non-blocking micro states that become part of a partially blocking macro state, since we calculate these states' probabilities in the upper bound. This happens less often in a nice "well-balanced" chain.

The heuristic "guess" value performs well in cases when both classes are elastic, and especially at low blocking values. Because the guess value is derived from the bounds, a similar reasoning applies to it as the one for the bounds above. Indeed, when the maximum slow down rates are exactly equal, the guess value gets remarkably close to the real blocking probabilities. This experience was actually confirmed by other numerical experiments (not shown here) as well.

Figures 20-23 show results for the case when the neighbor cell interferers with the cell under study. For this purpose we have set the parameter of the neighbor cell interference (see Table 1) to $\varphi = 0.2$. Recall that in our model this leads to a non-zero soft blocking in addition to the hard blocking that is present irrespective of the neighbor cell interference. For Case I, Figures 20 (Class-1) and 21 (Class-2) show that the total ((soft+hard) blocking probabilities, the lower and upper bounds and the guess value. For Case II, similar results are shown by Figures 22 and 23. In both cases we observe that both the bounds and the heuristic guess value are reasonable, showing similar behavior as in the case with no interference.

6 Conclusions

In this paper we considered a CDMA cell that supports elastic services. Sessions belonging to such service classes can dynamically adjust their transfer rates depending on the interference situation in the system. The dynamic adjustment of the session bit rates makes the calculation of the session-wise blocking probabilities non-trivial, because (1) the session holding times depend on the received throughput and (2) the relationship between the CDMA resource consumption and the instantaneous

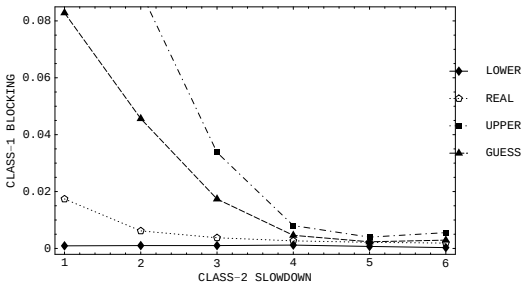

Fig. 12. Case I, $\hat{a}_1 = 5$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

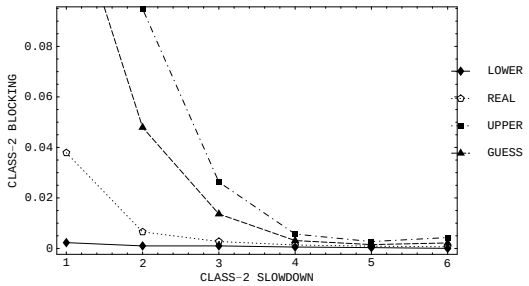

Fig. 13. Case I, $\hat{a}_1 = 5$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

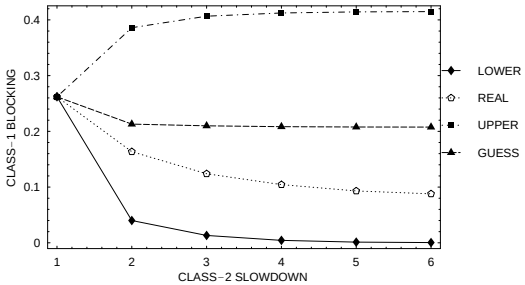

Fig. 14. Case II, $\hat{a}_1 = 1$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

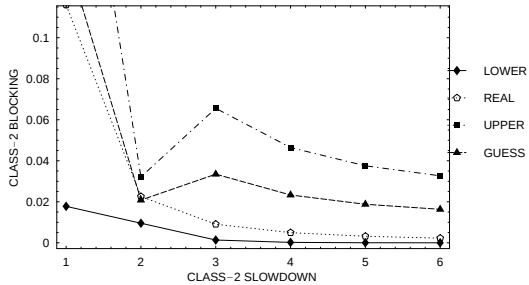

Fig. 15. Case II, $\hat{a}_1 = 1$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

