Modeling and Performance Analysis of Scheduling Policies for OFDMA Based Evolved UTRA

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***Abstract*— In orthogonal frequency division multiple access systems there is an intimate relationship between the packet scheduler and the inter-cell interference coordination (ICIC) functionalities: they determine the set of frequency channels (sub-carriers) that are used to carry the packets of in-progress sessions. In this paper we build on previous work - in which we compared the so called *random* and *coordinated* ICIC policies - and analyze three packet scheduling methods. The performance measures of interest are the session blocking probabilities and the overall throughput. We find that the performance of the so-called *Fifty-Fifty* and *What-It-Wants* scheduling policies is improved by coordinated sub-carrier allocation, especially in poor signal-to-noise-and-interference situations. The performance of the *All-Or-Nothing* scheduler is practically insensitive to the choice of the sub-carrier allocation policy.**

I. Introduction

The 3^{rd} Generation Partnership Project (3GPP) has selected orthogonal frequency division multiple access (OFDMA) as the radio access scheme for the evolving universal terrestrial radio access (E-UTRA). Packet scheduling (PSC) and inter-cell interference coordination (ICIC) are important radio resource management (RRM) techniques that together determine the set of OFDMA resource blocks (essentially the sub-carriers) that are taken into use when a packet is scheduled for transmission over the radio interface [1], [2]. Typically, PSC and ICIC operate on different time scales; in E-UTRA, for instance, PSC operates on the millisecond level to avoid fast fading dips, while ICIC operates at a time scale of hundreds of milliseconds (2-3 orders of magnitude higher). In broad terms, PSC is responsible for determining the session(s) that can send a packet during a scheduling interval and the number of sub-carriers that the session may use. The number of the assigned sub-carriers has a direct impact on the instantaneous bit-rate and thereby can be seen as part of the rate control mechanism. The ICIC function, in turn, is concerned with allocating the particular sub-carriers to the session taking into account the instantaneous channel conditions and the ICIC policy. Such ICIC policy may coordinate which sub-carriers should be taken into use by the schedulers in neighbor cells.

The impact of these two RRM functions on the session-wise and overall throughput has been for long recognized by the standardization and research communities. Sections 11.2.4 and 11.2.5 of [1] and Chapter 6.12 of [2] describe the roles of the PSC and ICIC functions and discuss their relation. From a performance analysis perspective, Letaief *et al.* developed a model that jointly optimizes the bit and power allocation in OFDMA schedulers [6] and [7]. ICIC has been the topic of research for long (for a classical overview paper, see [5]). Recently, specifically in OFDMA systems, [8] analyzed a reuse partitioning scheme without modeling the behavior of the packet scheduler. The paper by Liu and Li proposed a so called "Radio Network Controller algorithm" that determines the set of allowed resources in each base station under its control, while the "Base Station algorithm" schedules packets for transmission [14] (see also Chapter 8 of [15]). These works demonstrate that already with a single dominant interfering neighbor cell, the total throughput increases when an appropriate ICIC policy is employed by the packet scheduler.

The contribution of the current paper is that we (1) explicitly take into account that traffic is elastic and (2) propose a flexible model to capture the behavior of a wide range of schedulers under two different ICIC policies. With regards to (1) we allow the bitrates of the sessions to fluctuate between the associated minimum and maximum rates. This model allows the maximum rate to be large so that the behavior of TCP-like greedy sources can be captured. When the session is slowed down (with respect to its peak rate requirement), its holding time increases proportionally (similarly to what has been analyzed in a CDMA environment in [9] and [11]). Regarding (2), we introduce the notion of the scheduler *policy vector* that specifies the probability that a session is granted a certain amount of sub-carriers when there are competing sessions in the system. We add this rather general scheduler model to the interference coordination model described in [13] and analyze the model in the following steps. First, we derive the distribution of the number of colliding and collision (i.e. co-channel interference) free sub-carriers. We then employ the theory of the *effective* signal-to-noise-and-interference (SINR) [19] that helps determine the packet error rate and thereby the session-wise (useful) throughput given that that number of in-progress sessions is known. Finally, assuming that sessions arrive according to a Poisson process and stay in the system for a throughput dependent amount of time, we derive the performance measures of interest, which are the session blocking probabilities and the average overall throughput. This performance analysis gives insight into the potential gains that inter-cell interference coordination can give when employing different packet scheduling policies.

The paper is organized as follows. In the next section, we describe the scheduling and ICIC policies that we study and introduce the *policy vector* as a convenient tool to characterize these policies. Next, in Section III we state the performance analysis objective in terms of the input parameters and the performance measures of interest. The solution is summarized in a sequence of steps (as described above). Section IV discusses numerical results. We highlight our findings in Section V.

II. Scheduling and Inter-cell Interference Coordination Policies

We consider an OFDMA cell that comprises S orthogonal frequency channels (sub-carriers). The number of in-progress sessions is denoted by i and represents the state of the system. When the system is in state i, the scheduler determines the number of sub-carriers that are assigned to each session. For a particular session under study, this implies that the session is assigned s number of sub-carriers with probability $P(s)$; $\sum_{s=0}^{S} P(s) = 1$. We refer to the mechanism that (in each system state) establishes $P(s)$ as the *scheduling policy*. The *scheduling policy vector* is a vector of dimension $(S+1)$ whose s^{th} element specifies the probability that the session under study (and thereby *any* session) is allocated s channels, $s = 0 \dots S$. (We note that the indexing of the $(S+1)$ elements of the policy vector runs from 0 to S.) In the following subsections we describe three such scheduling polices.

Throughout we assume that the sessions belong to the same service class that is characterized by a peak rate requirement $\hat{R}$ and a maximum *slowdown factor* $\hat{a} \geq 1$. The minimum accepted (guaranteed) bit rate for a session is $R_{min} = \hat{R}/\hat{a}$. Also, when a session is granted s number of frequency channels, its ideal bit-rate (assuming a given and fixed modulation and coding scheme, MCS) and assuming zero packet error/loss rate ($PER = 0$) is denoted by R_s. When $\hat{R}$ is set to R_S (that is the peak bit-rate requirement is the bit-rate that is provided when all resources are assigned to a single session), we say that the session is *greedy*. We will also use the operator $\mathcal{S}(R)$ that returns the number of required channels in order for the session to experience R bit-rate (again assuming $PER = 0$). That is, when a session is admitted into the system, the number of allocated channels s (in the long term) must fulfil: $R_{min} \leq R_s \leq \hat{R}$. This implies that we assume that an admission control procedure operates in the system such that the maximum number of simultaneously admitted sessions remain under $\hat{I} \triangleq \lfloor \frac{S}{\mathcal{S}(\hat{R}/\hat{a})} \rfloor$. We say that state i is an under-loaded, critically loaded or overloaded state if $\mathcal{S}(i \cdot \hat{R})$ is less than, equal to or greater than S respectively.

A. The What-It-Wants Scheduling Policy

The What-It-Wants scheduling policy attempts to grant $\mathcal{S}(\hat{R})$ channels to the sessions as long as $i \cdot \mathcal{S}(\hat{R}) \leq S; \quad i > 0$. Otherwise, in overloaded states, it grants either $\lfloor \frac{S}{i} \rfloor$ or $\lceil \frac{S}{i} \rceil$ channels. Specifically, the What-It-Wants scheduling policy is defined by the following Policy Vector. If $i \cdot \mathcal{S}(\hat{R}) \leq S$:

$$\vec{P}_{WIW}(s) = \begin{cases} 1 & \text{if } s = \mathcal{S}(\hat{R}) \\ & \text{(granting peak rate with prob. 1),} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For overloaded states, we need to distinguish between two cases. If $\frac{S}{i}$ is an integer number, then:

$$\vec{P}_{WIW}(s) = \begin{cases} 1 & \text{if } s = \frac{S}{i} \\ & \text{(granting an equal share with prob. 1),} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

When $\frac{S}{i}$ is not an integer number, the following relations must hold. The scheduler grants $\lfloor \frac{S}{i} \rfloor$ channels with probability P_1 and $\lceil \frac{S}{i} \rceil$ number of channels with probability $1 - P_1$. Clearly:

$$P_1 \cdot \left\lfloor \frac{S}{i} \right\rfloor + (1 - P_1) \cdot \left\lceil \frac{S}{i} \right\rceil = \frac{S}{i};$$
$$P_1 = \left\lceil \frac{S}{i} \right\rceil - \frac{S}{i}. \tag{3}$$

Thus, the policy vector in this case takes the form:

$$\vec{P}_{WIW}(s) = \begin{cases} P_1 & \text{if } s = \lfloor \frac{S}{i} \rfloor \\ 1 - P_1 & \text{if } s = \lceil \frac{S}{i} \rceil \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

B. The All-Or-Nothing Scheduling Policy

In the All-Or-Nothing scheduling policy all resources are assigned to the scheduled session. This type of scheduling is employed in High Speed Downlink Packet Access (HSDPA) systems when code multiplexing is not used [16]. Thus, a session with peak rate requirement $\hat{R}$ would need to be scheduled with probability $\mathcal{S}(\hat{R})/S$ in order for it to receive its peak rate. However, when there are $i \geq 1$ on-going sessions, any given session cannot get scheduled with higher probability than $1/i$. That is, in the All-Or-Nothing scheduling policy, in system state i, a session gets scheduled with probability $P_2 = \text{Min}[\mathcal{S}(\hat{R})/S, 1/i]$. The scheduling policy takes the following form:

$$\vec{P}_{AoN}(s) = \begin{cases} P_2 & \text{if } \ s = S \\ 1 - P_2 & \text{if } \ s = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

C. The Fifty-Fifty Scheduling Policy

The Fifty-Fifty scheduling policy can be seen as a policy in between the What-It-Wants and All-Or-Nothing policies. When there are i sessions in the system, the scheduler divides the resources (almost) equally between the competing sessions (similarly to Fifty-Fifty). However, similarly to the All-Or-Nothing policy, in underloaded states this would mean that the sessions receive more resources in the long term than $\mathcal{S}(\hat{R})$. Thus, in this policy, in underloaded state i, if $\frac{S}{i}$ is not integer, a session receives $\lfloor \frac{S}{i} \rfloor$ channels with probability P_{31}, $\lceil \frac{S}{i} \rceil$ number of channels with probability P_{32} and no channels with probability $1 - P_{31} - P_{32}$. Clearly, in states for which $i \cdot \mathcal{S}(\hat{R}) < S$ and $\frac{S}{i}$ is not an integer number:

$$P_{31} \cdot \left\lfloor \frac{S}{i} \right\rfloor + P_{32} \cdot \left\lceil \frac{S}{i} \right\rceil = \mathcal{S}(\hat{R}), \quad \text{and:}$$
$$P_{31} : P_{32} = \left(\left\lceil \frac{S}{i} \right\rceil - \frac{S}{i} \right) : \left(\frac{S}{i} - \left\lfloor \frac{S}{i} \right\rfloor \right). \quad (6)$$

If $\frac{S}{i}$ is integer, the session is assigned $\frac{S}{i}$ number of channels with probability P_{33} and zero channels with probability $1 - P_{33}$:

$$P_{33} \cdot \frac{S}{i} = \mathcal{S}(\hat{R}); \quad \text{and:} \quad P_0 = 1 - P_{33}.$$

For critically and overloaded states ($i \cdot \mathcal{S}(\hat{R}) \geq S$) the channels are fully utilized ($P_{34} + P_{35} = 1$):

$$P_{34} \cdot i \cdot \left\lfloor \frac{S}{i} \right\rfloor + P_{35} \cdot i \cdot \left\lceil \frac{S}{i} \right\rceil = S.$$

In the critically loaded and overloaded states, if $\frac{S}{i}$ is integer, the number of allocated sessions for each session is $\frac{S}{i}$ with probability 1. Based on these observations, the scheduling policy vector for the Fifty-Fifty policy is straightforward to determine (although a bit tedious to formally specify it):

$$\vec{P}_{FF}(s) = \begin{cases} \mathcal{S}(\hat{R}) \cdot A \ \text{ if } \ s = \lceil \frac{S}{i} \rceil \ \text{ and } \ i \cdot \mathcal{S}(\hat{R}) < S \\ \quad \text{and} \ \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ \mathcal{S}(\hat{R}) \cdot B \ \text{ if } \ s = \lfloor \frac{S}{i} \rfloor \ \text{ and } \ i \cdot \mathcal{S}(\hat{R}) < S \\ \quad \text{and} \ \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 - \mathcal{S}(\hat{R}) \cdot A - \mathcal{S}(\hat{R}) \cdot B \ \text{ if } \ s = 0 \\ \quad \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) < S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 - \frac{\mathcal{S}(\hat{R})}{\frac{S}{i}} \ \text{ if } \ s = \frac{S}{i} = \text{Integer} \\ \quad \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) < S \\ \frac{\mathcal{S}(\hat{R})}{\frac{S}{i}} \ \text{ if } \ s = \frac{S}{i} = \text{Integer} \\ \quad \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) = S \\ \lceil \frac{S}{i} \rceil - \frac{S}{i} \ \text{ if } \ s = \lfloor \frac{S}{i} \rfloor \\ \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) > S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ \frac{S}{i} - \lfloor \frac{S}{i} \rfloor \ \text{ if } \ s = \lceil \frac{S}{i} \rceil \\ \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) > S \ \text{ and } \ (\lceil \frac{S}{i} \rceil > \lfloor \frac{S}{i} \rfloor) \\ 1 \ \text{ if } \ s = \frac{S}{i} = \text{Integer} \\ \quad \text{and} \ \ i \cdot \mathcal{S}(\hat{R}) > S \\ 0 \ \text{ otherwise,} \end{cases}$$

where:

$$A \triangleq 1 - \frac{i}{S} \cdot \left\lfloor \frac{S}{i} \right\rfloor,$$

$$B \triangleq \frac{i}{S} \cdot \left\lceil \frac{S}{i} \right\rceil - 1. \quad (7)$$