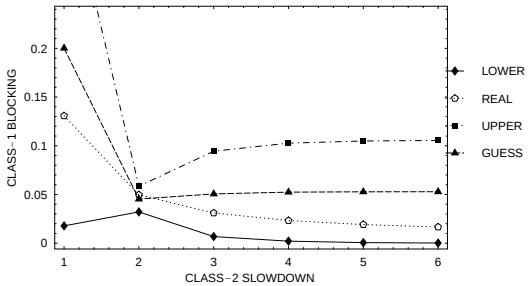

Fig. 16. Case II, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

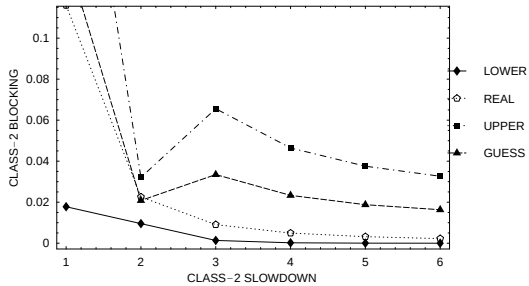

Fig. 17. Case II, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

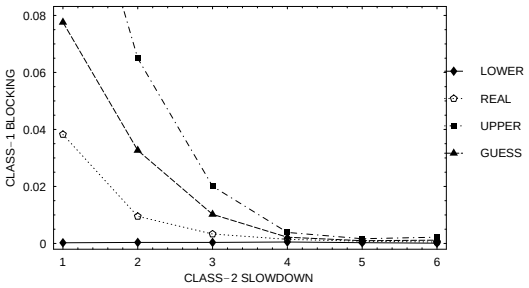

Fig. 18. Case II, $\hat{a}_1 = 5$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

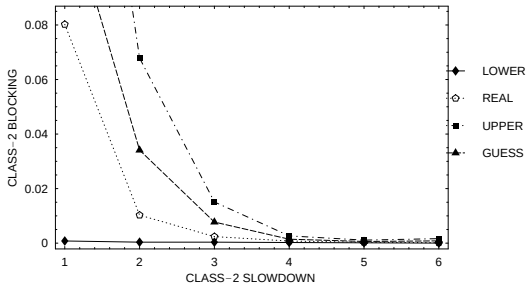

Fig. 19. Case II, $\hat{a}_1 = 5$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$.

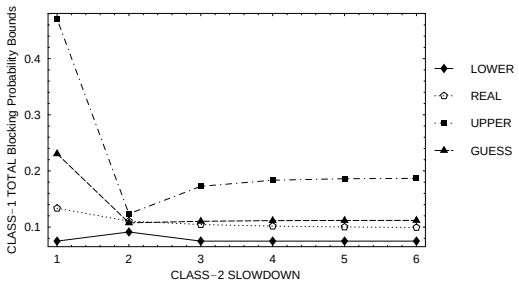

Fig. 20. Case I, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$. (Same setting as that for Figure 10 but now with neighbor cell interference.)

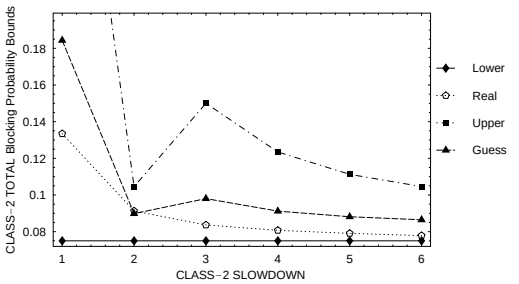

Fig. 21. Case I, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$. (Same setting as that for Figure 11 but now with neighbor cell interference.)

session-wise bit rates is non linear. In this fairly general setting the paper develops an extension of the Kaufman-Roberts recursion that has been the de facto standard to calculate the steady state of fixed multi-rate systems. In order to establish this recursion, we approximated the original irreversible Markovian system with a reversible one. It turns out that the system macro states are such that a single macro state may contain both blocking and non-blocking micro states. Because of this, we have proposed a method that gives bounds on the steady state and blocking probabilities rather than determining the exact values. Numerical examples indicate that slowing down the bit rates lead to the drastic decrease of the blocking probabilities, despite the necessary residency time increase. Our approximation performs especially well when the session-wise maximum slow down rates are equal. Based on our observations we proposed an intuitively appealing heuristic approximation method specifically for the blocking probabilities. This approximation works well in a large number of numerical cases (only part of which was presented here).

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We would like to thank the anonymous reviewers whose rigorous comments have inspired us to obtain some of the results of this paper and improved the presentation.