D. A Numerical Example

Consider an OFDMA cell that supports $S = 64$ subcarriers (channels). Sessions have a peak rate requirement that corresponds to $\mathcal{S}(\hat{R}) = 4$ channels. When there are 6 in-progress sessions, the system is underloaded ($6 \cdot 4 < 64$), the three scheduling policy vectors are as follows:

$$
\begin{aligned}
\underline{P}_{WIW} &= [0,0,0,0,1,0\ldots,0]\,; \\
\underline{P}_{AoN} &= \left[\frac{60}{64},0,\ldots,0,\frac{4}{64}\right]; \\
P_{FF} &= \left[\frac{40}{64},0,\ldots,0,\frac{8}{64},\frac{16}{64},0,\ldots,0\right], \quad (8)
\end{aligned}
$$

where the P_{FF} vector has non-zero elements at positions 0, 10 and 11 (corresponding to 1, 11 and 12 scheduled channels). Since the system is underloaded, the What-It-Wants policy grants the peak rate with probability 1 (4 channels), the All-Or-Nothing policy allocates all the 64 channels with probability 4/64. The Fifty-Fifty policy ($A = 0.0625$, $B = 0.03125$ so $P_{31} = 0.125$ and $P_{32} = 0.25$) either allocates 10 or 11 channels to any given session (with probabilities 8/64 and 16/64 respectively) or it does not schedule the session (zero channels with probability 40/64). It is easy to see that all three scheduling policies allocate the peak rate (4 channels) in the long term average (in this system state).

For an overloaded example, consider the above example with $i = 20$ in-progress sessions. The system is overloaded and so the peak rate cannot be granted in this system state. However, any one of the sessions can still receive (in long term average) 64/20=3.2 channels. The three policy vectors in this case are as follows:

$$
\begin{aligned}
\underline{P}_{WIW} &= \left[0,0,0,\frac{4}{5},\frac{1}{5},0\ldots,0\right]; \\
\underline{P}_{AoN} &= \left[\frac{19}{20},0,\ldots,0,\frac{1}{20}\right]; \\
P_{FF} &= \left[0,0,0,\frac{4}{5},\frac{1}{5},0\ldots,0\right].
\end{aligned}
$$

In this system state ($i = 20$), the What-It-Wants policy allocates 3 channels to 16 sessions and 4 to 4 sessions. Observing a single session, this means that this session under study is allocated either 3 or 4 channels. The All-Or-Nothing policy now allocates all the channels to the session under study during 1/20 of the time. We also realize that for critically or overloaded states ($i \cdot \mathcal{S}(\hat{R}) \geq S$), the What-It-Wants and the Fifty-Fifty policies have identical policy vectors.

E. Comments on the Reasoning Above and the Use of the Policy Vectors

Characterizing the number of assigned sub-carriers by means of the scheduling policy vectors basically assumes that the channel conditions within the cell are such that channel dependent scheduling does not much alter the resource shares between the in-progress sessions. For instance, in the All-Or-Nothing case, it is assumed that all sessions get an equal time share of the available channels. This assumption is not unrealistic if the mobile stations require service belonging to the same service class and their radio conditions are similar. We believe that this assumption does not distort the dependency of the performance measures of interest (as we shall define these later).

Another subtle assumption, which we will make use later on, is related to the independence of sessions and their share of the available resources. Under the assumption above on the channel conditions, an observer may indeed observe the probability vectors described above. However, assuming that the scheduler resides in the base station (both in downlink and uplink), the events that Session-A is assigned s_A and Session-B is assigned s_B number of channels are not independent. We will return to the issue of how our model takes account of this fact later.

F. ICIC Policies: Random and Coordinated Sub-carrier (Channel) Allocation

As we noted in the Introduction, ICIC operates at a much coarser time scale than packet scheduling [4]. Basically, there are two approaches as to *how* the sub-carriers out of the available ones are selected when a session requires a certain number of sub-carriers (see Figure 1). The simplest way is to pick sub-carriers out of the

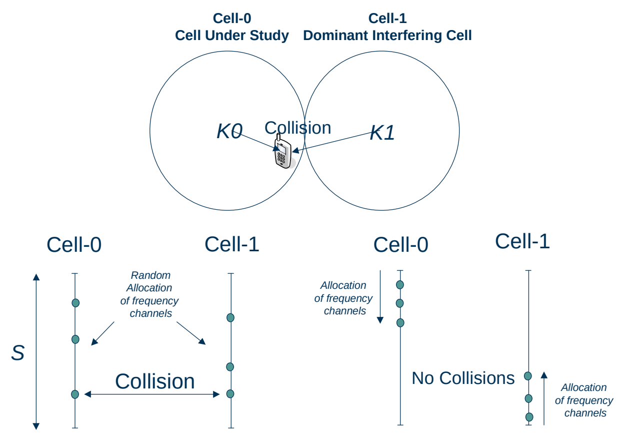

Fig. 1. Random and Coordinated ICIC policies. Coordinated ICIC that operates on the hundreds of milliseconds, seconds or at even slower time scale can be realized by assigning a cell specific ordered list of the frequency channels to each cell such that the "collisions" of frequency channels are avoided as long as there are non-colliding pairs. Assuming a single (dominant) interfering cell (as in [14] and [15]), devising such ordered lists is straightforward. For many cells, coordinated ICIC implies careful frequency planning, as described in for example [5].

ones that are available (i.e. scheduled) randomly such that any available sub-carrier has the same probability to get allocated to an arriving session. *Random allocation* of sub-carriers is attractive, because it does not require any coordination between cells, but it may cause collisions even when there are free sub-carriers. In contrast, a low complexity coordination can avoid collisions as long as there are non-colliding sub-carrier pairs in the two-cell case and non-colliding tuples in the multiple-cell case. We refer to this method as *coordinated* sub-carrier allocation (also called channel segregation [5]). (Further details about these ICIC policies can be found in [13].)

G. Summary

When different scheduling and interference coordination policies are employed in an OFDMA system, the impact of the interfering channels on the packet level becomes non-trivial, as illustrated in Figure 2. Assuming a fixed number of interfering frequency channels in the dominant interfering cell (which can be different for different sessions in a multicell environment), the packet error rate for the scheduled session depends on the interfering power level and also on the packet length distribution in the disturbed session. For instance, assuming a single large packet in Case A in Figure 2, the packet loss probability may be low, since most of the channels carrying the packet are collision free. In contrast, in the example of Case B, the "lower" packet suffers from a high packet error probability, while the "upper" packet is collision free. As we shall see, the packet error probability for a specific modulation and coding scheme is determined by the *effective* signal-to-noise-and-interference (SINR) ratio associated with the packet. The effective SINR, in turn depends on the individual SINR values of the channels assigned to a specific session and size of the packets used by that session. Although not explicitly shown in Figure 2, it is intuitively clear that for the All-Or-Nothing policy the session "size" in the frequency domain (i.e. the number of used channels when scheduled) is independent of the slowdown factor $a(n)$ for the All-Or-Nothing policy. In contrast, when sessions are slowed down, they occupy less channels under the What-It-Wants and Fifty-Fifty scheduling policies.

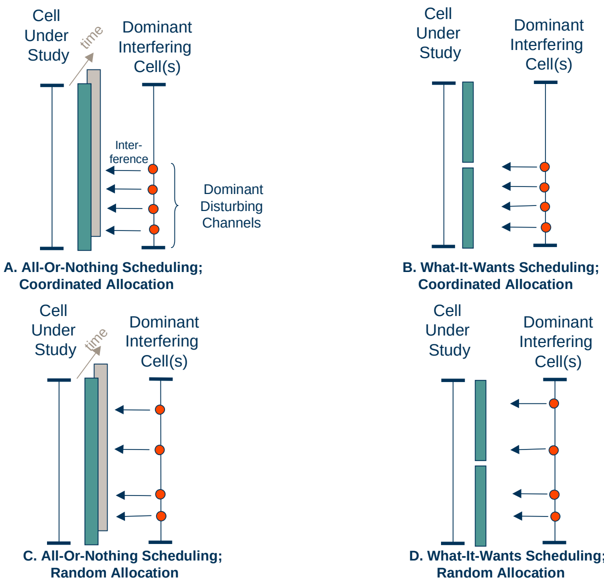

Fig. 2. Random and Coordinated ICIC policies when employing the All-Or-Nothing and the What-It-Wants Scheduling Policies. Scheduled sessions are hit by frequency domain collisions in a different way under the All-Or-Nothing and What-It-Wants scheduling policies. (The figure does not show the Fifty-Fifty policy.) Note that the scheduling algorithms operate on a different time scale than ICIC.

III. Performance Measures of Interest and Solution Approach

A. Input Parameters and Performance Measures of Interest

We consider a single OFDMA cell with S channels at which sessions belonging to the same (elastic) service class arrive according to a Poisson process of intensity λ. Each session brings with itself a file whose size is an exponentially distributed random variable with parameter μ. The session requests a radio bearer that is characterized by its peak rate $\hat{R}$ (for which: $\mathcal{S}(\hat{R}) \leq S$) and minimum rate $\hat{R}/\hat{a}$, where $\hat{a} \geq 1$ is the maximum slowdown factor associated with the session. If, at the time instant of the arrival of the new session, the admission of the new session brought the system into a state in which the minimum rate (governed by the particular scheduling policy) cannot be granted, the session is blocked and leaves the system. The single cell is disturbed (interfered) by a single dominant interferer cell, such as in [14]. In this paper we characterize the load in this dominant interfering cell by the number of used sub-carriers $K_1 \leq S$. When an allocated sub-carrier in the cell under study and one of the K_1 disturbing channels use the same sub-carrier frequency, we say that the two sub-carriers *collide* and suffer from co-channel interference [5]. We reuse the co-channel interference model in [13]. This model determines the distribution of the signal-to-noise-and-interference (SINR) of the colliding subcarrier in the cell under study.

The performance measures of interest are the session-wise blocking probability and the mean file transfer time. These two quantities represent a trade-off since more admitted sessions imply lower per-session throughput and thereby longer file transfer times. This trade-off in a WCDMA environment has been investigated by Altman in [9] (see also [10]) and subsequently by Fodor *et al.* in [11] and [12].

B. Step 1: Determining the Distribution of the Allocated Sub-carriers

Recall that in each system state the scheduling policy vector determines the probability that a given session is allocated s channels. When a session is given s channels (which happens with probability $\vec{P}(s)$), we need to calculate the conditional distribution of the number of the totally allocated number of channels in the cell (which we denote by K_0), given that the session under study is given s channels. This is because K_0 and the number of disturbing channels K_1 determine the distribution of the colliding and collision-free channels in the cell, which in turn determine the performance measures of interest.

We cannot give a closed form formula for the (conditional) distribution of K_0. However, in the Appendix we provide the pseudo code description of the algorithm that calculates it. The input of this algorithm includes the number of scheduled channels to the session under study (in the algorithm description denoted as `iBase` and the system state `iNoOfSessions`). The output of this algorithm is the vectors $\vec{K}_0$ and $\vec{P}_{K_0}$ and the value k_0^{MAX}. The values of $\vec{K}_0$ are the possible values of K_0 while the values of $\vec{P}_{K_0}$ are the associated probabilities. k_0^{MAX} gives the number of possible values of K_0 thereby the dimension of $\vec{K}_0$ and $\vec{P}_{K_0}$.

C. Step 2: Determining the Distribution of the Colliding Sub-carriers under the Random and Coordinated Sub-carrier Allocation Policies

Lemma 1: Let S denote the total number of available sub-carriers in each cell and let $K_0 \leq S$ and $K_1 \leq S$ denote the number of allocated channels in Cell-0 and Cell-1 respectively. Let $N_1(c)$ denote the number of possible channel allocations in Cell-0 and Cell-1 such that the number of collisions is c.

Then, the distribution and the mean of the number of collisions under the random allocation policy (γ_1) are as follows:

$$c_{MIN} = Max[0, K_0 + K_1 - S],$$
$$c_{MAX} = Min[K_0, K_1],$$

$$N_1(c) = \binom{S}{c} \cdot \binom{S-c}{K_0-c} \cdot \binom{S-K_0}{K_1-c}; \quad c = c_{MIN} \dots c_{MAX},$$

$$Pr\{\gamma_1 = c | K_0, K_1\} = \frac{N_1(c)}{TOT1},$$

$$E[\gamma_1 | K_0, K_1] = \sum_{c=c_{MIN}}^{c_{MAX}} \frac{c \cdot N_1(c)}{TOT1},$$

$$\text{where } TOT1 = \binom{S}{K_0} \cdot \binom{S}{K_1}.$$

Proof:
The three terms of $N_1(c)$ give the number of possible channel allocations for the c colliding channels out of the S available channels, the $K_0 - c$ non-colliding channels in Cell-0 and the $K_1 - c$ non-colliding channels in Cell-1 respectively. The other results immediately follow. We note that (as a possibility for verifying this result) $TOT1$ can also be calculated as $TOT1 = \sum_{c=c_{MIN}}^{c_{MAX}} N_1(c)$. ■

Lemma 2: Using similar notation as in Lemma 1, the distribution and the mean number of collisions under the coordinated allocation policy is given by:

$$N_2(c) = \begin{cases} 1 & \text{if } c = c_0 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$c_0 = \begin{cases} 0 & \text{if} \quad K_0 + K_1 < S, \\ K_0 + K_1 - S & \text{otherwise.} \end{cases}$$

$$Pr\{\gamma_2 = c | K_0, K_1\} = N_2(c),$$

and

$$E[\gamma_2 | K_0, K_1] = \sum_{c=c_{MIN}}^{c_{MAX}} c \cdot N_2(c).$$

Proof:
Under the assumption that we have a single dominating interfering cell, we may think of the coordinated channel allocation policy as one that allocates channels in Cell-0 and Cell-1 in "opposite order". That is, in Cell-0 channels are allocated in the order of $0, 1, \dots S$, while in Cell-1 in the order of $S, S-1, \dots 0$. Thus, for any (K_0, K_1) pair, the number of collisions is either 0 or $K_0 + K_1 - S$. ■

D. Step 3: Determining the Packet-wise Effective Signal-to-Noise-and-Interference-Ratio

The scheduling policy vector specifies the probability that s channels are used in Cell-0, whereas Lemmas 1-2 determine the probability that the number of colliding channels is c. We will use the following lemma to determine the probability that the number of colliding channels in a packet of size L is γ when the number of scheduled channels (for the session under study) is s and the total number of colliding channels is $c \leq s$.