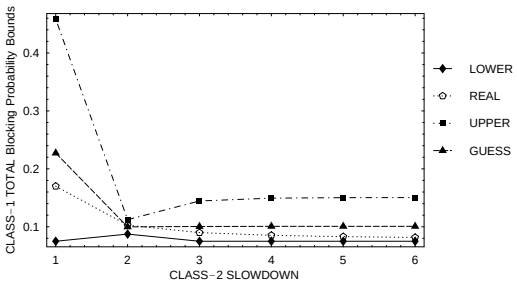

Fig. 22. Case II, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$. (Same setting as that for Figure 16 but now with neighbor cell interference.)

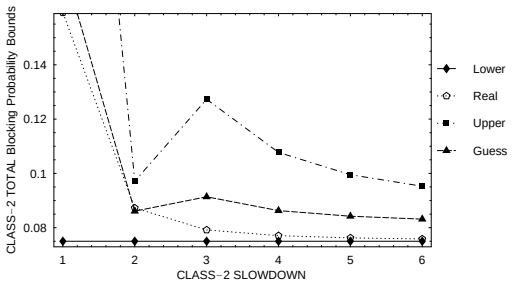

Fig. 23. Case II, $\hat{a}_1 = 2$ is kept fixed, $\hat{a}_2 = 1 \ldots 6$. (Same setting as that for Figure 17 but now with neighbor cell interference.)

Appendix I: Proof of Theorem 2

Proof: For $\forall \ell \leq J$ we introduce a series of normalized distributions, and accumulated distributions as follows:

$$Q(j,\ell) = \frac{q(j)}{\sum_{k=1}^{\ell} q(k)} = \frac{q(j)}{s(\ell)} \quad \text{and} \quad S(j,\ell) = \sum_{k=1}^{j} Q(k,\ell) = \frac{\sum_{k=1}^{j} q(k)}{\sum_{k=1}^{\ell} q(k)} = \frac{s(j)}{s(\ell)},$$

where $j \leq \ell$. Note that $Q(j,J) = Q(j)$ and $S(j,J) = S(j)$. We prove the theorem by induction. The theorem holds for $\ell = 1$ since $\bar{S}(1,1) = S(1,1) = \underline{S}(1,1) = 1$. Assuming that the theorem holds for $\ell \geq 1$, i.e., for $1 \leq j \leq \ell$

$$\bar{S}(j,\ell) \leq S(j,\ell) \leq \underline{S}(j,\ell), \tag{37}$$

we have

$$S(j,\ell+1) = \frac{s(j)}{s(\ell+1)} = \frac{s(j)}{s(\ell)} \frac{s(\ell)}{s(\ell)+q(\ell+1)} = S(j,\ell)\left(1 - \frac{q(\ell+1)}{s(\ell+1)}\right) \tag{38}$$

and using (30) and (37) it results

$$\begin{aligned} \bar{S}(j,\ell+1) &= \bar{S}(j,\ell)\left(1 - \frac{\bar{q}(\ell+1)}{\bar{s}(\ell+1)}\right) \leq \\ S(j,\ell+1) &= S(j,\ell)\left(1 - \frac{q(\ell+1)}{s(\ell+1)}\right) \leq \\ \underline{S}(j,\ell+1) &= \underline{S}(j,\ell)\left(1 - \frac{\underline{q}(\ell+1)}{\underline{s}(\ell+1)}\right) \end{aligned} \tag{39}$$

for $1 \leq j \leq \ell$.

Finally for $j = \ell + 1$ we have

$$\bar{S}(\ell+1,\ell+1) = S(\ell+1,\ell+1) = \underline{S}(\ell+1,\ell+1) = 1. \tag{40}$$

□

Appendix II: Proof of Theorem 3

Proof of $\underline{B}_i \leq B_i$ Because $S \leq \underline{S}$:

$$1 - \underline{S}(j_i^f) \leq 1 - S(j_i^f) + \sum_{j_i^p < j \leq j_i^f} \beta_i'(j)$$

Proof of $B_i \leq \bar{B}_i$ Because $\bar{S} \leq S$:

$$B_i \leq \sum_{j_i^p < j \leq j_i^f} \bar{\beta}_i'(j) + \sum_{j_i^f < j} Q(j) = \sum_{j_i^p < j \leq j_i^f} Q(j) + \sum_{j_i^f < j} Q(j) = \sum_{j > j_i^p} Q(j) =$$

$$= 1 - \sum_{j \leq j_i^p} Q(j) = 1 - S(j_i^p) \leq 1 - \bar{S}(j_i^p) = \bar{B}_i$$

□

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