Lemma 3:

$$Pr\{\gamma \text{ channels out of } L \text{ are colliding}\} =$$
$$= \begin{cases} 0 & \text{if } \gamma > c, \\ \binom{L}{\gamma} \cdot \frac{\binom{c}{\gamma}}{\binom{s}{\gamma}} \cdot \frac{\binom{s-c}{L-\gamma}}{\binom{s-\gamma}{L-\gamma}} & \text{otherwise.} \end{cases}$$

Proof:
Given that there are c number of colliding sub-carriers out of the total s that are taken into use in Cell-1, the probability that the first picked sub-carrier is colliding is c/s, that the second is colliding is $(c-1)/(s-1)$ and so forth. Similar reasoning applies to the $L - \gamma$ non-colliding channels. We also need to take into account the number of possible combinations for the γ colliding channels within the packet that is of size L. Finally, we notice that:

$$\prod_{i=0}^{\gamma-1} \frac{c-i}{s-i} = \frac{\binom{c}{\gamma}}{\binom{s}{\gamma}} \quad \text{and} \quad \prod_{i=0}^{L-\gamma-1} \frac{s-c-i}{s-\gamma-i} = \frac{\binom{s-c}{L-\gamma}}{\binom{s-\gamma}{L-\gamma}}.$$

■

E. Step 4: Calculating the SINR Level in Case of Collisions for the Downlink

Lemmas 1-3 determine the probability that the number of colliding channels is γ and the number of non-colliding channels is $L - \gamma$ in a packet of a session under study. We now need to determine the impact of the collision on a channel's signal-to-noise-and-interference (SINR) ratio.

For this, we use the path loss model recommended by the 3GPP (described in [22]) and a result from [13]. Let θ be a predefined threshold and let $X \triangleq \frac{r_0}{r_1}$ be a random variable representing the ratio between the mobile station distances from its serving and disturbing base station respectively. Also, let Q_0 and Q_1 denote the power that the serving and the neighbor base station uses on the colliding channels respectively. Furthermore, let G_0 and G_1 denote the path gains from the serving base station (that is in Cell-0) and the dominant neighbor base station (that is in Cell-1) respectively to the mobile station under study. Then, the probability that the SINR remains under this threshold can be approximated as follows [13]:

$$Pr\Big(\frac{G_0 \cdot Q_0}{G_1 \cdot Q_1 + N_0} < \theta\Big) \approx \int_0^{Max[X]} \Big(f_X(x) g(x)\Big) dx;$$

$$g(x) \triangleq \frac{1}{2}\text{erfc}\left(-\frac{5}{b\varsigma} \cdot \frac{ln \ \frac{x^{\mu}\theta}{Q_0/Q_1}}{ln \ 10}\right). \tag{9}$$

where $f_X(x)$ is the probability density function of X; b, ς and μ are the parameters of the 3GPP path loss model as described in [22].

F. Step 5: Calculating the Effective SINR and the Packet Loss Probability

We are now in the position that the packet loss probability in each system state can be determined.

When one or more of the channels that are used to carry a packet are hit by collisions, an efficient way to characterize the overall SINR quality of the packet is to use the notion of the *effective* SINR. This concept has been proposed in [18] and used in for instance [19], in which a method to calculate the packet error probability for a given value of the effective SINR was also proposed. A specific method to calculate the effective SINR (based on the SINR of the composing channels) that is applicable in cellular OFDM systems is also recommended by the 3GPP [17].

In this paper we employ the 3GPP method that can be summarized as follows. Suppose that there are L sub-carriers that carry a data packet and each has a SINR value of $SINR_i$. Than, the *effective* SINR that is assigned to the packet is given by:

$$SINR_{\text{eff}} = \alpha_1 \cdot I^{-1}\left(\frac{1}{L}\sum_{i=1}^{L} I\Big(\frac{SINR_i}{\alpha_2}\Big)\right), \tag{10}$$

where $I(\cdot)$ is a model specific function and $I^{-1}(\cdot)$ is its inverse. The parameters α_1 and α_2 allow to adapt the model to characteristics of the considered modulation and coding scheme. The exponential effective SINR metric proposed in [17] corresponds to:

$$I(x) = exp(-x).$$

In [19] it is shown that for QPSK and 16-QAM modulation, the parameters α_1 and α_2 can be chosen as follows: $\alpha_1 = 1$ and $\alpha_2 = 1$. In [19] a method to determine the packet error rate (σ) as a function of the effective SINR is presented. Essentially, this method maps (in a 1-1 fashion) the effective SINR onto a (modulation and coding scheme dependent) packet error rate.

G. Step 6: Determining the Performance Measures of Interest

We now make use of the assumption that the session arrivals form a Poisson process and that the session size is exponentially distributed. We choose the number of admitted sessions as the state variable and thus the number of states in the system is $\hat{I}+1$. The transitions between states are due to an arrival or a departure of a session. The arrival rates are given by the intensity of the Poisson arrival processes. Due to the memoryless property of the exponential distribution, the departure rate from each state depend on the nominal holding time of the in-progress sessions, and also on the slow down factor and the packet error rate in that state. Specifically, when the slow down factor is $a_i(n)$, and the packet error rate is $\sigma(n)$ its departure rate is $(1-\sigma_i(n))\mu_i/a_i(n)$.

The Markovian property for such systems was observed and formally proven by Altman *et al.* [20], and Nunez Queija *et al.* [21]. Thus, the system under these assumptions is a continuous time Markov chain (CTMC) whose state is uniquely characterized by the state variable n.

IV. Numerical Results

A. Input Parameters

In accordance with the 3GPP recommendation, we here (in a somewhat simplified fashion) assume that a downlink resource block (sometimes referred to as a *chunk*) occupies 300 kHz and 0.5 ms in the frequency and time domains respectively. A chunk carries 7 OFDM symbols on each sub-carrier; therefore the downlink symbol rate is 140 symbols/chunk/0.5ms. Assuming a 10 MHz spectrum band, and considering some overhead due to measurement reference symbols and other reasons, this corresponds to 30 chunks in the frequency domain ($S = 30$), that is 8400 ksymbol/s. The actual bit-rate depends on the applied modulation and coding scheme, in this paper we do not model adaptive modulation and coding (AMC), we simply assume a fixed binary phased shift keying (BPSK) so that each symbol carries 2 bits. Sessions arrive according to a Poisson process of intensity $\lambda = 1/8$ $[1/s]$. A session is characterized by the amount of bits that it transmits during its residency time in the system (we

TABLE I

MODEL (INPUT) PARAMETERS

$R_{symbol} = 280$	OFDM symbol rate per resource block;
$n_{MCS} = 2$	Number of bits per symbol (depending on the modulation and coding scheme)
$\lambda = 0.125$	Session arrival intensity
$S = 30$	Number of channels
$\mathcal{S}(\hat{R}) = 4$	Peak channel requirement
$\mathcal{S}(R_{min}) = S(\hat{R})/\hat{a} = 2$	Minimum channel requirement
$\lambda = 1/8$	Session arrival rate
$\nu = 4 * S * R_{symbol} * n_{MCS}$	Mean file size
$SINR_{good}$ =10 dB	Signal-to-Noise-and-Interference-Ratio without collision
$Q_0 = Q_1 = 20W/5MHz$	Power applied by the serving and neighbor base stations
$SINR_{bad}$ =0...3 dB	Signal-to-Noise-and-Interference-Ratio with collision
$K_1 = S/6 \dots S$	Number of used channels in the neighbor (disturbing) cell

may think of this quantity as the size of the file that is to be downloaded). We assume that this file size is an exponentially distributed random variable with mean value ν.

From the perspective of the radio access network (RAN), when a session is admitted into the system, a radio bearer associated with a minimum bit rate (also called the *guaranteed bit rate*, GBR) and a maximum bit rate (MBR) is set up. The GBR and the MBR bit rates correspond to the minimum and the maximum (peak) number of channels that the radio bearer must support. In our terminology, the MBR/GBR ratio corresponds to $\hat{a}$.

For each scheduled channel, the SINR depends on the distance between the base station and the mobile terminal, the channel conditions and whether the channel suffers from co-channel interference (collision) from neighbor cells or not. When there is no collision, we assume that the SINR value is a lognormally distributed random variable with mean 10 dB. When there is collision, the SINR value depends on the position of the mobile terminal and the applied power levels by the serving and the neighbor base stations (as described by (9) in Step 4).

The input parameters are summarized by Table I.

B. Discussion of Figures 3-6

In Figures 3-6 we study the impact of the (increasing) inter-cell interference on the session blocking probability and the file download time in the case when the associated radio bearer (RB) is peak allocated ($\hat{a} = 1$) (Figures 3-4) and when the GBR is set to the half of the PBR ($\hat{a} = 2$). On the x axis we let the number of disturbing channels (i.e. the occupied channels in the neighbor cell) increase ($K1/5 = 1 \dots 6$), while the y axis shows the blocking probabilities and the mean session residency times. The upper graphs in each figure correspond to the case when there is no channel allocation coordination between the cells, while the lower graphs assume coordination (channel segregation). We observe that when the sessions tolerate some slowdown, the blocking probability dramatically decreases (from 7% down to 0.06% !) without much increasing the download time (from around 33s to around 34s). Secondly, we note that coordinated allocation is beneficial when the What-It-Wants of the Fifty-Fifty scheduling method is employed, and has no effect when the All-Or-Nothing scheduling is used. The curve denoted "ideal" corresponds to the case when the packet error rate σ is zero in all system states.

C. Discussion of Figures 7-8

Figures 7-8 show the impact of the collisions on the download times as they happen at a mobile terminal that is gradually moved closer to the cell edge. When the mobile terminal is hit by an interfering downlink signal, the impact of this collision depends on the effect on the SINR. Here we let the SINR of the colliding channel decrease from 3dB (mobile terminal in the interior of the cell) down to 0 dB (equal distance from the serving and neighbor base stations). Figure 7 shows the result for the peak allocated case, while Figure 8 shows the result for the $\hat{a} = 2$ case. The download time increases in both cases, and - as expected -, this increase is greater with "elastic" bearers, that is when slowdown is accepted. More importantly, coordinated channel allocation significantly improves the system throughput performance when the scheduler is of type What-It-Wants or All-Or-Nothing.

D. Discussion on Figure 9

Figure 9 shows the time spent in the system when sessions are greedy, i.e. ($\mathcal{S}(\hat{R}) = S$) with a minimum resource requirement $R_{min} = 4$. For such sessions, the download time is the same for both ICIC policies. When the number of disturbing channels is low, the What-It-Wants and Fifty-Fifty policies perform somewhat better than All-Or-Nothing. This difference is due to the different behavior when multiple sessions receive service, that is how "slow down" the in-progress sessions is realized with the different schedulers. Since All-Or-Nothing is a pure time domain scheduler, slowing down the sessions does not have an impact on the occupied frequency channels. In contrast, What-It-Wants and Fifty-Fifty reduces the granted frequency channels per session.

V. Conclusions

Inter-cell interference coordination is an important radio resource management function for OFDMA based cellular systems in general [5] and for the evolving Universal Terrestrial Radio Access Network (E-UTRA) in particular [1], [2].

In a previous work we have showed that coordinated channel allocation (in [5] also called *channel segregation*) helps to improve the SINR and throughput performance of the system [13]. In this paper we built on the base model of that paper and investigated the performance of three scheduling disciplines with/without coordinated channel allocation.

The All-Or-Nothing scheduling method is a pure time domain scheduling technique, according to which a single session takes all available channels into use at any one time. The What-It-Wants scheduling method is a combined time and frequency domain technique: it allows the simultaneous transmission of different sessions. At every point in time, it allocates s channels to in-progress sessions such that $\mathcal{S}(R_{min}) \leq s \leq \mathcal{S}(\hat{R})$. The Fifty-Fifty scheduler can be seen as a scheduler that is "in between" these two scheduling methods.

We proposed the notion of the (scheduling) policy vector to model the behavior of the packet scheduler. Using the policy vector, we were able to derive the conditional distribution of the number of colliding and collision free channels in the cell under study for all three cases. This in turn allowed us to determine the distribution of the number of colliding and collision free (i.e. co-channel interference free) channels in each scheduled packet. We used this knowledge to calculate the *effective SINR* and from it the packet error rate and thereby the useful packet throughput of the system. This useful throughput determines the session wise blocking probabilities and the time it takes for elastic sessions to complete a file transfer.

Our major finding is that the performance of the ICIC function (its impact on the system throughput) depends on the employed scheduler. Specifically, when frequency domain scheduling is used in combination with time domain scheduling, it is useful to employ coordinated channel allocation in neighbor cells. Coordinated ICIC has little impact when the scheduler is pure time domain based. We also note that our numerical results indicate that ICIC is only necessary for cell edge users, whose SINR is negatively impacted by frequency domain collisions. This finding is in line with previous results (see for instance [13], [8], [23] and also [1] and [2]).

An important outstanding issue is the modeling of the scheduling gain that can be different for the different schedulers investigated in this paper. Time domain scheduling has been found beneficial in fast fading environments by allowing avoiding the fading dips. Studying the impact of such scheduling gains is left for future work.

Acknowledgments

We would like to thank Magnus Persson, Jessica Heyman and Sverker Magnusson (all at Ericsson Research) for encouraging discussions during this work.

APPENDIX

```
j = 1;
iNoOfSchedulingPositions = 0;
For[i = 1, i ≤ CH + 1, i++,
  If[tPolicyVector[[i]] ≠ 0,
    iNoOfSchedulingPositions = iNoOfSchedulingPositions + 1;
   ];
 ];
tSchedulingPositions = Table[0, {t1, 1, iNoOfSchedulingPositions}];
For[i = 1, i ≤ CH + 1, i++,
  If[tPolicyVector[[i]] ≠ 0,
    tSchedulingPositions[[j++]] = i;
   ];
 ];
For[i = 1, i ≤ iNoOfSessions, i++,
  tDenom[[i]] = (iNoOfSchedulingPositions^iNoOfSessions) / (iNoOfSchedulingPositions^i);
 ];
iRaw = iNoOfSchedulingPositions^iNoOfSessions;
mChannel = Table[0, {t1, 1, iRaw}, {t2, 1, iNoOfSessions}];
mChannel2 = Table[0, {t1, 1, iRaw}, {t2, 1, iNoOfSessions}];
For[i = 1, i ≤ iRaw, i++,
  For[j = 1, j ≤ iNoOfSessions, j++,
    mChannel[[i, j]] = Mod[Floor[(i - 1) / tDenom[[j]]], iNoOfSchedulingPositions] + 1;
    mChannel2[[i, j]] = tSchedulingPositions[[mChannel[[i, j]]]];
   ];
 ];
```

Fig. 10. Pseudo Code, Part I. This algorithm takes the `PolicyVector` and the number of currently served sessions (the system state, `iNoOfSessions`) as its input and generates the `mChannel2` matrix as the output. The `mChannel2` matrix has as many rows as there are combinations of the number of scheduled channels for each session. For instance, in system state 3, a row may contain `0, 5, 6` which corresponds to the case that Session-1 is given 0 channels, Session-2 is given 5 and Session-3 is given 6 channels. In the code `CH` is the number of available channels S.

```
For[i = 1, i ≤ iRaw, i++,
  dSum = 0;
  For[j = 1, j ≤ iNoOfSessions, j++,
   dSum = dSum + (mChannel2[[i, j]] - 1);
  ];
  dOccupationProb = 1.0;
  For[k = 1, k ≤ iNoOfSessions, k++,
   dOccupationProb = dOccupationProb * tPolicyVector[[mChannel2[[i, k]]]];
  ];
  If[dSum + iBase > CH,
   kk = CH - iBase;
   tNumberOfOccupiedChannels[[kk + 1]] = tNumberOfOccupiedChannels[[kk + 1]] + 1;
   tOccupationProbs[[kk + 1]] = tOccupationProbs[[kk + 1]] + dOccupationProb,
   (* Else *)
   tNumberOfOccupiedChannels[[dSum + 1]] ++;
   tOccupationProbs[[dSum + 1]] = tOccupationProbs[[dSum + 1]] + dOccupationProb;
  ];
 ];
j = 1;
For[i = 1, i ≤ iNoOfSessions * CH + 1, i++,
  If[tOccupationProbs[[i]] ≠ 0,
    tFinal[[j]] = tOccupationProbs[[i]];
    tShiftedFinalOccupiedChannels[[j]] = iBase + i - 1;
    j = j + 1;
   ];
 ];
```

Fig. 11. Pseudo Code, Part II. This algorithm takes `mChannel2` and the number of occupied channels by the session under study (`iBase`) as its input and generates the possible values of K_0 (`tShiftedFinalOccupiedChannels`) and the associated probabilities (`tFinal`). j gives the number of possible values of K_0. The pseudo code presented here is part of our running *Mathematica* [24] implementation.

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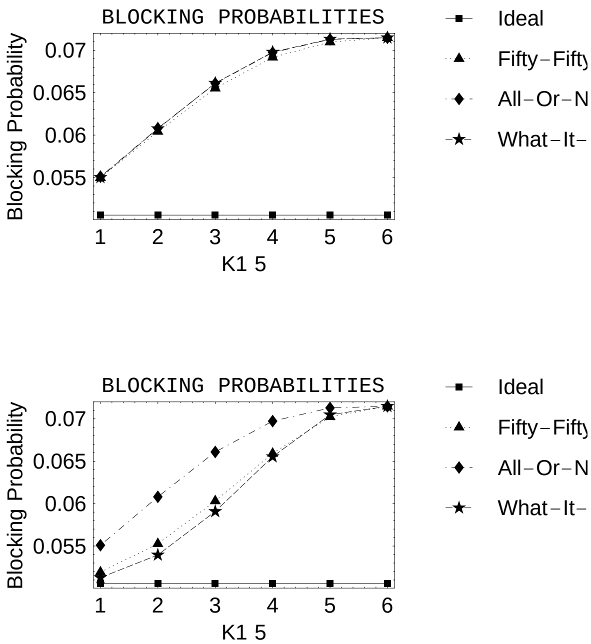

Fig. 3. As the number of occupied channels in Cell-1 increases from 5 to 30 ($K1/5 = 1\ldots6$), the blocking probability increases both under the random (upper) and the coordinated (lower) allocation policies. However, the Fifty-Fifty and the What-it-Wants scheduling method performs better than the All-Or-Nothing scheduling under coordinated allocation.

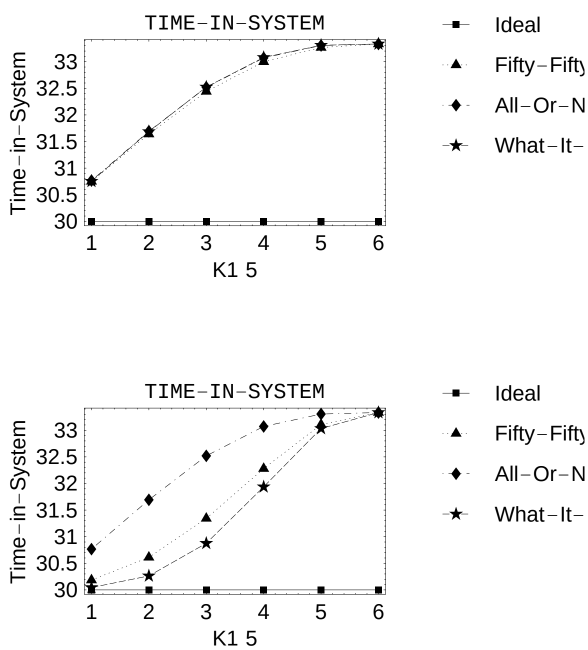

Fig. 4. As the number of occupied channels in Cell-1 increases from 5 to 30 ($K1/5 = 1\ldots6$), the download time increases both under the random (upper) and the coordinated (lower) allocation policies. However, the Fifty-Fifty and the What-it-Wants scheduling method performs better than the All-Or-Nothing scheduling under coordinated allocation.

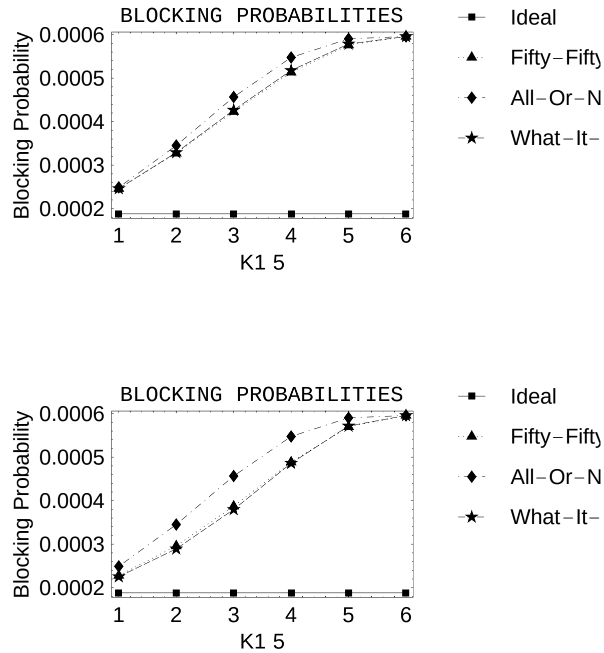

Fig. 5. When the sessions tolerate some slowdown (here $\hat{a} = 2$, that is $R_{min} = \hat{R}/2$), the blocking probabilities radically decrease, (here 2 orders of magnitude), and the coordinated allocation (lower) again performs somewhat better when the scheduling method is the All-Or-Nothing or What-It-Wants.

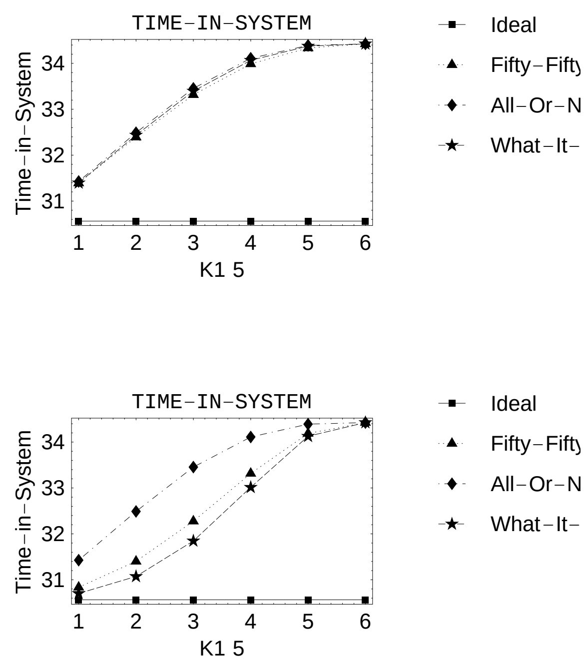

Fig. 6. When the sessions tolerate some slowdown (here $\hat{a} = 2$, that is $R_{min} = \hat{R}/2$, the session holding time increases somewhat, (a few procents), and the coordinated allocation again performs somewhat better when the scheduling method is the All-Or-Nothing or What-It-Wants.

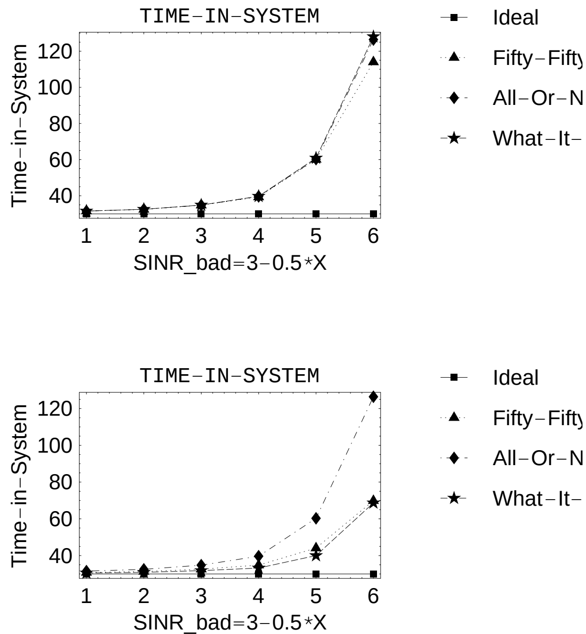

Fig. 7. Along the x axis, we let the value of the $SINR$ of colliding channels ($SINR_{bad}$) increase from 2.5dB to 0dB. In the downlink, this corresponds to the case the position of the mobile that is hit by the neighbor base station is in moved from the interior of the cell out to the cell edge. As the system throughput decreases, the average residency time of sessions increases dramatically, but when employing the coordinated channel allocation together with the What-It-Wants or Fifty-Fifty scheduling method, the system can be kept under normal operational conditions "closer to the cell edge".

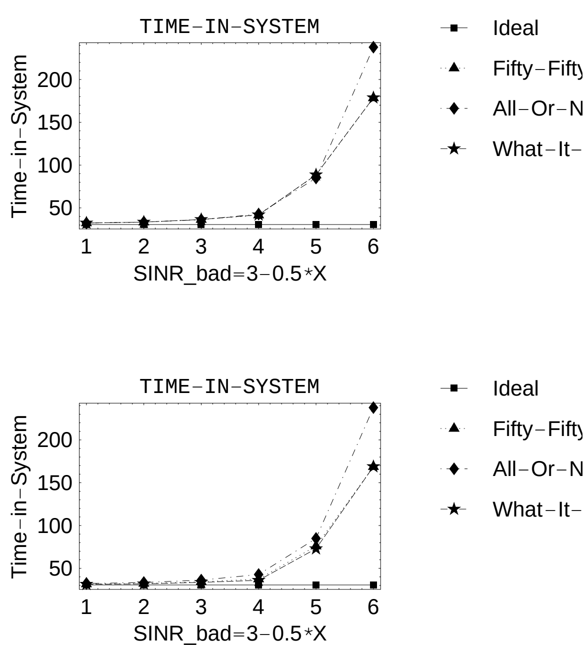

Fig. 8. This figure is similar to the previous figure, but now $\hat{a} = 2$. We notice the increase of the average session holding time and again the usefulness of the coordinated allocation policy when used together with the What-It-Wants or Fifty-Fifty scheduling.

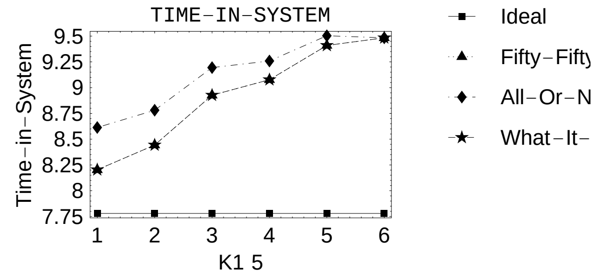

Fig. 9. Mean time in system for a greedy source ($\mathcal{S}(\hat{R}) = S$) under the random and coordinated ICIC policies. Fifty-Fifty and What-It-Wants perform somewhat better than All-Or-Nothing when the number of disturbing channels is low. There is no difference between Fifty-Fifty and What-It-Wants.

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